



Group Improvisation Based HS Algorithm for Designing T-S Type Fuzzy Controllers for a Class of Non-Linear Systems

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(Received 15 October, 2012 Accepted 01 December, 2012)

ABSTRACT: The present paper proposes a group improvisation based variant of harmony search (HS) algorithm, for solving optimization problems, conceptualizing the mutual cooperation of philharmonic orchestra. The proposed HS algorithm has been successfully applied to design a Takagi Sugeno (T-S) type fuzzy controller, so that the designed controller can guarantee desired stability and simultaneously it can provide satisfactory performance with a high degree of automation in the design process. The proposed variant and the original HS algorithm are implemented for two nonlinear benchmark systems in simulation case study and their results demonstrate that the proposed variant outperforms the original HS algorithm.

Keywords—Fuzzy logic controllers (FLC); Takagi Sugeno (T-S) type fuzzy controller; harmony search algorithm; harmony memory considering rate (HMCR); pitch adjusting rate (PAR).

I. INTRODUCTION

The relationship between music and mathematics dates back to the ancient civilization. The great Greek mathematician Pythagoras calculated the relations of sound wave in 6th century B.C. and that were utilized by the great music composer Mozart during the 18th century in “symphonic - harmonic”. In 2001, Z. W. Geem developed an optimization method, namely harmony search, for function optimization and engineering applications, inspired by the musical phenomena of harmony improvisations [1], [2], [3], [5].

In this present paper a new variant of HS algorithm is proposed where the improvisation of harmony in each iteration, is motivated from the group wise improvisation of harmony in rehearsal stage of a philharmonic orchestration performance. The mutual cooperation among the groups helps them to achieve an aesthetically better performance during the final concert.

The present paper also demonstrates a systematic design of T-S type fuzzy logic controllers employing basic HS algorithm and its proposed variant. The objective of this design strategy is to perform simultaneous adaptation of both the FLC structure and its free parameters, so that two competing requirements can be fulfilled: i) to guarantee stability of the controller designed and ii) to achieve very high degree of automation in the process of controller design by employing a global search method [10]. The HS based stochastic global optimization method is employed to determine the most suitable complete optimized vector containing the free parameters of the FLC e.g. positions of input MFs, scaling gains and positions of output singletons etc.

The rest of the paper is organized as follows: section II presents the HS algorithm and its proposed variant. Section III discusses about the design of stable adaptive fuzzy controller and section IV shows the simulation studies for two benchmark nonlinear systems. Section V concludes the paper.

II. HS ALGORITHM AND ITS PROPOSED VARIANT

A. Brief overview of Harmony Search (HS) algorithm

The Harmony Search (HS) is based on natural musical performance processes that occur when a musician searches for a better state of harmony, i.e. during jazz improvisation [1], [5]. Very similar to musical instruments, where a combination of pitches determine the quality of harmony generated, here a set of values of the decision variables is judged by the corresponding value of the objective function. As in musical harmony, if a combination of decision variables can produce good result, then this solution vector is stored in memory and this helps to produce an improved solution in near future [1].

B. Proposed improvements over the basic HS algorithm

The traditional HS algorithm uses fixed value for HMCR, PAR and bw. In HS algorithm these three are the very important parameters which are crucial in fine-tuning the optimized solution vectors, and can be potentially useful in adjusting the convergence rate of the algorithm to achieve the optimal solution. In an improvement suggested in [4], the PAR and bw are varied throughout the generations as linearly increasing and logarithmically decreasing respectively, but the HMCR parameter is kept constant. This

modification showed successful results for several constrained and unconstrained optimization problems.

