Assessment of THD in Induction Motor due to Unbalanced Supply

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ABSTRACT: Degradation in performance of a 3-phase induction motor has been a major concern in recent times due to unbalance in the power supply. This unbalance in supply has given rise to various infiltrates, percolating into the system thus causing disturbance in the input side of the workhorses of industry. This paper presents assessment of total harmonic distortion in stator current of an induction motor in presence of unbalance.

Index Terms: Fast Fourier Transform, d-q plane, Total Harmonic Distortion, Three Phase Squirrel Cage Induction Motor, Unbalance.

I. INTRODUCTION

The present deregulated industry has led to the introduction of power quality problem, the harmful effects of which is quite damaging in the long run and has become one of the major concerns in recent years [1], [4]. Voltage unbalance is regarded as a power quality problem of significant effect at the electricity distribution level [5], [2]. The level of current unbalance, that is present, is several times the level of voltage unbalance. Such an unbalance in the line currents can lead to excessive losses in the stator and rotor, which may cause protection systems to operate causing loss of production. Squirrel cage Ac induction motors are seemed to experience unbalance in supply voltages and currents due to the presence of various integer and sub-synchronous inter-harmonic components [3], [9], [8] present in the system. This paper presents a technique which justifies the occurrence of unbalance due to the presence of these infiltrates in power system. This analysis has been verified for data gathered via laboratory experimentation on a 1HP squirrel cage induction motor and Total Harmonic Distortions has been calculated in due course using Pattern Recognition Technique in Park plane. However, Fast Fourier Transform has been used for identification of the harmonic components in the real signal obtained [2], [6].

II. FAST FOURIER TRANSFORM

Fourier analysis, when applied to a continuous, period signal in the time domain, yields a series of discrete frequency components in the frequency domain [6]. By allowing the integration period to extend to infinity, the spacing between the harmonic frequencies, \( \omega \), tends to zero and the Fourier coefficients, \( c_n \), of the equation:-

\[ c_n = \frac{1}{\pi} \int_{-\pi}^{\pi} x(\omega t) e^{-jn\omega t} d(\omega t) \] …(1)

becomes a continuous function, such that:

\[ x(f) = \int_{-\infty}^{\infty} X(f)e^{j2\pi ft} \, df \] …(2)

X(f) is known as the spectral density function of x(t). Equations (1) and (2) form the Fourier transforms pair, whereas equation (2) is called forward transform.

The Limitations of Fourier Transform is given below [6], [9]:-

1. The function f(t) is single valued and continuous in well defined time interval T.
2. There is no discontinuity in time T, if there is any discontinuity, and then it will be finite.
3. Number of maxima in time T is finite.

Hence switching to Discrete Fourier Transform (DFT) is necessary where the continuous data is computed discretely. Yet DFT suffers from a serious disadvantage of large computational time due to large value of N (no. of samples). Therefore switching to Fast Fourier Transform is necessary where the value of N is usually taken to be 1024=210.

III. CLARKE AND PARK TRANSFORMATION

Park Transformation is used in high performance architecture in three phase power system analysis, wherein, phase currents and voltages are expressed in terms of current and voltage space vectors [6], [7]. These space vectors are represented in stationary reference frame. The space vectors are expressed utilizing the two axis theory, wherein the real part of the vector is equal to instantaneous value of the direct axis current component, \( i_d \), and imaginary part is equal to the quadrature axis current component \( i_q \).
In symmetrical 3-phase machines, the direct and quadrature axis currents $i_a$ and $i_b$ are fictitious quadrature-phase (2-phase) current components, which are related to the actual 3-phase currents as

$$i_a = k\left(i_x - \frac{1}{2}i_y - \frac{1}{2}i_z\right)$$

$$i_b = k\frac{\sqrt{3}}{2}(i_y - i_z)$$

...(3)

...(4)

In matrix form, the stator current in the stationary reference frame in terms of three phase currents can be written as

$$\begin{bmatrix} i_a \\ i_b \end{bmatrix} = \frac{k}{\sqrt{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} i_x \\ i_y \\ i_z \end{bmatrix}$$

...(5)

If the three phase system is symmetrical

$$i_a + i_y + i_z = 0$$

...(6)

Then,

$$i_a = k\left(i_x - \frac{1}{2}i_y - \frac{1}{2}i_z\right)$$

...(7)

The recommended value of ‘k’ is

$$k = \frac{2}{3}$$

...(8)

Thus,

$$\begin{bmatrix} i_a \\ i_b \end{bmatrix} = Clarke Matrix \times \begin{bmatrix} i_x \\ i_y \\ i_z \end{bmatrix}$$

...(9)

where,

$$Clarke Matrix = \frac{2}{3} \times \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$$

...(10)

Now, during conversion from ($\alpha,\beta$) stationary reference frame to (d,q) rotating reference frame, the angle (\theta) between the two frames must be known.

$$\sin \theta = \frac{\psi_d}{\psi_q}$$

...(11)

$$\cos \theta = \frac{\psi_q}{\psi_d}$$

...(12)

Thus (d,q) rotating reference frame can be represented as

$$\begin{bmatrix} d \\ q \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \times \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

...(13)

where,

$$Park Matrix = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$$

...(14)

Thus the transformation between R-Y-B to d-q rotating reference frame can be accomplished by the following mathematical equation

$$\begin{bmatrix} d \\ q \end{bmatrix} = \text{Park Matrix} \times \text{Clarke Matrix} \times \begin{bmatrix} R \\ Y \end{bmatrix}$$

...(15)

IV. RESULTS AND ANALYSIS

The work has been started by laboratory experimentation incorporating a 1HP, 415V, 0.75KW, 3 Phase Squirrel Cage Induction Motor. The picture of the setup has been shown in Figure 1.

