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Modelling and Optimizing Little Millet Yield through Interval Regression Analysis

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ABSTRACT: With the increase of people around the world, there is a shortage of land for agriculture. Increasing population on less land has increasingly become the subject of research. In this study our purpose is to identify the amount of fertilizers that should be applied to maximum production. Here we have utilized a quadratic programming approach to generate a yield response surface from the data provided from Little Milet Plantation. The response surface is a nonlinear interval valued function. Using interval valued response surface we have found the amount of fertilizers required for maximizing of yield.

Keywords: Interval regression, Optimum yield, Unconstrained Optimization, Nitrogen, Potassium.

Mathematics Subject Classification: 90C20, 90C30, 62J02,65G40

INTRODUCTION

Fertilizer applications in the soil are crucial to crop productivity. Food is an essential necessity for survival. Crop cultivation provides the majority of human dietary needs. Crop output must rise in order to feed the expanding population. Odisha's agricultural sector has stagnated during the past forty years. Odisha's agricultural growth rate from 1991 to 2008 was slower than the national average Reddy (2013). Many issues are putting a lot of strain on Odisha's agriculture industry. One of the main causes of the shortage of agricultural land for cultivation is the growing population. Numerous research have been conducted to ascertain the amount of fertilizer needed to optimize crop yield. Maurya (2017) conducted research on the conditions in central Uttar Pradesh to increase strawberry fruit yield and quality. Li *et al*. (2022) found the best fertilizer mixture for high production as well as biological and quality attributes by employing quadratic polynomial regression analysis. Paiman (2021) sought to ascertain the ideal dosage of NPK Mutiara fertilizer that would yield the most Ciherang type rice in alluvial soil. Shaocheng (1994) examined the impact of phosphorous (P) and nitrogen (N) on French bean seed yield maximization. Lad (2014) investigated the effects of phosphorus and nitrogen on French bean yield maximization on Vertisol soil in the Marwada region. Hariyadi (2021) has studied the increase in tomato fruit weight and total fruit number production. Abera *et al.* (2017) found that the interaction of maize varieties with nitrogen fertilizer rates had a substantial impact on all parts of maize yield and yield components. According to Nigatie (2021) research, grain yields in the common bean cultivars IPR 139 and perola improved linearly with higher N rates, reaching a rate of 180 kg N Per hector. By performing multiple tests and comparing the results of each experiment with those of other trials, the aforementioned works have all advised fertilizer dosages for crop productivity. Inferences drawn from observations might not be reliable.

Several scientists have been motivated to use mathematical optimization techniques to data from fertilization rate trials in order to provide reliable fertilizer recommendations. Vedooren (2003) used partial derivatives to calculate the amount of N and K fertilizers needed to maximize oil palm productivity. Kolbe (1990) used a nonlinear optimization technique to investigate how fertilizer affected the quantity and quality of tuber production from potatoes (Solanum tuberosum). To find the amount of fertilizers required to maximize the output of potatoes, garlic and onion, Behera and Mahanti (2021, 2022, 2022) have employed unconstrained optimization approaches including Newton's method and Marquadart's method. Dash and Kuila (2022, 2022, 2023) have used Quasi Newton method approximated solver and Marquardt's method to find maximization yield of rice and integrated crops.

To more precisely calculate the amount of fertilizer needed for maximum agronomic production, a response surface is created utilizing the data from a trial plantation by either linear or nonlinear statistical regression Gomez and Gomez (1984). Mathematical optimization approaches can be used to determine the fertilizer dosages required to maximize production because the response surface expresses crop yield as a nonlinear function of fertilizers utilized. The findings from the trial plantation are extrapolated to commercial plantations that are cultivated in similar conditions to the trial plantation, including similar soil types and weather patterns.

In order to generate the response surface, conventional linear and nonlinear regression models presume that there are enough data points from numerous tests. There are multiple instances where conducting enough experiments is not feasible. The validity of the functional relationship between yield and fertilizers is called into doubt if a response surface is generated from an inadequate amount of data. Shapiro has developed an alternate strategy that makes use of possibility theory in order to address the inaccuracy that results from regression models built using typical methods that use insufficient data Shapiro (2021). The most basic kind of potential regression analysis is called interval regression [Interval Regression Analysis by Quadratic Programming Approach], which is a regression model with interval coefficients. Ray and Panda (2020) have established a solution of constrained interval optimization problem with regularity. Bhurjee and Panda (2012) have found the solution of interval optimization problems. Bhurjee and Panda (2016) have discussed the optimal condition and duality theorem for interval optimization problem.

