



PSO Based H TCSC Controller with Comparison to its LMI Based Design in Mitigating Small Signal Stability Problem

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ABSTRACT: This paper aims to design a fixed structure H controller for a Thyristor Controlled Series Compensator (TCSC) in order to mitigate small signal oscillations in a multimachine power system. The structure of the controller is a basic lead-lag compensator whose parameters is to be optimized using Particle Swarm Optimization (PSO) method via minimization of the H -norm of the closed-loop plant transfer function. In another attempt Linear Matrix Inequality (LMI) based approach has been employed to design an internally stabilizing controller for the TCSC that satisfies H -norm constraint, while ensuring that the closed-loop poles lie in a certain region in the complex plane. The performance of both the PSO based and the LMI based H TCSC controllers are tested in an IEEE type 14-bus system. It has been revealed that the PSO based H TCSC controller is more effective compared to its LMI based design in mitigating small signal oscillations.

Index Terms— H Controller, Lead-Lag Compensator, Linear Matrix Inequality (LMI), Particle Swarm Optimization (PSO), Thyristor Controlled Series Compensator (TCSC).

I. INTRODUCTION

The problem of small signal oscillations (0.2-2.5 Hz) is a long-standing issue in electric power systems. These oscillations may sustain and grow up to cause severe system outage if adequate damping is not available [1]. The design and synthesis of conventional damping controllers-Power System Stabilizer (PSS) and Flexible AC Transmission System (FACTS) devices [2]-[3] are simple but these controllers lack robustness even after careful tuning. Out of many FACTS devices, Thyristor Controlled Series Compensator (TCSC) has been proven to be effective means [4]-[5] for mitigating small signal oscillations in long transmission lines of modern power systems.

Robust controllers based on the optimization of the H -norm of the transfer function matrix between the system disturbance and its output, via Algebraic Riccati Equations (ARE) or Linear Matrix Inequality (LMI) techniques have been widely applied in control theory and applications [6]-[7]. The mixed-sensitivity H -control techniques based on LMI approach has been applied in [8] to design an inter-area damping controller employing Superconducting Magnetic Energy Storage (SMES) device. A multiple-input, single-

output (MISO) LMI based H robust controller design has been illustrated in [9] for a TCSC to improve damping of the inter-area modes employing global stabilizing signal.

Such controllers show robustness against disturbance but may have a large size that may give rise to complex structure. Therefore, reduction of the controller's model is normally adopted in practical implementation [10]. However, such reduction often produces some degradation of performance and robustness control as H -norm increases. To overcome these difficulties design of a specific controller structure like Proportional-Integral-Derivative (PID) or Lead-Lag compensator has been performed whose parameters can be determined via optimization of the system H -norm. Some heuristic based approaches such as Artificial Neural Network (ANN), Genetic Algorithm (GA) have been used [11]-[12] to solve this optimization problem.

Recently, Particle Swarm Optimization (PSO) [13] technique is especially more adopted as a new and efficient design tool to solve the fixed-structure and reduced-order H control problem [14]-[15]. The method reported in [16] is based on the synthesis of various fixed-structure mixed-sensitivity H controllers using a constrained PSO algorithm. In [17] it has been shown that the reduced order PSO based H

controller allows a smaller H -norm in comparison with Hankel-reduction controller. In this paper, a PSO based fixed-structure lead-lag controller for a TCSC has been designed through minimization of the H -norm specifications which ensures satisfactory damping of the critical swing mode following possible power system disturbances (e.g. change in load and transmission line outage) in a multimachine system. To the best of author's knowledge this work has not been explored in the existing literatures. To show the effectiveness of the proposed PSO based approach, the results are compared with its LMI based design. It has been observed that the PSO based H TCSC controller is more effective and has the advantages of simple structure, less computation complexity compared to the LMI based H TCSC controller.

