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Finite Element Analysis of a Beam with Piezoelectrics Using Third order theory—Part I Static Analysis-Shape Control

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ABSTRACT : The use of discrete piezoelectric patches in Smart structures for controlling the shape has been of considerable interest in recent years. Such systems of one or more dimensions can nowadays be fully modeled with the help of finite element codes. The shape control of a smart structure using surface bonded piezoelectric patches acting as sensors and actuators is examined in this work. The finite element model based on Reddy's third order laminate theory has been developed for a beam having both ends fixed. The simulation results show that both the number and location of piezoelectric patches play an important role in shape control of the structure. The shape control of the beam improves as the number of sensor/actuator pairs. The location of the sensors/actuators on the control system is more critical than their number. The sensors/actuators pairs are much more effective in shape control when placed near the regions of highest strains.

Keywords : Smart Structure, Piezoelectrics, Fixed ends beam, Reddu's third order theory, Shape control.

I. INTRODUCTION

A smart structure can be defined as a structure with bonded or embedded sensors and actuators with an associated control system, which enables the structure to respond simultaneously to external stimuli exerted on it and then suppress undesired effects. Smart structures have found application in monitoring and controlling the deformation of structures in a variety of engineering systems, such as aerospace, automobiles, bridges and precision machining etc. Advances in smart materials technology have produced much smaller actuators and sensors with high integrity in structures and an increase in the application of smart materials for passive and active structural damping. Some typical applications are aero-elastic control of aircraft lifting components, health monitoring of bridges and shape control of large space trusses.

Several investigators have developed analytical and numerical, linear and non-linear models for the response of integrated piezoelectric structures. These models provide platform for exploring the shape and active vibration control in smart structures. The experimental work of Bailey and Hubbard, 1985 [1] is usually cited as the first application of piezoelectric materials as actuators. They successfully used piezoelectric sensors and actuators in the vibration control of isotropic cantilever beams. Crawley and de Luis, 1987 [2] formulated static and dynamic analytical models for extension and bending in beams with attached and embedded piezoelectric actuators. Heyliger and Reddy, 1988 [3] developed a finite element model for bending and vibration problems using third order shear deformation theory. Ha, Keilers and Chang, 1992 [4] developed a model based on the classical laminated plate theory for the dynamic and static response of laminated composites with distributed piezoelectrics. Chandrashekhara and Varadarajan, 1997 [5] gave a finite element model based on higher order shear deformation theory for laminated composite beams with integrated piezoelectric actuators. Valoor *et al.*, 2000 [6] used neural network-based control system for vibration control of laminated plates with piezoelectrics. Lee and Reddy, 2004 [7] used the third-order shear deformation theory to control static and dynamic deflections of laminated composite plates. Prasad *et al.*, 2005 [8] developed a criterion for the evaluation and selection of piezoelectric materials and actuator configurations.

The Euler-Bernoulli classical theory used to model the beam/plate deformation neglects the transverse shear deformation effects. The shear deformation theory has a disadvantage as it needs a shear correction factor, which is very difficult to determine especially for arbitrarily laminated composite structures with piezoelectric layers. To overcome the above mentioned drawbacks, Reddy, 1984 [9] developed a third order laminate theory, which takes into account the quadratic variation of transverse shear strains, eliminates the transverse shear stresses on the top and bottom of a laminated composite structure and requires no shear correction factor. Since the accuracy of the models developed depends on the perfection of the mechanical interaction between the piezoelectrics and the underlying structure, Reddy's third order theory has been used to develop the model.

