



Voltage Stability Assessment Using Sensitivity Under Line Outage Condition

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ABSTRACT : This paper presents a procedure for voltage stability margin evaluation under line outage condition considering static voltage stability limit. The proposed methodology is carried out in three steps. The first step of the study mode consists of obtaining sensitivity relation which relate loadability margin with real and reactive power injections at buses. In step two line outage is simulated using compensation injections, compensation injections and sensitivity relations have been utilized in the last step to determine change in loadability margin

Keywords : Voltage stability, Loadability margin sensitivity, Compensation injections, Eigen vectors.

I. INTRODUCTION

Voltage stability problem is emerging as a new challenge to power system planning and operation due to increased loading of transmission lines as well as difficulty of erecting new generating plants near the load centers. There have been a number of incidents in the past years which were diagnosed to have been caused by increased loading and reduced stability margin. The stability margin may be defined as the distance between the loading of the systems and the maximum loading limit of the systems. As this margin decreases the system approaches a very critical stage, since if one of the contingency such as unexpected line outages occurs that may often result in voltage instability leading the systems to a total collapse. Therefore in recent years there has been a lot of research in how to determine the voltage stability of the system or to determine the margin between operating point of the system and maximum loading point. This loading margin changes as system parameters or controls are altered i.e. load shedding, reactive power support, load increase, load model, line susceptance, generator redispatch etc. There are a number of methods developed for voltage stability assessment viewing voltage stability problem as static one. Determinant of the smallest eigenvalue or singular value of the system Jacobian [2-4] is some of the good indicators for assessing loadability margin.

Operationally a power system is said to be secure when all operating constraints are satisfied with sufficient loadability margin. Line flows and voltage magnitudes are important considerations. Now a days the trend is to evaluate voltage security from voltage stability view point since voltage magnitude alone may not provide sufficient indicators about a voltage security [5]. The overall aim of economy security function is to operate the system at the lowest cost, with the guaranteed alleviation of emergency condition. A power system is in emergency condition of varying degree of severity when operating limits are violated

resulting from contingencies e.g. line outages. An important part of security studies, therefore, moves around the power system ability to withstand the effects of contingencies under steady state condition. Security assessment and control are two important aspects of security studies. Security assessment is the process whereby limit violations are detected in post contingency operating states and then constraints are generated for security control [6].

An important component of security assessment is contingency selection. The purpose of contingency ranking is to determine which contingencies may cause power system limit violations and/or system instability according to voltage stability criteria. The margin between the voltage collapse point and the current operating point is used as the voltage stability criterion. The goal of the methods for contingency ranking is to estimate rapidly and accurately the voltage stability margin for all contingencies. Jasman and Lee [5] were probably the first to develop a line outage ranking algorithm based on a voltage stability criterion. Vaahedi et al [7] used reactive support index and iterative filtering technique for voltage security evaluation. Arya et al [8] developed a contingency selection algorithm based on vanishing eigenvalue concept. Bao et al [9] developed an on line voltage stability monitoring algorithm using correlation between VAR reserves and loadability margin of the system. Aumullar and Saha [10] presented sensitivity based technique for voltage stability assessment by determining groups of coherent buses. Amjady and Esmaili [11] presented an application of a new sensitivity analysis framework for voltage contingency ranking Arya et al [12] developed a line voltage stability index and this index is used to estimate system loadability and line outage ranking. Stott et al [13] have shown linear and quadratic estimates to the variation of loading margin with respect to any power system parameter or control. These estimates can be quickly used to assess the change in loadability margin for given change in control. 1/MVA sensitivity ranking

algorithm as developed in ref. [14] for voltage collapse can rank all contingencies based on estimates of the contingency bifurcation point. An approach is presented in ref. [15] for contingency ranking based on static security assessment which employs weighted performance index with the application of fuzzy logic analytical hierarchy process in order to select appropriate weighting factors to be imposed.