II. PROBLEM FORMULATIONS BASED ON H CONTROL THEORY

The problem here is to find the optimal parameter set of the TCSC controller (Fig. 1) using PSO algorithm, which

minimizes the H -norm constraint of the closed-loop plant transfer function. The configuration of the closed-loop plant together with the fixed-structure lead-lag controller is proposed in Fig. 2. Here $G(s)$ is the open-loop plant, $K(s)$ is the controller whose parameters are to be optimized, and $W_1(s)$ and $W_2(s)$ are frequency dependent weights for shaping the characteristics of the closed-loop plant. The input to the controller is the normalized speed deviation (Δv), and the output signal is the deviation in thyristor conduction angle ($\Delta \sigma$). Here, (1) and (2) represent the linearized differential equations and stator algebraic equations of the machine. Equations (3) and (4) are the linearized network equations pertaining to the generator buses and the load buses. The installation of a TCSC in this model results in addition of state variables of the TCSC power flow equations. The basic TCSC module consists of a fixed series capacitor bank C in parallel with a Thyristor Controlled Reactor (TCR) as shown in Fig. 3 [20]. The size of the TCSC is chosen as $X_L = 0.0049$ pu, $X_C = 0.0284$ pu.

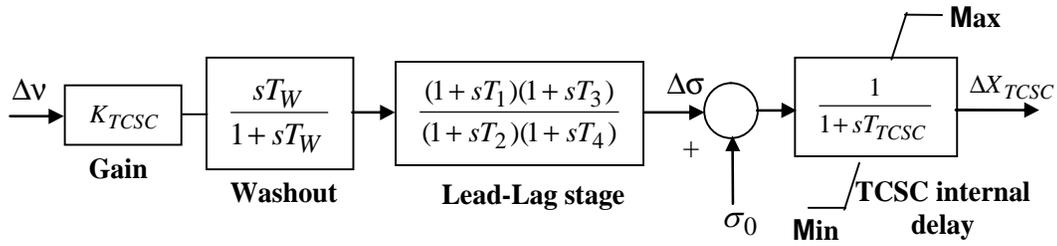


Fig. 1. Block-diagram model of the TCSC controller, $K(s)$.

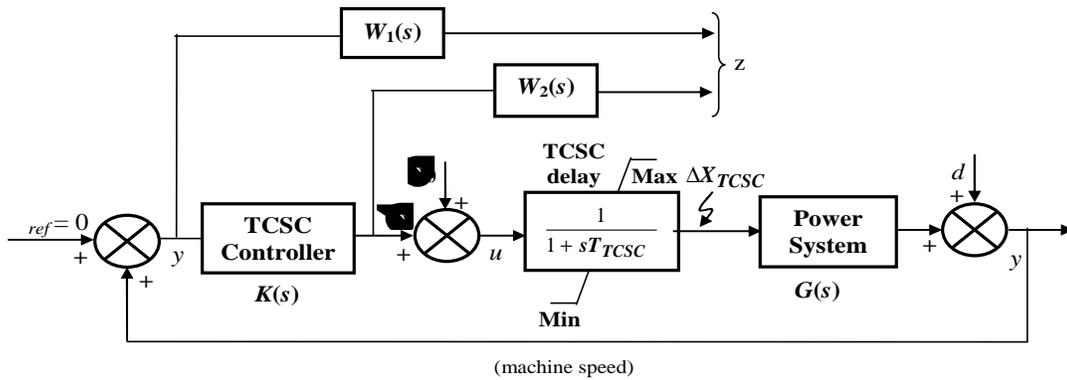


Fig. 2. The closed-loop system along with the controller $K(s)$.

The problem is to minimize the weighted Sensitivity transfer function $S(s) = [I - G(s)K(s)]^{-1}$, which ensures disturbance rejection and Complementary sensitivity transfer function $K(s)S(s) = [K(s)I - G(s)]^{-1}$ that ensures robustness in design and minimizes the control effort [18].