II. PIEZOELECTRIC EQUATIONS

Assuming that a beam consists of a number of layers (including the piezoelectric layers) and each layer possesses a plane of material symmetrically parallel to the x-y plane and a linear piezoelectric coupling between the elastic field and the electric field the constitutive equations for the layer can be written as,

$$\begin{Bmatrix} D_1 \\ D_3 \end{Bmatrix}_k = \begin{bmatrix} 0 & e_{15} \\ e_{31} & 0 \end{bmatrix}_k \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_5 \end{Bmatrix}_k + \begin{bmatrix} \bar{\varepsilon}_{11} & 0 \\ 0 & \bar{\varepsilon}_{33} \end{bmatrix}_k \begin{Bmatrix} E_1 \\ E_3 \end{Bmatrix}_k \quad \text{... (1)}$$

$$\begin{Bmatrix} \sigma_1 \\ \sigma_5 \end{Bmatrix}_k = \begin{bmatrix} Q_{11} & 0 \\ 0 & Q_{55} \end{bmatrix}_k \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_5 \end{Bmatrix}_k - \begin{bmatrix} 0 & e_{31} \\ e_{15} & 0 \end{bmatrix}_k \begin{Bmatrix} E_1 \\ E_3 \end{Bmatrix}_k \quad \text{... (2)}$$

The thermal effects are not considered in the analysis.

The piezoelectric constant matrix $[e]$ can be expressed in terms of the piezoelectric strain constant matrix $[d]$ as

$$[e] = [d][Q] \quad \text{... (3)}$$

where,

$$[d] = \begin{bmatrix} 0 & d_{15} \\ d_{31} & 0 \end{bmatrix} \quad \text{... (4)}$$

III. DISPLACEMENT FIELD OF THE THIRD ORDER THEORY

The displacement field based on the third order beam theory of Reddy [9] is given by

$$u(x,z,t) = u_0(x,t) + z\phi_x(x,t) - \alpha z^3\left(\phi_x + \frac{\partial w_0}{\partial x}\right) \quad \text{... (5)}$$

$$w(x,\ z,\ t) = w_0(x,\ t) \quad \text{... (6)}$$

where $\alpha = \frac{4}{3t^2}$ and t is the total thickness of the beam.

The displacement functions are approximated over each finite element by

$$u_0(x,t) = \sum_{i=1}^{2} u_i(t)\psi_i(x) \quad \text{... (7)}$$

$$\phi_x(x,t) = \sum_{i=1}^{2} \phi_i(t)\psi_i(x) \quad \text{... (8)}$$

$$w_0(x,t) = \sum_{i=1}^{4} \Delta_i(t)\varphi_i(x) \quad \text{... (9)}$$

Using finite element formulation equations (5) and (6) can be expressed as,

$$\{u\} = [N]\{\bar{u}\}$$

$$\{\bar{u}\} = \{u_1 \quad \phi_1 \quad \Delta_1 \quad \Delta_2 \quad u_2 \quad \phi_2 \quad \Delta_3 \quad \Delta_4\}^T \quad \text{... (10)}$$

where,

$$[N] = \begin{bmatrix} \psi_1 & 0 \\ (z-\alpha z^3)\psi_1 & 0 \\ -\alpha z^3 \frac{\partial \varphi_1}{\partial x} & \varphi_1 \\ -\alpha z^3 \frac{\partial \varphi_2}{\partial x} & \varphi_2 \\ \psi_2 & 0 \\ (z-\alpha z^3)\psi_2 & 0 \\ -\alpha z^3 \frac{\partial \varphi_3}{\partial x} & \varphi_3 \\ -\alpha z^3 \frac{\partial \varphi_4}{\partial x} & \varphi_4 \end{bmatrix} \quad \text{... (11)}$$

