



The Evaluation of Different Statistical Distributions in Order to Fit *Alnus subcordata* C.A.M. Species Diameter in Mountainous Forests North of Iran

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ABSTRACT: In order to study the diameter of *Alnus subcordata* trees at breast height (D.B.H.) distribution in Iran's north forests a stand with one hectare and 201 trees as a random sample had been selected, and the diameters of all trees were measured. The stand under study had trees in all diameter classes; moreover, the trees were healthy. According to the existing studies and experiences and after comparing the real amounts with some statistical models in order to fit these trees' diameter, Gamma(2p), distribution, Gamma (3p) distribution, chi-square (2p), Error, Dagum (3p), Normal, Johnson SB, Gen.Extreme value and Logistic distributions were applied. The results from Kolmogorof-Smirnov, (k.s.), Anderson-Darling(A.D) and chi-square (χ^2) tests at the probability level of 5% showed that the Dagum (3p) statistical distribution is the most appropriate model in order to fit the *Alnus subcordata* trees' diameter in our study area. The mean and the standard deviation of these trees in order are 47/85 cm and 24/83 cm.

Key words: Distribution, Fit, Diameter, *Alnus subcordata*, North forests of Iran

INTRODUCTION

Alnus subcordata species in Iran's north forests is considered to be one of the most precious forest trees. The enduring decrease of this species for different reasons has made the necessity of accomplishing research projects about this precious species. This study, also, aims to get more knowledge about *Alnus subcordata* forests species and its diameter breast height with statistical distributions, so by using them a better management could be done in these forests. An important characteristic such as diameter changes by time and acquaintance with the quality of these changes can help researchers a lot in comparing the current and ideal positions. The study of trees diameter at breast height in a forest stand has a very important role in a stand's growth and production (Burnham 2002; Li-feng, *et al.*, 2010; Lu, *et al.*, 2003). In the past decades quantitative systematic studies been done on trees diameter frequency distribution models in even aged stands (Alder 1995; Chen, *et al.*, 2004; Li, *et al.*, 2006; Shi 2006). In a study different methods exist to fit the important characteristics of species by the use of statistical distributions (Zhang and Lei, 2010).

As an example full stand models are models that use stands as a sample unit (Curtis, *et al.*, 1981; Li, *et al.*, 1998; Newton, *et al.*, 2005; Wei, 2006). This is while tree individual study models use each tree as the study target (Cao 2000; Cao, *et al.*, 2002; Zhang, *et al.*, 1997; Zhang and Lei, 2009).

Diameter distribution models, also, contrary to these models use statistical probabilities such as the Weibull distribution (Bailey and Dell 1973; Liu, *et al.*, 2004; Meng 1988; Newton, *et al.*, 2005) or Beta distribution (Gorgoso, *et al.*, 2008). In this research also in order to fit the *Alnus subcordata* trees diameter, statistical models such as Gamma (2p), Gamma (3p), Chi-squared (2p), Error, Dagum, Normal, Johnson SB, Gen.Extreme value and logistic were applied. The studies done can be used to fit Beech trees diameter in Iran's north forests; moreover, in this study Normal distribution was found to be appropriate to fit Beech trees diameter (Fallahchai, 2011).

In another study that was done to fit the Oak trees diameter in Iran's north forests among the applied distributions the Gamma distribution has more descriptive ability in order to explain Oak trees diameter and other distributions lack such ability (Fallahchai *et al.* 2012).

In Denmark, also, the Weibull distribution has been studied, and its usage has been considered appropriate in order to present a model for even-aged Beech trees diameter distribution (Nord- Larson *et al.* 2006). In another study that has been taken place in the natural forests of Jian province in China by use of four statistical models namely Weibull, Beta, Gamma, and Exponential, the conclusion has been reached that the Weibull distribution model, better than other models, has the potentiality to fit trees diameter in different

congestions (Li-feng and Xin-nian, 2010). Also in the north western forests of Beijing province *Pinus tabulae formis* diameter fit was studied, and the conclusion has been reached that among the applied models the Weibull distribution has more potentiality for explaining *Pinus tabulae formis* trees diameter with stable sample pieces (Zhang and Lei, 2010). Probability density function applied in the study area are given in Table 1.