This paper proposes a method for contingency evaluation from voltage stability considerations. For a practical system, the number of contingencies (line outage) could be in thousands. The purpose of security evaluation is to reduce the list of contingencies for further processing by eliminating the cases causing no violation / threat. This is essential for on line applications. Limited accuracy results with lesser computational burden are sufficient. Hence, in this paper, for voltage security evaluation linear power system model has been used. To reduce the computational time base case sensitivities of loadability margin and compensation injection method has been used. The proposed method is applied to IEEE 25 bus system and the results are compared with continuation power flow technique [16].

II. SENSITIVITY DERIVATIONS FOR EVALUATING CHANGE IN LOADABILITY MARGIN

Power flow equations are given as follows.

$$\begin{aligned} P(V, \delta) - (P_{gi} - \alpha_i P_d) &= 0 \quad \dots(1) \\ Q(V, \delta) + \alpha_i \beta_i P_d &= 0 \end{aligned}$$

Incremental model at solution point can be written as follows:

$$[J_x] \begin{bmatrix} \Delta\delta \\ \Delta v \end{bmatrix} - [J_p] \begin{bmatrix} \Delta P_g \\ \Delta Q_g \end{bmatrix} + [J_d] \Delta P_d = 0 \quad \dots(2)$$

Where $[J_x]$ is load flow solution at solution point and

$$J_p = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & & 0 \\ \vdots & & \vdots & & \vdots \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, J_d = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \vdots \\ \alpha_{NB} \\ \alpha_1 \beta_1 \\ \alpha_2 \beta_2 \\ \alpha_3 \beta_3 \\ \vdots \\ \vdots \\ \alpha_{NB} \beta_{NB} \end{bmatrix}$$

Where α_i is the load participation factor of i^{th} bus and β_i is tangent of power factor angle at i^{th} bus. Now assume that system is stressed and very nearer to collapse point

where minimum eigen value of load flow Jacobian is very small in magnitude and further let us assume that η^T is corresponding left eigenvector. Premultiplying equation 2 by η^T one obtain

$$\eta^T [J_x] \begin{bmatrix} \Delta\delta \\ \Delta v \end{bmatrix} - \eta^T [J_p] \begin{bmatrix} \Delta P_g \\ \Delta Q_g \end{bmatrix} + \eta^T [J_d] \Delta P_d = 0 \quad \dots(3)$$

Since eigen value corresponding to eigenvector η^T is quite small the first term in (3) is neglected and relation (3) can be written as follows.

$$\Delta P_d = \frac{\eta^T [J_p]}{\eta^T [J_d]} \begin{bmatrix} \Delta P_g \\ \Delta Q_g \end{bmatrix} \quad \dots(4)$$

$$\Delta P_d = \begin{bmatrix} SP \\ SQ \end{bmatrix} \begin{bmatrix} \Delta P_g \\ \Delta Q_g \end{bmatrix} \quad \dots(5)$$

where,

$$\begin{bmatrix} SP \\ SQ \end{bmatrix} = \frac{\eta^T [J_p]}{\eta^T [J_d]} \quad \dots(6)$$

Since perturbation is very near to collapse point, ΔP_d is approximately change in loadability margin and each element of vector $[SP \ SQ]^T$ represents as sensitivity i.e.

$$\frac{\Delta P_d}{\Delta P_{gi}} = SP_i, \frac{\Delta P_d}{\Delta Q_{gi}} = SQ_i \quad \dots(7)$$

Closed form relations for SP_i and SQ_i are written from (6) and observing the structure of $[J_p]$ and $[J_d]$ as follows.

$$SP_i = \frac{\eta_i}{\delta}, i = 2, \dots, NB$$

$$SQ_i = \frac{\eta_i}{\delta}, i = NG + 1, \dots, NB$$

where δ is expressed as follows

$$\delta = \sum_{i=2}^{NB} \eta_i \alpha_i + \sum_{i=NG+1}^{NB} \eta_i \alpha_i \beta_i$$

In fact change in loadability margin with respect to changes in real and reactive power injections at buses can be calculated using equation (7).