NOMENCLATURE

$[d]$	Piezoelectric strain constant matrix
D	Electric Displacement field
$[e]$	Piezoelectric constant matrix
e	Piezo electric constant
E	Electric field; Young's Modulus of elasticity
$\{F_v\}$	Global electrical force vector
$\{F\}$	Global external mechanical force
G_c	Gain of the current amplifier vector
$[G]$	Control gain matrix
G_i	Gain to provide feedback control
$[K]$	Global Stiffness matrix
$[K^e]$	Elemental Stiffness matrix
$[M]$	Global mass matrix
$[M^e]$	Elemental mass matrix
N_i	Shape function the i^{th} element
Q	General Stiffness of the material
S_i	Strain energy of the j^{th} element
t	Total thickness of the beam
u, v, w	Displacements of a point along x, y and z directions respectively
V	Applied voltage to Piezo actuator
V_S	Open circuit sensor voltage
u_0, w_0	Displacement of a point on the mid-plane along the x and z direction respectively
$\{\bar{u}\}$	Nodal displacement vector
$\{\ddot{\bar{u}}\}$	Nodal acceleration vector
$\{x\}$	Generalized displacements
x, y, z	Cartesian coordinates
ϕ_x	Bending rotation of x-axis
ε_i	Strain of i^{th} element in strain tensor
$\bar{\varepsilon}$	Absolute permittivity of the dielectric
$\{\sigma\}$	stress vector
$\{\varepsilon\}$	strain vector
ϕ	Rotation of the transverse normal about y-axis
φ	Cubic Hermit interpolation polynomial
Δ_1, Δ_3	Nodal values of ω_0
Δ_2, Δ_4	Nodal values of $\frac{\partial \omega_0}{\partial x}$
ψ	Linear Lagrangian interpolation polynomial

The strain-displacement relations are given by

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_5 \end{Bmatrix} = [B]\{\bar{u}\} \quad \text{... (12)}$$

where,

$$\{B\} = \begin{bmatrix} \frac{\partial\psi_1}{\partial x} & 0 \\ (z-\alpha z^3)\frac{\partial\psi_1}{\partial x} & (1-3\alpha z^2)\psi_1 \\ -\alpha z^3\frac{\partial^2\varphi_1}{\partial x^2} & (1-3\alpha z^2)\frac{\partial\varphi_1}{\partial x} \\ -\alpha z^3\frac{\partial^2\varphi_2}{\partial x^2} & (1-3\alpha z^2)\frac{\partial\varphi_2}{\partial x} \\ \frac{\partial\psi_2}{\partial x} & 0 \\ (z-\alpha z^3)\frac{\partial\psi_2}{\partial x} & (1-3\alpha z^2)\psi_2 \\ -\alpha z^3\frac{\partial^2\varphi_3}{\partial x^2} & (1-3\alpha z^2)\frac{\partial\varphi_3}{\partial x} \\ -\alpha z^3\frac{\partial^2\varphi_4}{\partial x^2} & (1-3\alpha z^2)\frac{\partial\varphi_4}{\partial x} \end{bmatrix}$$

IV. EQUATIONS OF MOTION

The dynamic equations of the piezoelectric structure are derived using Hamilton's principle. To develop the equation of motion of the system, we consider the dynamic behavior of the system and drive the associated dynamic equations using Hamilton's principle. These equations also provide coupling between electrical and mechanical terms. The electric force due to the applied charge of the actuator is not considered in the analysis. The equation of motion in the matrix form can be written as,

$$[M^e]\{\ddot{\bar{u}}\}^e + [K^e]\{\bar{u}\}^e = \{F\}^e + [K^e_{uv}]V^e \quad \text{... (13)}$$

where,

$$[M^e] = \int_{V_e} \rho[N]^T[N]dV \quad \text{... (14)}$$

$$[K^e] = \int_{V_e} [B][Q][B]dV \quad \text{... (15)}$$

$$[K^e_{uv}] = \int_{V_e} [B]^T[e]^T[B_v]dV \quad \text{... (16)}$$

Assembling all the elemental equations gives the global dynamic equation,

$$[M]\{\ddot{\bar{u}}\} + [K]\{\bar{u}\} = \{F\} + \{F_V\} \quad \text{... (17)}$$

where,

$$\{F\} = [K_{uv}]\{V\} \quad \text{... (18)}$$

V. SENSOR EQUATIONS

Since no external electric field is applied to the sensor layer and as charge is collected only in the thickness direction, only the electric displacement is of interest and can be written as

$$D_3 = e_{31}\varepsilon_1 \quad \text{... (19)}$$

Assuming that the sensor patch covers several elements, the total charge the total charge developed on the sensor surface is

$$q(t) = \sum_{j=1}^{N_s}\frac{1}{2}\left[\int_{S_j}\left([B_1]_{(z=z_k)} + [B_1]_{(z=z_{k+1})}\right)e_{31}dS\{\bar{u}_j\}\right] \quad \text{.. (20)}$$