Inspired by their modifications, this paper proposes a modification of the basic HS algorithm, where HMCR and PAR parameters are varied simultaneously throughout the generation of harmony improvisations in different ways to obtain the optimal solution vector. In this modification, the HMCR and PAR parameters are increased linearly from a minimum value to a maximum value and bw is kept fixed all through the generation of improvisations of harmony memory. The physical implication of this strategy is that as the generation of harmony improvisation increases the optimization approach relies more on the harmony memory because the harmony memory would be rich in experience due to the improvisations throughout the generations. The PAR parameter is also increased because the fine tuning is required as the optimization algorithm relies more on the past values of the harmonies stored in the memory. This signifies the inspiration behind the modifications of the HMCR and PAR parameters of the original HS algorithm.

Along with the HMCR, PAR and bw parameters for controlling convergence and its rate, another parameter group memory considering rate (GMCR) is introduced with the proposed concept of group wise improvisation of harmony memory. In this proposed concept the harmony memory is divided into number of groups and the member harmony for each group is being selected in each iterations. Each harmony belonging to a group is improvised in each iteration and a particular decision variable, which are collectively forms a harmony, is adjusted either from its own group or from the others. A particular harmony may belong to different group in different iteration, thus making the exploration of the solution space in more stochastic manner. The whole new concept of harmony improvisation is conceptualized from the mutual cooperation of different music groups and their combined performance in a philharmonic orchestra. In HS based engineering design optimization, this group wise improvisation of harmony will bring the essence of local neighborhood topological concept used in other meta heuristics like swarm intelligence based optimizations etc. The proposed group improvisation based variant of HS algorithm is summarized in Algorithm 1.

III. DESIGN OF STABLE ADAPTIVE FUZZY CONTROLLERS

A. Stable Fuzzy Controllers – Theoretical Preliminaries

Let the n th order SISO non-linear plant under consideration is [7], [9], [10], [11]:

$$\begin{cases} \dot{x}^{(n)} = f(x) + bu \\ y = x \end{cases} \quad (1)$$

where, $f(\cdot)$ is an unknown continuous function, $u \in R$ is the input and $y \in R$ is the output of the plant. Here b is an unknown positive constant. The state vector under consideration is given as

Algorithm 1: Group Improvisation based Harmony Search algorithm

BEGIN

Define objective function $g(\underline{z})$, $z_i \in Z_i$, $i = 1, 2, \dots, q$, \underline{z} is a candidate solution vector comprising the set of decision variable z_i , $\underline{z} = [z_1, z_2, \dots, z_q]^T \in R^q$, Z_i is the universe of discourse of the i th decision variable z_i i.e. $z_{i-\min} \leq z_i \leq z_{i-\max}$ and q is the number of decision variable.

Define maximum and minimum values of harmony memory considering rate (HMCR).

Define group memory considering rate (GMCR).

Define maximum and minimum values of pitch adjusting rate (PAR) and other parameters.

Generate Harmony Memory (HM) with random HMS number of harmonies.

WHILE ($t < \text{max number of iterations (iter}_{\max})$)

Set the HS parameters as:

$$HMCR(t) = HMCR_{\min} + \frac{(HMCR_{\max} - HMCR_{\min}) \times t}{iter_{\max}},$$

GMCR = fixed value,

$$PAR(t) = PAR_{\min} + \frac{(PAR_{\max} - PAR_{\min}) \times t}{iter_{\max}},$$

bw = fixed value.

Select number of group $G(t)$.

WHILE ($g < G(t)$)

Select the member harmony for each group from HM.

WHILE ($i \leq \text{number of variables}$)

IF ($rand < HMCR(t)$)

IF ($rand < GMCR$), Choose a value from own group member harmony for the variable i .

IF ($rand < PAR(t)$), Adjust the value by adding certain amount based on a bandwidth (bw) uniformly distributed over [-1, 1].

END IF

ELSE Choose a value from other groups for the variable i .

IF ($rand < PAR(t)$), Adjust the value by adding certain amount based on a bandwidth (bw) uniformly distributed over [-1, 1].

END IF

END IF

ELSE Choose a random value.

END IF

END WHILE

END WHILE

Accept the new harmony (solution) if better.

END WHILE

Find the current best solution.