Here, we have calculated how much fertilizer should be needed during the crop's planting stage in order to maximize Little Millet's (*Panicum sumatrense*) agronomic yield. To examine the impact of nitrogen and potassium levels on the growth and yield of little millet (*Panicum sumatrense*) a model has been developed using the data from an investigation that was conducted during the Kharif season of 2016 at the AICRP on dry land agriculture, University of Agricultural Sciences, GKVK, Bangalore, Karnataka. It is evident that there are not enough experiments conducted on the experimental plantation.

Tanaka & Lee (2002) established a method for creating an interval regression model for the limited amount of data from the Little Millet plantation that Charate examined in 2018 Charate *et al*. (2018). An interval regression model has been created using the data from the Millet crop.

Since our goal is to maximize yield of little millet, we must resolve the unconstrained optimization problem. Max y (N, K).

(1) It is impossible to solve the problem (1) using traditional optimization approach because of the intervals. So, we require a method that was covered in in order to address the problem (1).

METHODS AND METHODOLOGY

Here, quadratic programming interval regression model was discussed. An interval regression model was developed for maximum yield of Little Millet and finally a nonlinear optimization was converted into deterministic form to find optimum yield.

A. A quadratic programming interval regression Model (Tanaka & Lee *1998*)

Let the U_i be a closed interval $[u_i - \xi_i, u_i + \xi_i]$ and bedenoted as (u_i, ξ_i), where u_i is called as the centre and ξ_i the spread of the interval. Consider a set of n number of input-output data points $(x_j, y_j) = (1, x_1j, x_2j, x_{nj}, y_j)$, where $xj = [1, x_1j, \ldots, x_{nj}]^T$, j=1,…,n.

An interval regression model for this input output data points is given by

$Y(x_j) = U_0 + U_1 x_{j1} + \ldots + U_n x_{jn}$
$= (u_0 + u_1 x_{j1} + \cdots + u_n x_{jn}, \xi_0 + \xi_1 |x_{j1}| + \cdots + \xi_n |x_{jn}|)$
$= (u^t x_j, \xi^t |x_j|)$ (2)

where $u = (u_0, u_1, \cdots, u_n)^t$,
$\xi = (\xi_0, \xi_1, \cdots, \xi_n)^t$ and
$|x_j| = (1, |x_{j1}|, \cdots, |x_{jn}|)^t$.

Here $u^t x_j$ and $\xi^t x_j$ represents a centre and radius/spread of the predicted interval $Y(x_j)$ corresponding to input data $x_j = (1, x_{1j}, x_{2j}, \ldots x_{nj})$.

It follows that $y_j \in Y(x_j) \Rightarrow \begin{Bmatrix} u^t x_j - \xi^t |x_j| \le y_j \\ y_j \le u^t x_j + \xi^t |x_j| \end{Bmatrix}$

The objective function of the interval regression model is constructed in such a manner that the sum of squares of the spreads of the intervals present in (2) is minimised. Thus the objective function is to minimise $\xi^t \left(\sum_{j=1}^{p} |x_j| \, |x_j|^t\right) \xi$.

It is necessary to add the quadratic term $\delta u^t u$ to the objective function so that the modified objective function becomes a quadratic expression involving variables u_i, $\xi_i = 0, 1, 2 \ldots, n$. Based on the above mentioned reasoning, the objective of interval regression model is to minimize the spread of the predicted interval $Y(x_j)$. The basic formulation of quadratic programming problem is therefore (Tanaka & Lee 2002).

$\min_{u,\xi} \xi^t \left(\sum_{j=1}^{p} |x_j| \, |x_j|^t\right) \xi + \delta u^t u$ (3)

Subject to $u^t x_j + \xi^t |x_j| \ge y_j$ (4)

$u^t x_j - \xi^t |x_j| \le y_j \ , j = 1, 2, \cdots, n$ (5)

$\xi_i \ge 0$ and δ is a significantly small positive number.