The linearized small signal model of a multimachine system and its state-space formulations relating to the

performance of the machine with exciter and network power flow equations have been discussed in literature [19] and are represented here by the following equations

$$\dot{X} = A_1 X + B_1 I_g + B_2 V_g + E_1 U \quad (1)$$

$$0 = C_1 X + D_1 I_g + D_2 V_g \quad (2)$$

$$0 = C_2 X + D_3 I_g + D_4 V_g + D_5 V_l \quad (3)$$

$$0 = D_6 V_g + D_7 V_l \quad (4)$$

The series reactance of the TCSC is adjusted through appropriate variation of the firing angle, $\alpha = (\cdot)$ to keep the specified amount of active power flow across the series compensated line. With the installations of a TCSC device, the TCSC power flow equations are to be additionally included with the network equation (3) and (4). The TCSC linearized power flow equations at the node 's' can be obtained by the following expression [21].

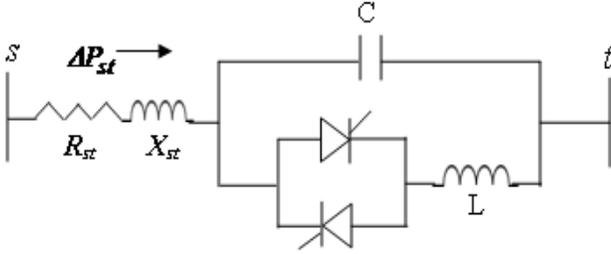


Fig. 3. TCSC module between node s and t.

$$0 = \begin{bmatrix} \frac{\partial P_s}{\partial s} & \frac{\partial P_s}{\partial V_s} & \frac{\partial P_s}{\partial \alpha} \\ \frac{\partial Q_s}{\partial s} & \frac{\partial Q_s}{\partial V_s} & \frac{\partial Q_s}{\partial \alpha} \\ \frac{\partial P_{st}}{\partial s} & \frac{\partial P_{st}}{\partial V_s} & \frac{\partial P_{st}}{\partial \alpha} \end{bmatrix} \begin{bmatrix} V_s^s \\ \alpha \end{bmatrix} \quad (5)$$

$$\text{where } P_{st} = V_s^2 g_{st} - V_s V_t (g_{st} \cos \theta_{st} + b_{st} \sin \theta_{st}) \quad (6)$$

$$\text{and } Q_{st} = -V_s^2 b_{st} - V_s V_t (g_{st} \sin \theta_{st} - b_{st} \cos \theta_{st}) \quad (7)$$

Similarly, the linearized power flow equations for the node 't' can be obtained by replacing t for s.

This linearized state-space model of the power system given by (1)-(4) can be augmented following the mixed-sensitivity based H - control theory [10]

$$\dot{x}_p = A_p x_p + B_{p1} d + B_{p2} u \quad (8)$$

$$z = C_{p1} x_p + D_{p11} d + D_{p12} u \quad (9)$$

$$y = C_{p2} x_p + D_{p21} d + D_{p22} u \quad (10)$$

where x_p is the state vector of the augmented plant, u is the plant input, y is the measured signal modulated by the disturbance input d and z is the controlled output.

The TCSC controller, $K(s)$ depicted in block-diagram (Fig. 1) can be realized by the following state-space equations

$$\dot{\hat{x}} = A_k \hat{x} + B_k y \quad (11)$$

$$u = C_k \hat{x} + D_k y \quad (12)$$

The overall state-space representation of the closed-loop plant with TCSC controller is then given by

$$\dot{z} = A_{cl} z + B_{cl} d \quad (13)$$

$$z = C_{cl} z + D_{cl} d \quad (14)$$

$$\text{where } \dot{z} = \begin{bmatrix} \dot{x}_p \\ \dot{\hat{x}} \end{bmatrix}, A_{cl} = \begin{bmatrix} A_p + B_{p2} D_k C_{p2} & B_{p2} C_k \\ B_k C_{p2} & A_k \end{bmatrix};$$

$$B_{cl} = \begin{bmatrix} B_{p1} + B_{p2} D_k D_{p21} \\ B_k D_{p21} \end{bmatrix}; D_{cl} = D_{p11} + D_{p12} D_k D_{p21};$$