END

$\underline{x} = (x_1, x_2, \dots, x_n)^T = (x, \dot{x}, \dots, x^{(n-1)})^T \in R^n$. The system under control has a reference model given as:

$$\begin{cases} \dot{x}_m^{(n)} = f(x_m, w) \\ y_m = x_m \end{cases} \quad (2)$$

where $w(t)$ is the excitation signal input to the reference model, and the state vector of the reference model is

$$\underline{x}_m = (x_m, \dot{x}_m, \dots, x_m^{(n-1)})^T \in R^n.$$

The control objective is to force the plant output $y(t)$ to follow a given bounded reference signal $y_m(t)$ under the constraints that all closed-loop variables involved must be bounded to guarantee the closed loop stability of the system. The tracking error is $e = y_m - y$. We now try to

design a T-S type fuzzy controller for the SISO system under consideration.

We attempt to find a feedback control strategy $u = u(x|\theta)$ and the supervisory control law $u_s(x)$ so that i) the closed loop system must be globally stable and ii) the tracking error $e(t)$ should be as small as possible [7], [8]. To accomplish these control objectives, let the error vector be $\underline{e} = (e, \dot{e}, \dots, e^{(n-1)})^T$ and $\underline{k} = (k_1, k_2, \dots, k_n)^T \in R^n$ be such that all the roots of the Hurwitz polynomial $s^n + k_n s^{n-1} + \dots + k_2 s + k_1$ are in the open left half of s -plane.

Now the ideal control law for the system in (1) and (2) is given as [7], [10]:

$$u^* = \frac{1}{b} \left[-f(x) + \dot{y}_m^{(n)} + \underline{k}^T \underline{e} \right] \quad (3)$$

where $y_m^{(n)}$ is the n th derivative of the desired reference signal. For some specific class of plants as in (1) this leads to the error differential equation as $e^{(n)} = -k_1 e - k_2 \dot{e} - \dots - k_n e^{(n-1)}$ and the definition implies that u^* guarantees perfect tracking i.e. $y(t) \equiv y_m(t)$ if $\lim_{t \rightarrow \infty} e(t) = 0$. A fuzzy logic system can suitably approximate the ideal control law u^* as f and b are not known precisely. To ensure stability, $u(t)$ is assumed to be composed by the additive contribution of a fuzzy control, $u_c(x|\theta)$ and an additional supervisory control [9], [10], [11] strategy $u_s(x)$. This implies

$$u(t) = u_c(x|\theta) + u_s(x) \quad (4)$$

For a zero order Takagi-Sugeno (TS) fuzzy system based AFLC [6] the $u_c(x|\theta)$ is given in the form

$$u_c(x|\theta) = \underline{\theta}^T * \underline{\xi}(x) \quad (5)$$

where $\underline{\theta} = [\theta_1 \theta_2 \dots \theta_N]^T$ = the vector of the output singletons, $\underline{\xi}(x)$ = vector containing normalized firing strength of all fuzzy IF-THEN rules = $(\xi_1(x), \xi_2(x), \dots, \xi_N(x))^T$.

Let us define a quadratic form of tracking error as $V_e = \frac{1}{2} \underline{e}^T P \underline{e}$ where P is a symmetric positive definite matrix satisfying the Lyapunov equation [6], [7].

Here $u_s(x)$ is constructed as given in [9], [10] and can be presented as

$$u_s(x) = I_1^* \operatorname{sgn} \left(e^T P b_c \left[|u_c| + \frac{1}{b_L} \left(f^U + |y_m^{(n)}| + |\underline{k}^T \underline{e}| \right) \right] \right) \quad (6)$$