The yield data of Little Millet is presented in the following table

By solving the constrained optimisation problem (3)-(5) we can calculate the pair

(u_i, ξ_i) $i = 0, 1, \ldots, n$ so that the interval regression equation can be obtained as

$Y(x) = U_0 + U_1 x_1 + U_n x_n$.

B. An interval regression model For Crop (Millet) Yield

The experiment, as mentioned in Table 1, was conducted in randomized block design with factorial fertilisation trials consisting of three nitrogen levels (20, 40, 60) in Kgha^{-1} and four potassium levels (0, 10, 20, 30) in Kgha^{-1}. The yield data of Little Millet is presented in the following table 1.

Table 1: Effect of different level of N (nitrogen) &K(potassium) on yield of little Millet.

Nitrogen(N)in kg/ha	Potassium (K)in kg/ha	Yield in kg/ha
20	0	448
20	10	471
20	20	575
20	30	621
40	0	652
40	10	676
40	20	730
40	30	683
60	0	664
60	10	692
60	20	728
60	30	727

The number of experiments, conducted is twelve. Since the number of experimental data is small, we need to construct an interval regression model to make annual fertilizer doses recommendations based on fertilizer trials. To construct interval regression model we first determine the variable that should be included in the interval regression model. To this end, we determine the regression model which is free from interval coefficients and fits the data involving crop yield and applied fertilizer doses as accurately as possible. A quadratic function is often used Verdooren (2003) as response surface because higher order interaction usually have little effect. The response surface for yield in response to different levels of N (Nitrogen fertiliser) and K (Pottasium fertiliser) after application of multiple linear regression using data Charate *et al.* (2018), is given by

$y(N, K) = a + bK + cN + dN^2 + eNK$ (6)

Where y (N, K) represents the yield of little Millet, a, b, c, d and e are coefficients of regression. The p-value of each of them is less than 0.05. Coefficients of determination of yield surface

$R^2 = 0.93913$ and Adjusted

$R^2 = 0.904347$ with significance F<.001

Taking in to account the functional relationship (6), we now construct an interval regression model which has the factors N, K, N^2, NK alongwith interval coefficients. Minimizing (3) subject to the constraints (4) and (5) we obtained the following interval regression model taking into account the data provided in Table 1.

$y(N, K) = A_1 + A_2K + A_3N + A_4N^2 + A_5NK$ (7)

where

$A_1 = [233.5, 390.5]$, $A_2 = 2.2167, 4.6167]$,

$A_3 = [1.6875, 19.2875]$, $A_4 = [0, 0.1525]$ and $A_5 = [0, 0.0050]$.

The function $y(N, K)$ represents the yield of Little Millet as a function of *N and K.*

Here, we have determined amount of fertilizers that need to be applied at sowing stage of crop for optimization of agronomic yield of Little Millet (*Panicum sumatrense*).

C. Interval Optimisation

Our objective is to determine the quantities of N and K that will maximise the yield (N, K). Due to presence of interval coefficients, there is a degree of uncertainty associated with the response surface (6) and, therefore, it is not possible to find the optimum value of $y(N, K)$ by usual optimisation techniques. A function which has intervals as parameters is called an interval valued function. We require interval optimisation techniques to find local minimum of interval valued function. Interval optimization problem has been studied by Hladik (2011), Hladik (2012), Jayswal (2011), Li and Tian (2008), Shaocheng (1994), Wu (2008), Roy and Panda (2021), Liao *et al.* (2023), Jiang *et al.* (2022), Jiang *et al.* (2008), Jiang *et al.* (2014), Wu (2009).

D. Conversion of an interval in to normal distributed random variable

To optimise an interval valued function, we shall follow the steps suggested by Kumar and Panda (2017). Accordingly, we shall first convert the intervals present in an interval valued function into normally distributed random variables and then optimise the resulting function which has random variables as coefficients.

Thus, replacing the intervals $A_1, A_2, A_3, A_4, and\ A_5$ and by random variables , which we denote as R_1, R_2, R_3, R_4 *and* R_5 respectively, (7) can be expressed in the following manner

$Y(N, K) = R_1 + R_2K + R_3N + R_4N^2 - R_5NK,$ (8)

where $Y(N, K)$ is the random variable corresponding to the interval $y(N, K)$.