Table 1. Probability density function applied in the study area.

| row | Distribution name | density function |
|-----|---------------------------|---|
| 1 | Gamma (2p) | $f(x) = \frac{x^{\alpha-1}}{\beta^{\alpha}\Gamma(\alpha)} \exp(-x/\beta)$ |
| 2 | Gamma (3p) | $f(x) = \frac{(x-y)^{\alpha-1}}{\beta^{\alpha}\Gamma(\alpha)} \exp(-(x-y)/\beta)$ |
| 3 | Chi-squared (2p) | $f(x) = \frac{(x-y)^{V/2-1}}{2^{V/2}\Gamma(V/2)} \exp(-(x-y)/2)$ |
| 4 | Error | $f(x) = C_1 \sigma^{-1} \exp(- C_0 Z ^K)$ |
| 5 | Dagum(3p) | $f(x) = \frac{\alpha K (\frac{x}{\beta})^{\alpha K-1}}{\beta (1 + (\frac{x}{\beta})^{\alpha})^{K+1}}$ |
| 6 | Normal | $f(x) = \frac{\exp(-\frac{1}{2}(\frac{x-\mu}{\sigma})^2)}{\sigma\sqrt{2\pi}}$ |
| 7 | Johnson SB | $f(x) = \frac{\sigma}{\lambda\sqrt{2\pi}Z(1-Z)} \exp(-\frac{1}{2}(\gamma + \sigma \ln(\frac{Z}{1-Z}))^2)$ |
| 8 | Generalized Extreme Value | $f(x) = \begin{cases} \frac{1}{\sigma} \exp(-(1+KZ)^{-1/K})(1+KZ)^{1-1/K} & K \neq 0 \\ \frac{1}{\sigma} \exp(-Z - \exp(-Z)) & K = 0 \end{cases}$ |
| 9 | Logistic | $f(x) = \frac{\exp(-Z)}{\sigma(1 + \exp(-Z))^2}$ |

MATERIALS AND METHODS

The area under study is located in Nave Asalem series one in Iran's north forests and its geographical coordinates according to UTM system is among 48° 48' 51" to 48° 52' 27" E and 37° 37' 51" to 37° 41' 16" N. The under study piece has the height limits of 900 to 1000 m above the sea level, and its slope average is about 25 percent.

From the pedology point of view the soil type is forest brown, and its pH is acidic. The under study annual average rainfall is 945 mm, and the average annual temperature is 12.4 C, and it has a humid climate. From the view point of tree cover, the under study area consists of uneven aged *Fagus orientalis*, *Alnus subcordata* and accompanied species such as *Acer velutinum*, *Sorbus torminalis*, *prunos avium*, *Ulmus glabra* and *Tilia begunifolia*. In this study 201 *Alnus subcordata* trees were randomly selected from an area with of one hectare with a dimention of 62.5 ×160 m² that naturally had the most *Alnus subcordata* species in different diameter classes, and the diameter at breast height of all *Alnus subcordata* trees that were thicker than 7/5 cm which had been measured.

The Easy fit software was used in order to fit the collected data.

A. Applied statistical distributions

In this study Gamma (2p), Gamma (3p), Chi-squared (2p), Error, Dagum (3p), Normal, Johnson SB, Gen.Extreme value the least quantity is from more than 95cm diameter class.

and logistic statistical distributions were applied whose mathematical formula will be presented (Table 1). In all these distributions, according to the type of evaluation, X is the symbol of diameter at breast height, and the Greek alphabets are probability density function parameters (Zwillinger and Kokoka 2000).

B. Considering goodness fit

For selecting the best fit, it is necessary for each distribution used to be tested by goodness fit. Kolmogrof-Smirnov, Chi-squared and Anderson-Darling tests have been used in this study (Jerrold, 1999).

III. RESULTS AND DISCUSSION

The results related to descriptive statistics of *Alnus subcordata* trees diameter breast height character in the study area have been presented in Table 2.

According to this table, the least diameter is 7.5 cm, and the greatest diameter is 128cm. The mean diameter breast height of all measured trees was 47.85cm, and the standard deviation was 24.83cm.

Fig. 1, also, shows the quantity distribution in *Alnus subcordata* diameter classes in the study area which has detracting procedure, and has extensive amplitude of trees in diameter classes and clearly presents an uneven-aged structure. According to the frequency in diameter classes the most quantity is observed in 60cm diameter class, and

Table 2. Diameter at breast height statistical results of *Alnus subcordata* species.

| Statistics | Mean | Standard of deviation | Variance | The least diameter | The most diameter | Coefficient of variation | Skewness | Kurtosis |
|--------------------------------|-------|-----------------------|----------|--------------------|-------------------|--------------------------|----------|----------|
| Diameter at breast height (cm) | 47/85 | 24/83 | 616/88 | 7/5 | 128 | 0/519 | 0/53 | 0/43 |