$C_{cl} = [C_{p1} + D_{p12} D_k C_{p2} \quad D_{p12} C_k]$. Without loss of generality, D_{p22} can be set to zero to make the derivation simpler and then plant becomes strictly proper. The closed-loop transfer function between 'd' to 'z' can be found as

$$T_{zd} = \begin{bmatrix} W_1(s)S(s) \\ W_2(s)K(s)S(s) \end{bmatrix} = C_{cl}(sI - A_{cl})^{-1} B_{cl} + D_{cl} \quad (15)$$

A. H Controller through PSO

The objective of the H optimization problem is to minimize $\|T_{zd}\|_{\infty} < \gamma$, where $\gamma > 0$ is a designable parameter. There are five tuning parameters of the TCSC controller the controller gain (K_{TCSC}), lead-lag time constants (T_1, T_2, T_3 and T_4). These parameters are to be optimized by minimizing the desired objective function ($J = \|T_{zd}\|_{\infty} < \gamma$) applying PSO based technique. The washout stage of the controller is designed with $T_w = 10$ sec. The problem constraints are the bounds on the possible parameters of the TCSC controller.

The optimization problem can then be formulated as:

$$\text{Minimize } J = \|T_{zd}\|_{\infty} < \gamma; \quad \gamma > 0 \quad (16)$$

Subject to:

$$K_{TCSC}^{\min} \leq K \leq K_{TCSC}^{\max}; \quad T_1^{\min} \leq T_1 \leq T_1^{\max};$$

$$T_2^{\min} \leq T_2 \leq T_2^{\max}; \quad T_3^{\min} \leq T_3 \leq T_3^{\max};$$

$$T_4^{\min} \leq T_4 \leq T_4^{\max}.$$

It may be appreciated at this point that an analytical solution of this optimization problem is difficult to obtain and this numerical method produces solution more efficiently.

B. H Controller through LMI

In LMI formulations, the proposed objective, *Minimize* $J = \|T_{zd}\|_{\infty} < \gamma$ given by (16) can be achieved in a sub-optimal sense if there exist an internally stabilizing controller $K(s)$ such that the following *bounded real lemma* [7] given by: *Lemma- (H performance) the closed loop gain $\|T_{zd}\|_{\infty}$ does not exceed the performance index ' γ ' if and only if there exist a solution $X_{cl} = X_{cl}^T > 0$ such that,*

$$\begin{bmatrix} A_{cl}^T X_{cl} + X_{cl} A_{cl} & B_{cl} & X_{cl} C_{cl}^T \\ B_{cl}^T & -I & D_{cl}^T \\ C_{cl} X_{cl} & D_{cl} & -\gamma^2 I \end{bmatrix} < 0 \quad (17)$$

is satisfied and the controller is said to be sub-optimal. It is to be noted that such solutions do not require pre-specified controller structure.

In LMI frame work an ‘LMI region’ can be assigned by clustering all the closed-loop poles inside a conic sector centred at the origin with inner angle ‘ θ ’ in the left-half of s -plane (Fig. 4) which ensures that damping ratio of the poles lying in this sector is at least $\zeta = \cos \frac{\theta}{2}$. It has been reported in [22] that the state matrix, A_{cl} of the closed-loop plant, has all its poles inside the conical sector if and only if there exists $X_c = X_c^T > 0$ such that

$$\begin{bmatrix} \sin \frac{\theta}{2} (A_{cl} X_c + X_c A_{cl}^T) & \cos \frac{\theta}{2} (A_{cl} X_c - X_c A_{cl}^T) \\ \cos \frac{\theta}{2} (X_c A_{cl}^T - A_{cl} X_c) & \sin \frac{\theta}{2} (A_{cl} X_c + X_c A_{cl}^T) \end{bmatrix} < 0 \quad (18)$$

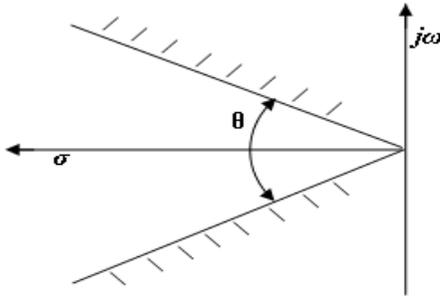


Fig. 4. Conic sector LMI region of closed loop poles.