An interval $A = [a^{-L}, a^{-R}]$ be alternatively can represented as ξ $\langle m(A)|W(A)\rangle$, where $m(A) = \frac{\bar{a}^L + \bar{a}^R}{2}$ is called the mean and $w(A) = \frac{\bar{a}^L - \bar{a}^R}{2}$ is called the spread of the interval A. Then A will be approximated by $R \sim N\left(m(A), \left(\frac{\omega(A)}{3}\right)^2\right)$, a normally distributed random variable R whose mean $m(R)$ is taken as $m(A)$ and the variance σ is taken as $\left(\frac{\omega(A)}{3}\right)$.

Accordingly, R_1, R_2, R_3, R_4 and R_5 are respectively the normally distributed random variables

$N(312, 684.69)$, $N(3.41, 0.16)$, $N(10.48, 8.60)$, $N(0.07, 6.4516e\text{-}04)$ and $N(0.0025, 6.9444e\ \text{-}07)$

E. Expectation and Variance of an Interval valued function in probabilistic sense

Let $A^k = (A_1, A_2, \ldots, A_k)^T$ be a vector of intervals. Suppose that $A_1, A_2, \ldots, A_k$ are associated with random variables $R_1, R_2, \ldots R_k$ respectively where

$R_j \sim N\left(m(A_j), \left(\frac{\omega(A_j)}{3}\right)^2\right)$, $j = 1, 2 \ldots, k$. The interval vector A^k is associated with a vector of random variables $R_v^k = (R_1, R_2, \ldots R_k)$ with mean mR_v^k and variance σR_v^k are respectively given by

$$m_{R_v^k} = \left(m(A_1), m(A_2), \cdots, m(A_k)\right)^T, \quad (9)$$

$$\sigma_{R_v^k} = \left(\frac{\omega(A_1)}{3}, \frac{\omega(A_2)}{3}, \cdots, \frac{\omega(A_k)}{3}\right)^T. \quad (10)$$

Consider an interval valued function that has n-variables $x_1, x_2, \ldots, x_n$. Let X be the vector $(x_1, x_2, \ldots, x_n)^T$. We shall denote an interval valued function of variables $x_1, x_2, \ldots, x_n$ and intervals $A, A_2, \ldots A_k$ as $F(X; A^k)$. Let $f(X; R_v^k)$ be the function obtained by replacing the intervals $A_1, A_2, \ldots, A_k$ in $F(X; A^k)$ by random variables $R_1, R_2, \ldots R_k$ respectively. In order to find out the expectation and variance of the function $f(X; R_v^k)$, we need to expand $f(X; R_v^k)$ about mR_v^k. Assuming that the variances of the random variables $R_1, R_2, \ldots R_k$ are to very small, the Taylor's series expansion of $f(X; R_v^k)$ is given by

$$f(X; R_v^k) \approx f\left(X; m_{R_v^k}\right) - \sum_{j=1}^{k}\left(\frac{\partial f(X; R_v^k)}{\partial R_j}\bigg|_{m_{R_v^k}}\right) m(A_j) + \sum_{j=1}^{k}\left(\frac{\partial f(X; R_v^k)}{\partial R_j}\bigg|_{m_{R_v^k}}\right) R_j$$

$$\approx \Psi(X; R_v^k). \quad (11)$$

Since by definition $m(A_j) = E(R_j)$, expectation of $f(X; R_v^k)$ is given by

$$E\left(f(X; R_v^k)\right) \approx E\left(\Psi(X; R_v^k)\right)$$

$$= f\left(X; m_{R_v^k}\right) - \sum_{j=1}^{k}\left(\frac{\partial f(X; R_v^k)}{\partial R_j}\bigg|_{m_{R_v^k}}\right) m(A_j) + \sum_{j=1}^{k}\left(\frac{\partial f(X; R_v^k)}{\partial R_j}\bigg|_{m_{R_v^k}}\right) E(R_j)$$

$$= f\left(X; m_{R_v^k}\right) \quad (12)$$

Where E (.) is the expectation operator and $f(X; m_{R_v}^k)$ is obtained from $f(X; R_v^k)$ after replacing R_v^k by $m_{R_v}^k$. Furthermore, the variance $Var(\psi(X; R_v^k))$ of $f(X; R_v^k)$ is taken as

$$Var\left(f(X; R_v^k)\right) \approx Var\left(\Psi(X; R_v^k)\right) \approx \sum_{j=1}^{k}\left(\frac{\partial f(X; R_v^k)}{\partial R_j}\bigg|_{m_{R_v^k}}\right)^2 Var(R_j)$$

$$= \sum_{j=1}^{k}\left(\frac{\partial f(X; R_v^k)}{\partial R_j}\bigg|_{m_{R_v^k}}\right)^2 \left(\frac{\omega(A_j)}{3}\right)^2 \quad (13)$$