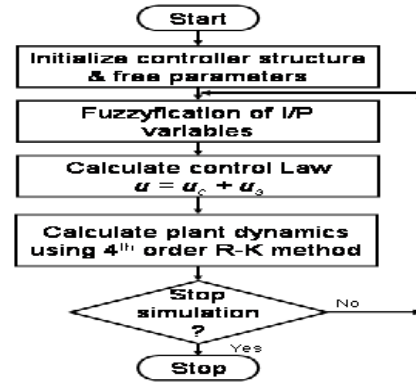


Fig. 1. Flowchart representation of CCS algorithm

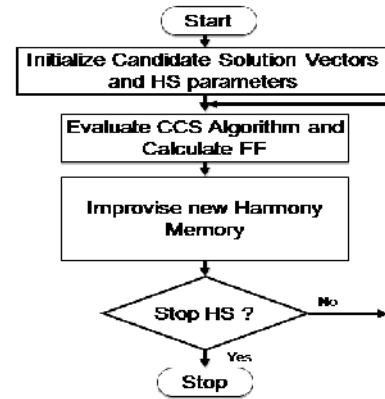


Fig. 2. Flowchart representation of HSBA algorithm

where $\begin{cases} I_1^* = 1 & \text{if } V_e > \bar{V} \\ I_1^* = 0 & \text{if } V_e \leq \bar{V} \end{cases}$, \bar{V} is a constant specified by

the designer, $f^U \geq |f(x)|$ and $0 < b_L < b$. With this

$u_s(x)$ and (6) it can be shown that $\dot{V}_e \leq -\frac{1}{2} e^T Q e \leq 0$.

Thus, as $P > 0$, boundedness of V_e implies the boundedness in \underline{x} . Hence, the closed loop stability is guaranteed.

B. HS Algorithm Based Stable Adaptive Fuzzy Controller Design Methodology

Earlier in Lyapunov theory based direct AFLC design, only the output singletons of the AFLC are adapted and the other free parameters e.g. supports of the input MFs, input and output scaling gains of the AFLC are chosen a priori. However, while designing the HS algorithm based optimal controller, all these parameters are obtained automatically by encoding them as part of the solution vector i.e. each of these parameters forms a decision variable. This helps to determine the optimal structure of the FLC.

In HS algorithm based design, a harmony i.e. candidate solution vector (CSV) in solution space is formed as [10]:

$$\underline{Z} = [\text{center locations of the MFs} \mid \text{scaling gains} \mid \text{positions of the output singletons}] \quad (7)$$

To design the AFLC utilizing HS algorithms first the population of CSV chosen, then the candidate controller simulation (CCS) algorithm, as shown in Fig. 1, is implemented for each on the candidate controller to calculate the fitness function (FF) which is the integral absolute error (IAE) in this case. Then, according to the value of the fitness function (IAE) of each CSV in each iteration, the best solution is achieved using the HS algorithm. The HS algorithms will stop searching the solution space when the number of iterations specified by the designer is reached or a pre specified error is attained by the controller. The flowchart representation of HS based approach (HSBA) of AFLC design is shown in Fig. 2.

IV. SIMULATION CASE STUDY

At the onset we consider a case of model referencing control of the DC motor with non-linear characteristics and this can be described as [7]:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{f(x_2)}{J} + \frac{C_T}{J}u \\ y = x_1 \end{cases} \quad (8)$$

where $y = x_1$ is the angular position of the rotor (in rad), x_2 is the angular speed (in rad/sec) and u is the current fed to the motor (in A). The plant parameters are $C_T = 10$ Nm/A, $J = 0.1$ kgm² and the non-linear friction torque $f(x_2) = 5 \tan^{-1}(5x_2)$ Nm. The control objective is to make the angular position y follow the output of a reference model given by

$$\ddot{y}_m = 400w(t) - 400y_m - 40\dot{y}_m \quad (9)$$

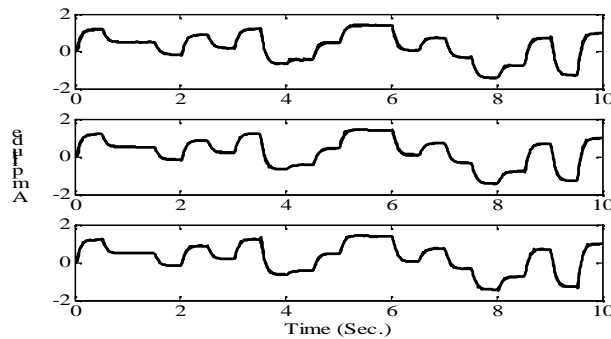


Fig. 3. Evaluation period responses of the AFLC for non-linear DC motor system for original HS [1], HS variant [4] and proposed group improvisation HS based controller (GrHSBA) (from top to bottom of the figure).