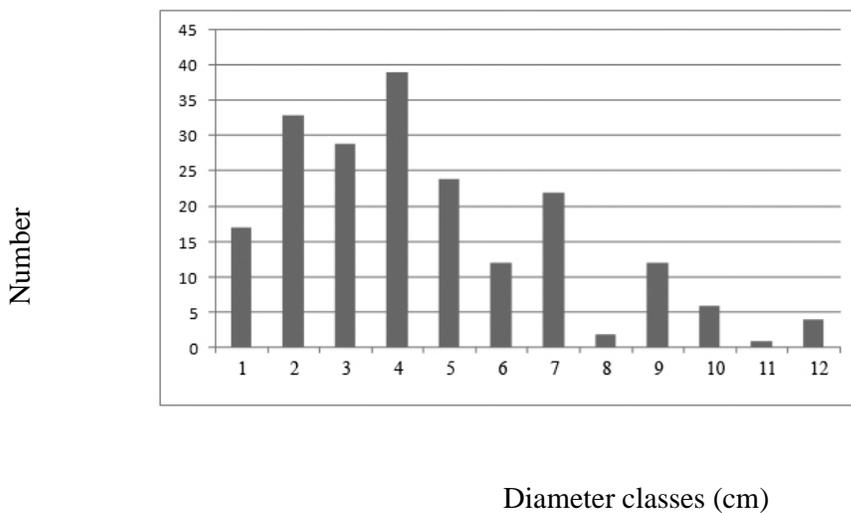


Fig. 1. Quantity distribution in *Alnus subcordata* species of different diameter classes.

Fig. 2 also shows the curves related to comparing observed frequencies and evaluated frequencies from diameter probability distributions of *Alnus subcordata* species in the study area.

Kolmogorof-Smirnov, Anderson-Darling and Chi-squared were used in order to consider the fit probability distributions applied in this study. By considering the obtained statistics quantities it has been determined that the most appropriate distribution in order to fit the *Alnus subcordata* trees diameter in the study area is the Dagum(3p) statistical distribution, but Normal, Error, Gen. Extreme Value, Chi-squared (2p), Logistic and Jonson SB also have the ability to fit the *Alnus subcordata* trees diameter (Table 3). Having in mind the importance of diameter at breast height as one of the forest trees' most essential biometrics variables, studying them has a lot of benefits. Using appropriate statistical distributions in forest planning is very beneficial and has a lot of importance in predicting the trees' situation in forest stand (Nanang 1998). In order to compare the results of this study with identical studies, since as yet

there has been no studies done on *Alnus subcordata* trees diameter fit, it would be impossible to compare the results of this study with the results of other researchers. Regarding the diameter fit of some other species, some studies have been done, and we will mention some of them. Amanzadeh, *et al.*, (2011) Studied the quality of *Fagus orientalis* species stands diameter distribution in different transformation stages in Iran's north forests and concluded that statistical distributions aren't identical in different stages of forest transformation, so the Lognormal (3p) distribution has a good fit at the initial stage, and for the optimal stage Beta distribution, and for decay stage Johnson SB have been evaluated to be appropriate. In the current study, also, the applied distributions don't have identical ability to fit *Alnus subcordata* species diameter, but according to the non-parametric tests that have been done, the most appropriate one would be Dagum (3p) statistical distribution and then Normal, Error, Gen.Extreme value, chi-squares(2p), Logistic and Jonson SB are appropriate.