It follows from (12) and (13) that $f(X; R_v^k)$ is a normally distributed random variable whose mean and variance is given by (12) and (13) respectively.

F. Conversion of Nonlinear Interval Programming to Deterministic form

Consider the unconstrained interval optimisation problem

Min $F(X; A^k)$, (14)

Let R_v^k be the vector of random variables representing the vector of intervals A^k. Let $f(X; R_v^k)$, be the function of random variables corresponding to $F(X; A^k)$. Let $m = E(f(X; R_v^k))$ and

Standard deviation

$$\sigma = \sqrt{var\left(f(X; R_v^k)\right)}.$$

Then, by Chance Constrained Programming Technique Rao(2009), the deterministic equivalence of (14) is

Min $\alpha m + \beta\sigma$, (15)

Where $\alpha \geq 0$ and $\beta \geq 0$, and their numerical values indicate the relative importance of m and σ for minimization.

The problem (15) is solved by any deterministic optimisation technique of a function of several variables. (9) (10)

RESULT AND DISCUSSION

Let $R_v^5 = R_1, R_2, R_3, R_4$ and R_5) where, R_1, R_2, R_3, R_4 and R_5 are the random variables in (8). Let $\psi(N, K; R_v^5)$ be the expression obtained by applying the formula (11) to (N, K).

By (9) and (10) we have

$$m_{R_v^5} = \begin{pmatrix} 312 \\ 3.41 \\ 10.48 \\ 0.07 \\ 0.0825 \end{pmatrix},$$

$$\sigma_{R_v^k} = \begin{pmatrix} 684.69 \\ 0.16 \\ 8.60 \\ 6.4516e-04 \\ 6.9444e-07 \end{pmatrix}.$$

By applying the formula (12) we find that

$E[\psi(N, K; R_v^5)] = 312 + 3.41K + 10.48N - 0.07N^2 - 0.0825NK$ (16)

By applying the formula (13) we have

$Var(\psi(N, K; R_v^5)) = 684.69 + 0.16K^2 + 8.60N^2 - 6.4516e\text{-}04N^4 - 6.9444e\text{-}07N^2K^2$ (17)

So $Y(N, K)$ is a normally distributed random variable whose mean and variance are respectively are given by (16) and (17).

The nonlinear interval programming converted in to the following form

Minimize

$$\alpha(312 + 3.41K + 10.48N - 0.07N^2 - 0.0825NK) + \beta\sqrt{684.69 + 0.16K^2 + 8.60N^2 - 6.4516e - 04N^4 - 6.9444e - 07N^2K^2}$$

This is now a general nonlinear programming problem for different values of α and β. We solve this programming for maximization we got the yield response surface $y(N, K; A)$ will have optimum yield when K = 35.8172(Kg/ha) and N = 115.8937(*Kg/ha*).

CONCLUSION

The application of fertilizer to the soil has a significant impact on crop output since naturally occurring nutrients are typically insufficient to support a healthy yield. Increases in fertilizer application can be attributed to higher crop yields as long as there is a shortage of fertilizers in the soil. If maximizing crop productivity in a particular field is necessary, the quantity of fertilizer required must be understood beforehand. In order to calculate the amount of fertilizer, a response surface is often created with yield as the response variable and fertilizers as the explanatory variables. Finding the amount of fertilizer that will produce the best yield is simple when one studies the response surface. Regression analysis is used to formulate the response surface. The latter method necessitates sufficient experimental data in tabular form. Interval regression equations are required when sufficient data is unavailable to formulate a proper response surface. In this case, we have shown how to build an interval regression model and how to maximize an interval valued function to calculate the fertilizer prescription for maximum yield. We may use this model for optimizing the yield of different types of single crop or integrated crops.

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