where $[B_1]$ is the first row of $[B]$

The distributed sensor generates a voltage when the structure is oscillating; and this signal is fed back into the distributed actuator using a control algorithm, as shown in Fig. 1. The actuating voltage under a constant gain control algorithm can be expressed as,

$$V^e = G_i V_s = G_i G_c \frac{dq}{dt} \quad \text{... (21)}$$

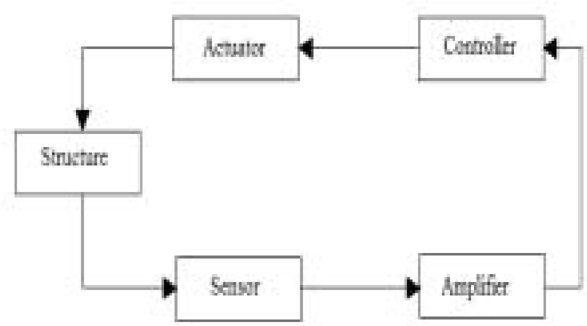

Fig. 1. Block Diagram of Feedback Control System.

The system actuating voltages can be written as

$$\{V\} = [G][K_v]\{\dot{\bar{u}}\} \quad \text{... (22)}$$

where $[G]$ is the control gain matrix and G = G_iG_c.

In the feedback control, the electrical force vector $\{F_v\}$ can be regarded as a feedback force. Substituting equation (22) into equation (18) gives

$$[F_v] = [K_{uv}][G][K_v]\{\dot{\bar{u}}\} \quad \text{... (23)}$$

VI. BEAM WITH SURFACE BONDED PIEZOELECTRICS

A beam having both ends fixed with both the upper and lower surfaces bonded by piezoelectric ceramics is shown in Fig. 2. The beam is made of T300/976 Graphite/ Epoxy composites and the Piezoceramic is PZT G1195N. The adhesive layers are considered to be of Isotac. The material properties are given in Table 1. The total thickness of the beam is 10 mm and the thickness of each Piezoceramic and adhesive layers are 0.2 mm and 0.1 mm respectively. The lower Piezoceramics serve as sensors and the upper ones as actuators. The relative sensors and actuators form sensor/ actuator (S/A) pairs through closed control loops.

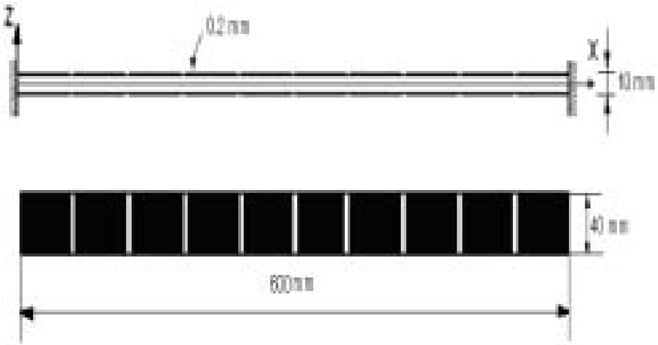

Fig. 2. Fixed ends beam with ten pairs of surface bonded piezoelectric sensors and actuators.

The beam as shown in Fig. 2, is subjected to a steady concentrated force of 4N at the middle. In the analysis, the beam is divided evenly into 40 elements. In the case of shape control, all the Piezoceramics on the upper and lower surfaces of the beam are used as actuators. Equal-amplitude voltages with an opposite sign are applied to the upper and lower piezoelectric layers respectively to control the deformation of the composite beam. Due to the converse piezoelectric effect, the distributed piezoelectric actuators contract or expand depending on negative or positive active voltage. In general, for an upward displacement, the upper actuators need a negative voltage and the lower actuators need a positive one.