III. DETERMINATION OF COMPENSATION INJECTIONS FOR LINE OUTAGE SIMULATION

It is assumed that outaged line is connected between buses k and m line outage is simulated with the help of four injections $\Delta P_k, \Delta Q_k, \Delta P_m$ and ΔQ_m as shown in Fig. 1, the compensation injections are obtained such that net real and reactive power flows in the line is equal to injection at respective end [11]. Following relations are written to evaluate the compensation injection using superposition.

$$\left. \begin{aligned} \Delta P_k &= f_{km}^0 + \Delta f_{km} \\ \Delta Q_k &= g_{km}^0 + \Delta g_{km} \\ \Delta P_m &= f_{mk}^0 + \Delta f_{mk} \\ \Delta Q_m &= g_{mk}^0 + \Delta g_{mk} \end{aligned} \right\} \dots(8)$$

Where f_{km}^0 and g_{km}^0 are pre-outage real and reactive power flows from k^{th} to m^{th} bus. f_{mk}^0 and g_{mk}^0 are power flows from m^{th} to k^{th} bus. Δf_{km} , Δg_{km} , Δf_{mk} and Δg_{mk} are real and reactive power flows at both ends due to compensation injections. Compensation injections are evaluated using incremental power flow equations. After some algebraic manipulation compensation injections are evaluated using

$$\begin{bmatrix} \Delta P_k \\ \Delta Q_k \\ \Delta P_m \\ \Delta Q_m \end{bmatrix} = \begin{bmatrix} C_{1-j} & C_{2-j} & C_{3-j} & C_{4-j} \\ C_{5-j} & C_{6-j} & C_{7-j} & C_{8-j} \\ C_{9-j} & C_{10-j} & C_{11-j} & C_{12-j} \\ C_{13-j} & C_{14-j} & C_{15-j} & C_{16-j} \end{bmatrix} \begin{bmatrix} f_{km}^0 \\ g_{km}^0 \\ f_{mk}^0 \\ g_{mk}^0 \end{bmatrix} \dots (9)$$

Where $C_{1-j} - C_{16-j}$ are constants obtained using base case load flow Jacobian and partial derivatives of real and reactive power flow equations for outaged j^{th} line with respect to bus voltages V_k , V_m and phase angle difference $(\delta_k - \delta_m)$ [11].

IV. COMPUTATIONAL ALGORITHM

Change in loadability margin is calculated using the sensitivity relation (7) and compensation injections (9). The advantage of line outage simulation using compensation injections is that, the topology of the network remains same. Hence, base case sensitivities are applicable for all line outage simulation. Change in loadability margin is evaluated as follows

$$\Delta P_{dj} = SP_k \Delta P_k + SQ_k \Delta Q_k + SP_m \Delta P_m + SQ_m \Delta Q_m \dots(10)$$

Where ΔP_{dj} is the change in loadability margin for the outage of j^{th} line is connected between k^{th} and m^{th} buses. The computational sequence is given in following steps :

Step-1: Obtain load flow solution near to collapse point.

Step-2: Determine left eigen vector η^T corresponding to minimum eigen value λ_{\min} which is very small in magnitude.

Step-3: Obtain sensitivities SP_i and SQ_i for all buses as given in eqns. (6) & (7).

Step-4: Select a contingent line $j = 1$.

Step-5: Obtain compensation injection as given in equation (9).

Step-6: Obtain change in loadability margin ΔP_{dj} using equation (10).

Step-7: Select $j = j + 1$

Step-8: if $j > NL$, go to step 9, otherwise repeat from Step-5.

Step-9: Arrange all changes in loadability margins in descending order of magnitudes and prepare a list of line outages according to severity.

Step-10: Identify the line outage, which gives loadability margin less than a specified value (say 15% of the current operating points). These lines outages are credible line outage and may require pre or post outage corrective rescheduling.

The accuracy of the methodology can be verified with the help of continuation power flow program under each line outage case.