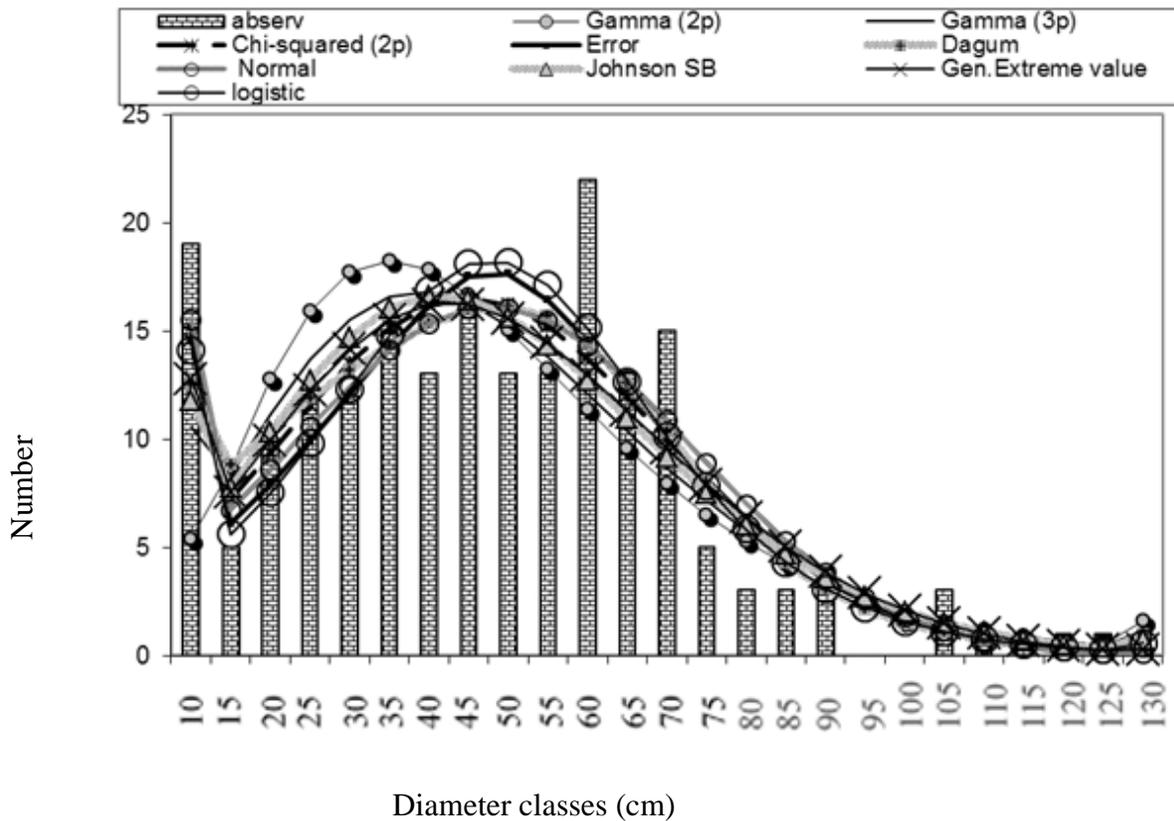


Fig. 2. *Alnus subcordata* species diameter distribution presentation in the study area.

Table 3. Comparing different statistical models in order to fit the *Alnus subcordata* trees diameter.

| Row | Distribution name | The statistical amounts of goodness fit tests | | |
|-----|-------------------|---|----------------------|-----------------|
| | | χ^2 | A.D | K.S |
| 1 | Gamma (2p) | 23/531 ^{ns} | 3/3112 ^{ns} | 0/09031* |
| 2 | Gamma (3p) | 14/813 ^{ns} | 1/292* | 0/07075* |
| 3 | Chi-squared (2p) | 6/2678* | 0/85227* | 0/05592* |
| 4 | Error | 8/7053* | 1/0346* | 0/05254* |
| 5 | Dagum | 3/4154* | 0/592* | 0/0483* |
| 6 | Normal | 4/7987* | 0/99822* | 0/05211* |
| 7 | Johnson SB | 12/087* | 0/99565* | 0/06207* |
| 8 | Gen.Extreme value | 6/1483* | 0/9159* | 0/05303* |
| 9 | logistic | 8/4862* | 1/155* | 0/05713* |

*significant at the level of 5 percent

n.s = non-significant

In another study done by Fallahchai (2011) in order to consider the quality of *Fagus orientalis* species diameter at breast height and its fit by the use of statistical distributions in Iran's north forests, it has been shown that Beta distribution in southern, eastern and western directions and Exponential distribution also in western direction of *Fagus orientalis* stands have made a proper fit for this species tree diameter distribution. Oak species trees diameter fit also in Iran's north forests showed that Gamma distribution has the diameter explaining ability. (Fallahchai, *et al.*, 2012). The results from Cao studies in 2004 also showed that the Weibull distribution (3p) is appropriate for *Pinus teade* trees distribution description (Cao *et al.* 2004). Also, Li-feng and Xin-nian (2010) have claimed that Weibull distribution compared to other distributions has a more fit ability in trees diameter in different accumulations. Zhang and Lei (2010) also introduced the Weibull distribution as the distribution with more ability in explaining *Pinus tabulae formis* trees diameter in north western forests of China. As it has been observed different statistical models could be expected in forest stands from the view point of structure, the Mixture and social situation of trees are different.

Alnus subcordata species is one of the fast growing hard wood trees in Iran's north forests that is severely exploited and are in danger of extinction.

For this reason scientific studies about this valuable species seems to be necessary. According to Full-callingering (Fallahchai, *et al.*, 2013) from this species in the study area it has become cleared that the number of *Alnus subcordata* trees with high diameters in the study area is decreasing which is due to the immethodical exploitation. So in tending operations on these stands we should act in a way that while preserving un-even aged structure, we could have a Logical distribution of all *Alnus subcordata* trees in different diameter classes. Also according to the current study we could conclude that probability distributions is applicable in trees diameter distribution manner and evaluation, and statistical models could be used in order to present the best trees distribution pattern. Besides it seems that accessing the appropriate model in a natural forest is different according to its sites characteristics and conditions, and there would be reason for a distribution to be the best in all conditions (Fallahchai *et al.* 2011). For this reason it should be admitted that the results of this study is influenced by its information, and in other studies different results may be obtained.

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