The calculated centerline deflections of the beam with different arrangements of actuator pairs and different active voltages are shown in Figs. 3-5. From the figures it can be seen that a lower voltage is needed to eliminate the deflection caused by the external load when more actuators are used. It is shown in Fig. 3, that the beam cannot be smoothly flattened when it is fully covered actuators. When two pairs of actuators at each end are used, as shown in Fig. 4, a very high active voltage is needed to quell the deformation and the beam is also not smoothly flattened. In Fig. 5, it is shown that, under a certain active voltage, the beam can be flattened quite smoothly by two pairs of actuators located at the middle of the beam. This fact indicates that, under some conditions, it is not appropriate to cover structures entirely with piezoelectric materials from the view of efficiency and economy.

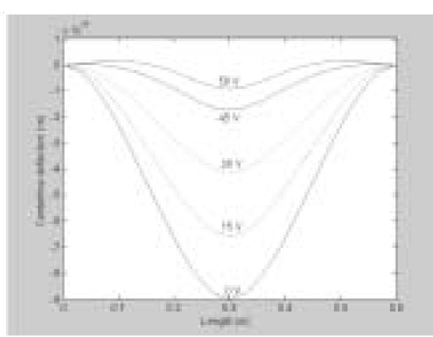

Fig. 3. The centerline deflection of the beam with ten pairs of actuators evenly distributed.

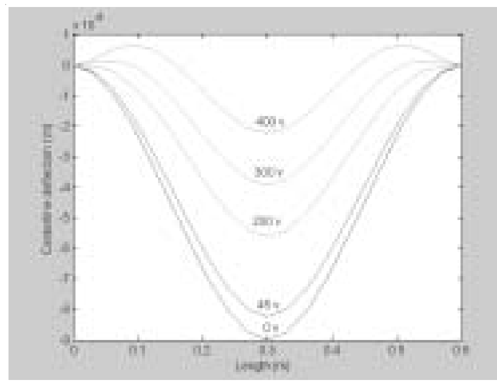

Fig. 4. The centerline deflection of the beam with two pairs of actuators located at the fixed ends.

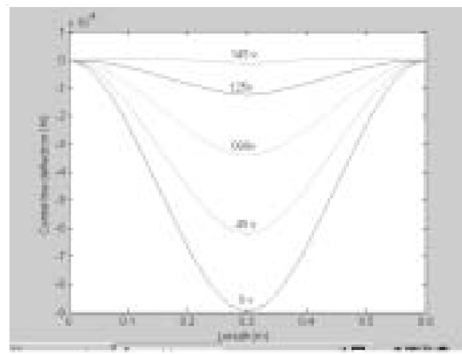

Fig. 5. The centerline deflection of the beam with two pairs of actuators located at the middle span.

VII. CONCLUSION

A finite element model and computer codes (in Matlab), based on the third order laminate theory, are developed for a beam having both ends fixed with distributed piezoelectric ceramics. The shape control of the beam is investigated by using the model. The investigation shows that the number and location of the sensor/actuator play a very important role in the shape control of the beam. The positions of sensor/actuator have a critical influence on the shape control of smart structures. For maximum effectiveness, the sensor/actuator pairs must be placed in high strain regions and away from areas of low strains. The number of sensor/actuator has a great effect on the performance of smart structures. An increase in the number of sensor/actuator gives a better performance in shape control. However the effect of number of sensor/actuator is not as critical as their location.

Table 1. Material properties PZT G1195N Piezoceramic and T300/976 Graphite/Epoxy composites and Adhesive layer.

	PZT	T30/976	Isotac
Young's moduli (GPa)			1.1
E_{11}	63.0	150.0	
$E_{22} = E_{33}$	63.0	9.0	
Shear moduli (GPa)			
$G_{12} = G_{13}$	24.2	7.10	
G_{23}	24.2	2.50	
Density, ρ (kg/m3)	7600	1600	890
Piezoelectric constants (m/V)			
$d_{11} = d_{22}$	254×10^{-12}		
Electrical permittivity (F/m)			
$\varepsilon_{11} = \varepsilon_{22}$	15.3×10^{-9}		
ε_{33}	15.0×10^{-9}		
First mode damping coefficient, ξ	---	0.009	

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