V. RESULTS AND DISCUSSIONS

The developed algorithm for voltage stability margin evaluation has been implemented on a IEEE 25-bus standard test system [16]. Total base case real load is 6.38pu and reactive load is 2.28 pu. The system consists of 35 lines. Only non islanding branch contingencies are studied. Typically not all branches are considered in contingency analysis due to enormous number of individual cases, rather only a few areas of larger power system network are considered. Given the base case network with a branch removed, continuation power flow method is applied to calculate the exact bifurcation point by tracing a new solution curve for each contingency. This repeated continuation power flow approach is time consuming. However the exact bifurcation location information is necessary for comparison purpose.

Table-1 reveals the loadability margin for line outage using newly developed sensitivity relations and is compared with continuation power flow method for 25-bus system. Based on loadability margin, ranking of each line outage is performed as indicated in Table-2. This table shows that line number 2 is most vulnerable branch whose outage results inadequate loadability margin. Loadability margin is the difference between present loadability (Pd) and maximum loading under line outage condition. Results obtained using developed algorithm completely in agreement with those obtained using continuation power flow program. Fig. 2 displays bar chart for loadability margin after each line outage.

Table 1: Loadability margin under line outage condition for 25- bus system.

Outaged Line No.	Loadability margin (pu) obtained using sensitivity technique	Loadability margin obtained using continuation power flow (CPF)
1	1.104	1.182
2	-0.090	-0.126
3	0.917	0.913
4	0.954	0.928
5	0.862	0.855
6	0.249	0.263
7	1.129	1.245
8	1.243	1.263
9	1.123	1.262
10	1.000	1.012
11	1.190	1.166
12	0.325	0.327
13	0.829	0.887
14	1.191	1.165
15	1.304	1.292
16	1.206	1.241
17	1.198	1.212
18	1.204	1.168
19	1.239	1.290
20	1.301	1.288
21	1.306	1.281
22	1.284	1.293
23	1.279	1.295
24	1.285	1.290
25	1.290	1.288
26	1.284	1.294
27	1.274	1.280
28	0.485	0.467
29	1.280	1.261
30	1.300	1.304
31	1.231	1.220
32	1.010	1.006
33	1.067	1.126
34	1.164	1.172
35	1.092	1.081

Table 2: Line outages ranking and detection of vulnerable line outages for 25-bus system.

Rank	Ranking using developed algorithm	Ranking with CPF	Comment
1	2	2	Most vulnerable
2	6	6	Vulnerable
3	12	12	Vulnerable
4	28	28	
5	13	5	
6	5	13	
7	3	3	
8	4	4	
9	10	32	
10	32	10	
11	33	35	
12	35	33	
13	1	14	
14	34	11	

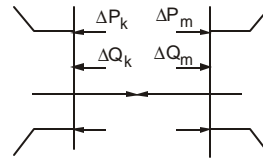


Fig. 1 Line outage simulation

Loadability margin (pu) obtained using sensitivity technique
 Loadability margin (pu) obtained using continuation power flow (CPF)

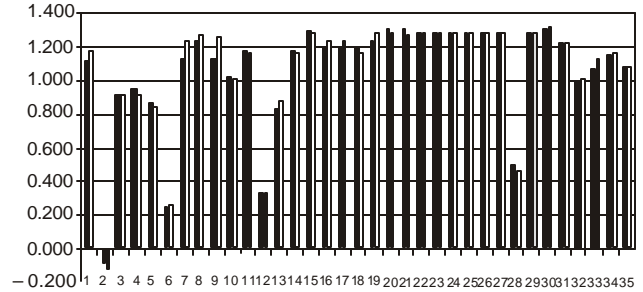


Fig. 2. Comparison of Loadability Margin for 25-bus system.

VI. CONCLUSIONS

A computationally efficient algorithm has been developed for evaluating loadability margin. This has been achieved with the help of derived sensitivity relation and compensation injections. The advantage of line outage simulation using compensation injections is that same pre outaged sensitivities can be used for all line outages, since topology of power network does not change. The evaluated loadability margin has been used for identifying vulnerable line outage. Developed algorithm has been implemented on standard test system. Results obtained have been compared with those obtained using continuation power flow.

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