The second nonlinear system considered in this paper is the Duffing's oscillatory system and the mathematical model of this system can be described as [10], [11]:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -0.1x_2 - x_1^3 + 12\cos(t) + u(t) \\ y = x_1 \end{cases} \quad (10)$$

The control objective is to track the reference signal y_m , where $y_m = \sin(t)$.

To study the performances in terms of IAE, the model referenced non-linear DC motor with the configuration of $5 \times 5 \times 5$ input MFs (two fuzzy input for second ordered plant and one for the excitation signal input to the reference model) and Duffing's non-linear system with the configuration of 5×5 input MFs (two fuzzy input for second ordered plant) are simulated, each using a fixed step 4th order Runge-Kutta method, with sampling time $\Delta t_c = 0.01$ sec. Each process model is evaluated for 10 sec. duration. The results of the comparative study, in terms of average IAE between the reference and output signal, are tabulated in Table-I and from there it is observed that the proposed modification with group wise improvisation concept shows a superior result than the other methods for both the systems under consideration. Evaluation period Responses of the optimal fuzzy controller for non-linear DC motor system and Duffing's system are shown in Fig. 3 and Fig. 4 respectively.

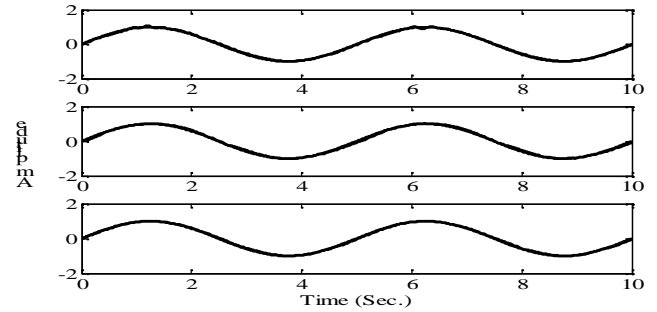


Fig. 4. Evaluation period responses of the AFLC for Duffing's system for original HS [1], HS variant [4] and proposed group improvisation HS based controller (GrHSBA) (from top to bottom of the figure).

V. CONCLUSION

This paper has introduced an improved group wise improvisation based variant of harmony search algorithm which has the power of the HS algorithm with mutual cooperation among the groups conceptualizing the performance of a philharmonic orchestration concert. The proposed variant of HS algorithm is utilized to design the adaptive fuzzy controllers for the benchmark nonlinear systems and the comparative results demonstrates that the usefulness of the proposed modifications of the basic HS algorithm.

TABLE I. COMPARATIVE STUDY OF DIFFERENT HS ALGORITHM BASED AFLC DESIGN STRATEGIES

Control Strategy Evaluation time = 10 sec. HMS = 30 No. of iterations = 500	Average IAE	
	Non-linear DC motor system	Duffing's oscillatory system
HSBA [1] (Single harmony improvisation) <i>HMCR</i> : fixed, <i>PAR</i> : fixed, <i>bw</i> : fixed	0.4362	0.5078
HSBA [4] (Single harmony improvisation) <i>HMCR</i> : fixed, <i>PAR</i> : increasing linearly, <i>bw</i> : decreasing logarithmically	0.3996	0.4908
GrHSBA: (Group harmony improvisation) <i>HMCR</i> : increasing linearly, <i>PAR</i> : increasing linearly, <i>bw</i> : fixed	0.3354	0.4095

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