The optimization problem therefore reduces to minimization of under LMI based H control with pole placement constraints. The inequalities in (17) and (18) are not jointly convex as the solutions $X_{cl} \neq X_c$. The convexity can be accomplished by seeking a common solution, $X_{cl} = X_c = X_d$. It is to be noted that the inequalities in (17) and (18) contain non-linear terms $A_{cl} X_d$ and $C_{cl} X_d$ where A_{cl} and C_{cl} contain unknown matrices of the controller and the resulting problem therefore cannot be handled by LMI optimization directly. To convert the problem into a linear one, a change of new controller variables (\hat{A} , \hat{B} , \hat{C} and \hat{D}) is executed following the transformations given in [22]-[23]. Once the new variables \hat{A} , \hat{B} , \hat{C} and \hat{D} are solved from the LMIs, the actual controller variables A_k , B_k , C_k and D_k are then recovered from these \hat{A} , \hat{B} , \hat{C} and \hat{D} matrices.

III. CONTROLLER DESIGN

A. PSO Based Approach

To optimize (16), routines from ‘PSO toolbox’ [24] are used. The PSO toolbox consists of a main program associated with a bunch of useful sub-programs and routines which are utilized as per requirements. In this work the main program ‘*pso_Trelea_vectorized.m*’ has been implemented for ‘Common’ type PSO as a generic particle swarm optimizer. To find the optimal value of the controller parameters and the objective function (J), this main program uses the user defined small signal stability and eigenvalue computation program as a sub-program. A default plotting routine ‘*gplotpso.m*’ is used by the PSO algorithm to plot the best value of the objective function ‘*gbest*’ for the specified generation (epochs) limit. In PSO based algorithm, the particle is defined as a vector which contains the TCSC controller parameters as shown in (19)

$$\text{Particle: } [K_{TCSC} \ T_1 \ T_2 \ T_3 \ T_4] \quad (19)$$

The initial population is generated randomly for each particle and is kept within a typical range following [19]-[20] and mentioned in Table I. The values of the TCSC controller parameters are updated in each generation within this specified range. The ‘PSO parameters’ set in the PSO algorithm are dimension of inputs 5, number of iterations 200, swarm size 15 etc. Choice of these parameters affects the performance and the speed of convergence of the algorithm.

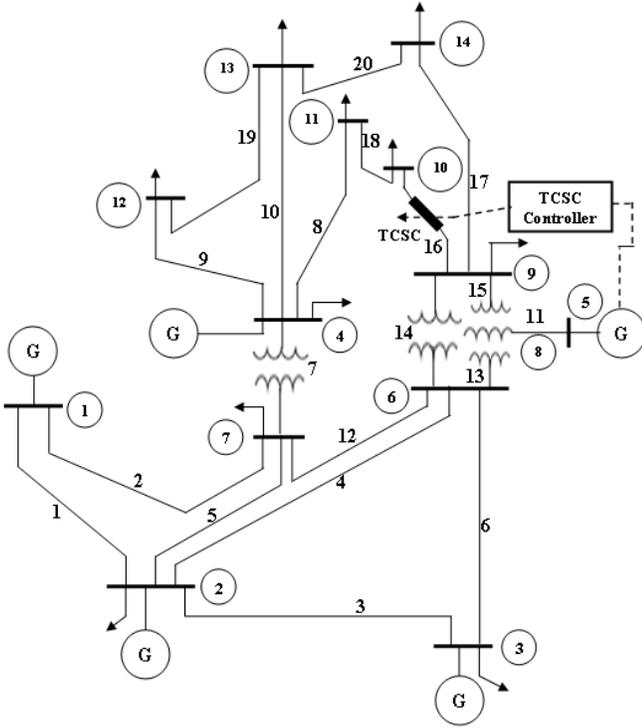
Table 1. Range of TCSC controller parameters.