![Fig. 1. Picture of experimental setup.](image-url)

The variac (s) has been firstly set to supply balanced voltage to the three phases of the induction motor. In each phase current transformers of 10/1A rating has been used for instantaneous current measurement. The instantaneous current spectrum for the three phases, for 1 sec, has been recorded in Digital Storage Oscilloscope (DSO), which is shown in Figures 2-4. Thereafter, the variac (s) has been set to provide unbalanced voltages to the three phases of the induction motor, whose current spectrums have been recorded in DSO, as shown in Figures 5-7. Both the healthy and unbalanced current spectrums have been analyzed using Fast Fourier Transform to determine the frequency contents of the signal, as shown in Figures 8-13. The frequencies above 50Hz has only been considered for the analysis here. Total Harmonic Distortion has been calculated for the Fast Fourier Transform coefficients in case of both healthy and unbalanced conditions. Then both the healthy and unbalanced current spectrums have been converted to Park Rotating Reference Frame wherein the Total Harmonic Distortion (THD) has been calculated for each of the cases concerned.
Fig. 2. Current spectrum of healthy R-phase of Induction motor

Fig. 3. Current spectrum of healthy Y-phase of Induction motor

Fig. 4. Current spectrum of healthy B-phase of Induction motor.

Fig. 5. Current spectrum of unbalanced R-phase of Induction motor.

Fig. 6. Current spectrum of unbalanced Y-phase of Induction motor.

Fig. 7. Current spectrum of unbalanced B-phase of Induction motor.

Fig. 8. FFT spectrum of healthy R-phase of Induction motor.

Fig. 9. FFT spectrum of unbalanced R-phase of Induction motor.
Fig. 10. FFT spectrum of healthy Y-phase of Induction motor.

Fig. 11. FFT spectrum of unbalanced Y-phase of Induction motor.

Fig. 12. FFT spectrum of healthy B-phase of Induction motor.

Fig. 13. FFT spectrum of unbalanced B-phase of Induction motor.

Fig. 14. D-q spectrum of currents for healthy induction motor.

Fig. 15. D-q spectrum of currents for induction motor under unbalance.

The D-q spectrums for both healthy and unbalanced conditions of the induction motor have been provided in Figures 14-15. The FFT spectrums seems to have much incipient disturbances for both healthy and unbalanced conditions, which make us to conclude about the existence of sub-synchronous inter-harmonic components [3], [9] in a real signal. It is very clear from the Figures 14-15 that, at healthy condition the Dq axis spectrums show normal attributes, but for unbalanced condition, the phase difference between D and q frames becomes 180°.

Table 1. Comparative study between % THDs for R, Y, B phases.

<table>
<thead>
<tr>
<th></th>
<th>R</th>
<th></th>
<th>Y</th>
<th></th>
<th>B</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>healthy</td>
<td>33%</td>
<td>unbalance</td>
<td>healthy</td>
<td>6.66%</td>
<td>unbalance</td>
<td>healthy</td>
</tr>
<tr>
<td>1.42949</td>
<td>5</td>
<td>1.61440</td>
<td>4</td>
<td>1.2990</td>
<td>68</td>
<td>1.45595</td>
</tr>
</tbody>
</table>
Table 2. Comparative study between %THDs for D and q axes reference frames.

<table>
<thead>
<tr>
<th></th>
<th>D</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>healthy</td>
<td>25.51426</td>
<td>37.82556</td>
</tr>
<tr>
<td>unbalance</td>
<td>40.835588</td>
<td>40.1348</td>
</tr>
</tbody>
</table>

The above analysis shows that the percentage deviation of the THDs, in case of D axis, for healthy and unbalanced spectrums is -37.52%, and in case of Q axis, for healthy and unbalanced spectrums is -5.75%. Thus from the above analysis the THDs for different percentage unbalance of the phases can be calculated and a comparison can be made between healthy and unbalanced conditions, in due course, for the percentage unbalance provided by actual experimentation. Also the D axis spectrum shows much more deviation in terms of total harmonic distortion than the q axis spectrum for the voltage unbalance provided in this case. Thus it is seen from the above analysis that, for voltage unbalances, the D axis spectrum is much more affected than the q axis spectrum in terms of total harmonic distortion, which has been verified for different voltage unbalance conditions of this induction motor after data acquisition in laboratory.

V. CONCLUSION

This analysis shows a technique for finding the total harmonic distortion of the phases of an induction motor operating under unbalance. The analysis can be applied both for finding the FFT spectrum as well as the D-q spectrum of the healthy and unbalanced signals. This analysis also shows that for voltage magnitude unbalances, the D axis spectrum is much more affected than the q axis spectrum, in terms of percentage Total Harmonic Distortion. The future scope of this work will aspire for determination of rule sets for determining percentage unbalance in phases pertaining to percentage total harmonic distortions in an induction motor operating under unbalance to assist in condition monitoring to industrial workhorses.

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REFERENCES