Variable	Minimum range	Maximum range
K_{TCSC}	0.1	10
T_1, T_2	0.2, 0.02	1.5, 0.15
T_3, T_4	0.01, 0.1	0.1, 1.0

Following the standard guidelines of mixed-sensitivity based design [18], weights $W_1(s)$ and $W_2(s)$ are chosen as low and high pass filters, respectively. The weights $W_1(s)$ and $W_2(s)$ are worked out to be: $W_1(s) = \frac{2}{s+1.5}$; $W_2(s) = \frac{0.5s+10}{0.25s+1}$. The H -norm of the closed-loop transfer function $\|T_{zd}\|_{\infty}$ is computed in MATLAB for the proposed IEEE type 14-bus study system (Fig. 5) [25]. The PSO algorithm generates the optimal values of the TCSC controller parameters by minimizing the objective function ‘ J ’ and the output results are presented in Table II. The convergence rate of objective function ‘ J ’ for *gbest* with the number of generations for 200 has been shown in Fig. 6. The convergence is guaranteed by observing the value of ‘ J ’, which remains unchanged up-to 8 decimal places.

Table 2. Pso based tsc controller parameters.

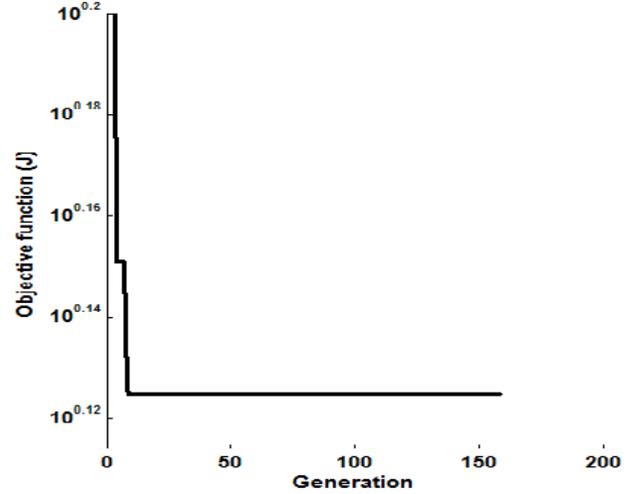
Parameters	PSO based value	Value of ‘ σ ’
K_{TCSC}	0.52714	1.333
T_1, T_2	0.27309, 0.02002	
T_3, T_4	0.09762, 0.96707	

**Fig. 5.** IEEE-14 bus system with TCSC controller.

B. LMI Based Approach

The LMI formulations described in Section II.B are now applied to the study system. The original system has total 36 states including one state for the TCSC delay. The corresponding LMI based controller would be of a higher order than this. The plant model is hence reduced to a 10-th order equivalent using square-root balanced truncation [10]. The multi-objective H_∞ synthesis program for disturbance rejection and control effort optimization features of LMI was accessed by suitably chosen arguments of the function ‘*hinfmix*’ of the LMI Toolbox in MATLAB. The pole placement objective in LMI has been achieved by defining the conical sector with $\frac{\sigma}{2} = 67.5^\circ$, which provides a desired minimum damping $\zeta = 0.39$ for all the closed-loop poles.

The order of the controller obtained from the LMI solution is quite high (12-th order) posing difficulty in practical implementation. Therefore, the controller is further reduced to a third-order one by the balanced truncation without significantly affecting the frequency response.

**Fig. 6.** Convergence rate of objective function for ‘*gbest*’.

The TCSC controller variables A_k, B_k, C_k and D_k are obtained as

$$A_k = \begin{bmatrix} -3.253 & 1932.5 & 81.847 \\ -1932.5 & -52700.0 & -3249.0 \\ -81.847 & -3249.0 & -202.13 \end{bmatrix}; B_k = \begin{bmatrix} 126.08 \\ 13018 \\ 732.78 \end{bmatrix};$$

$$C_k = [-126.08 \quad 13018 \quad 732.78]; D_k = [9.4571].$$

This reduced-order controller has been tested on the full-order system against said power system disturbances.

IV. PERFORMANCE EVALUATION OF THE CONTROLLER

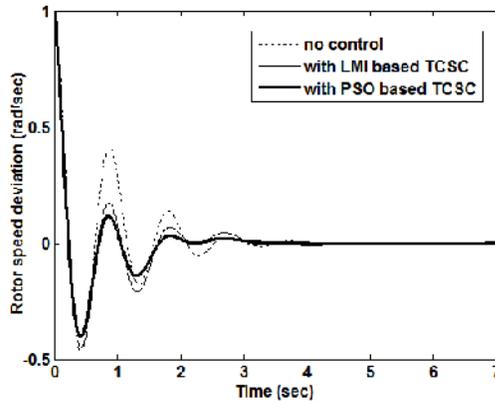
The performance of both the PSO based and the LMI based TCSC controllers are evaluated here in the face of two commonly occurring power system disturbances; that include real and reactive load increase (15% more than nominal ($P_L=0.339$ pu, $Q_L=0.190$ pu)) in a selected bus #9 and outage of a transmission lines (#10). The damping ratio of the swing modes without controller has been presented in Table III. It has been observed that the swing mode #4 is the critical one as the damping ratio of this mode is smallest compared to other modes. Therefore, stabilization of this mode is essential in order to improve small signal stability. The PSO based and the LMI based TCSC controllers are separately installed in the proposed 14-bus test system and the results are shown in Table IV. It has been found that damping ratio of the critical swing mode #4 is improved substantially with application of both the LMI based and the PSO based TCSC controller but the later improves 6-10% more damping compared to the earlier one. The time response plot of rotor speed deviation of the machine #1 also indicates similar results (Fig. 7). In view of these observations it is reasonable to conclude that the PSO based TCSC controller exhibits relatively good damping characteristics and is more effective compared to the LMI based TCSC controller.

Table III. Swing modes without controller.

#	Load increased at bus # 9		Transmission line (#10) outage	
	Swing modes	Dampin g ratio	Swing modes	Dampin g ratio
1	-1.5446 ± j7.5274	0.2010	- 1.5482 ± j7.5222	0.2015
2	-1.4244 ± j6.5313	0.2130	- 1.4291 ± j6.5339	0.2136
3	-1.1590 ± j6.1460	0.1853	- 1.1501 ± j6.1659	0.1833
4	-0.8831 ± j5.8324	0.1497	- 0.8845 ± j5.8336	0.1499

Table 4. Critical swing mode #4 with TCSC controller.

Power system disturbances	With LMI based H TCSC controller		With PSO based H TCSC controller	
	Load increased at bus # 9	Transmissi on line outage	Load increased at bus # 9	Transmissi on line outage
Critical swing mode #4	-1.1583 ± j6.8822	-1.0719 ± j6.1341	-1.2276 ± j6.8783	-1.1642 ± j6.0791
Damping ratio	0.1659	0.1721	0.1757	0.1880

**Fig. 7.** Rotor speed deviation response of machine #1.

V. CONCLUSION

This paper presents a new approach of design of a fixed-structure H TCSC controller based on PSO algorithm in order to mitigate small signal oscillations in a multimachine power system. The eigenvalue and time response analysis revealed that the PSO based H controller is more effective and superior compared to the LMI based H controller. The PSO based

method appears to have fast convergence and simple algorithm and results reduced order controller with a less complicated structure which makes easier in practical implementation. The proposed approach can be applied for the design of other FACTS controllers.

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