



## Analysis of Fluid Film Stiffness and Damping coefficient for A Circular Journal Bearing with Micropolar Fluid

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**ABSTRACT:** This paper presents the effect of additives in lubricants on the fluid film stiffness and damping coefficient of a 25mm radius journal bearing with clearance of 0.04mm. The effect of micropolarity is evaluated by introducing two non-dimensional parameters, the coupling number ( $N^2$ ) and characteristic length ( $L_m$ ).

**Keywords:** Fluid Film Stiffness, Damping coefficient, Circular Journal Bearing, Micropolar Fluid

### I. INTRODUCTION

The design of journal bearings is of considerable importance to the development of rotating machinery. Journal bearings are essential machine components for compressors, pumps, turbines, internal-combustion-engines, motors, generators, etc. In its most basic form a journal bearing consists of a rotating shaft (the journal) contained within a close fitting cylindrical casing (the bearing). Generally, but not always, the bearing is fixed in a housing. The journal and bearing surfaces are separated by a film of lubricant (liquid or gas) that is supplied to the clearance space between these two surfaces. The clearance space is generally quite small and has four major functions:

- To permit assembly of the journal and bearing.
- To provide space for the lubricant.
- To accommodate unavoidable thermal expansions, and
- To tolerate any shaft misalignment or deflection.

The fundamental purpose of a journal bearing is to provide radial support to a rotating shaft. Under load, the centers of the journal and the bearing are not coincident but are separated by a distance called the eccentricity. This eccentric arrangement establishes a converging-wedge geometry which in conjunction with the relative motion of the journal and the bearing permits a pressure to be developed by viscous effects within the thin film of lubricant and thus produces a load carrying capability. Hydrodynamic journal

bearings, also called self-acting bearings, depend entirely on the relative motion of the journal and the bearing to produce film pressure for load support.

Earlier studies have been carried out for journal bearings considering the behavior of lubricant as Newtonian fluids and neglecting the deformation of bearing shell [1,2] later on studies were conducted considering the deformation of the bearing shell (Elastohydrodynamic lubrication) [3,6].

Further the improvement in various properties of commercially available Lubricants by way of mixing different additives paved the ways for mathematical studies on Micropolar fluids. [4,5]. The additives are considered as suspended micro structure particles in the lubricant and in case of journal bearing where clearance space and film thickness are quite low the effect of these cannot be ignored.

### II. ANALYSIS

In this work studies are carried out on the performance characteristics of a rigid circular bearing with micropolar lubricant.

The Reynolds equation is used for the hydrodynamic flow field and the Non-Newtonian effect is introduced by deriving a generalized Reynolds equation. The transient motion trajectories of the rigid bearing with non Newtonian lubricants have been obtained using linearized equation of the disturbed motion of the journal.

The static performance characteristics in term of eccentricity ratio, attitude angle, minimum fluid film thickness, power loss, and load has been obtained by using a MATLAB program.

#### A. Modified Reynolds equation

The general form of the governing equations for the steady state motion of incompressible micropolar fluids, as given Eringen's theory are

$$\nabla \cdot (\rho \mathbf{V}) = 0 \quad (1)$$

$$-\nabla P + (\lambda_o + \mu + \mu_r) \nabla(\mathbf{V} \cdot \mathbf{V}) + (\mu + \mu_r) \Delta \mathbf{V} + 2\mu_r (\mathbf{V} \times \mathbf{W}) = 0 \quad (2)$$

$$(c_o + c_d - c_a) \nabla(\mathbf{V} \cdot \mathbf{W}) + (c_d + c_a) \Delta \mathbf{W} + 2\mu_r (\mathbf{V} + \mathbf{V} - 2\mathbf{W}) = 0 \quad (3)$$

The above equations are the equation of conservation of mass, conservation of linear momentum, and conservation of angular momentum, respectively, where  $\mathbf{V}$  is the velocity vector and  $\mathbf{W}$  is the microrotational velocity vector,  $p$  is the pressure  $\rho$  is the mass density,  $\mu$  and  $\lambda_o$  are dynamic Newtonian viscosity and the second Viscosity coefficient; respectively, while  $\mu_r$  represents the dynamic microrotation viscosity.  $c_o$ ,  $c_a$  and  $c_d$  are the coefficients of angular velocity. Now

$$\mathbf{V} = (v_x, v_y), \mathbf{W} = (w_x, w_y) \quad (4)$$

Since the height of the fluid film is very small compared to the bearing radius we can neglect the curvature of the fluid film, then Eqs. (2) and (3) are reduced to

$$(\mu + \mu_r) \frac{\partial^2 v_x}{\partial y^2} + 2\mu_r \frac{\partial w_z}{\partial y} - \frac{\partial P}{\partial x} = 0 \quad (5a)$$

$$(\mu + \mu_r) \frac{\partial^2 v_z}{\partial y^2} - 2\mu_r \frac{\partial w_x}{\partial y} - \frac{\partial P}{\partial z} = 0 \quad (5b)$$

$$(c_a + c_d) \frac{\partial^2 w_x}{\partial y^2} + 2\mu_r \frac{\partial v_z}{\partial y} - 4\mu_r w_x = 0 \quad (5c)$$

$$(c_a + c_d) \frac{\partial^2 w_z}{\partial y^2} - 2\mu_r \frac{\partial v_x}{\partial y} - 4\mu_r w_z = 0 \quad (5d)$$

The boundary conditions at bearing and journal surfaces are

$$V_{x/y=0} = U, V_{z/y=0} = W_{x/y=0} = W_{z/y=0} = 0 \quad (6)$$

$$V_{x/y=h} = V_{z/y=h} = W_{x/y=h} = W_{z/y=h} = 0 \quad (7)$$

When we solve Eqs. (5a)-(5d) with the above boundary conditions, the velocity components are obtained. Substituting the velocity components in the Eqn. (1), and integrating across the film results in the generalized form of the Reynolds equation for micropolar fluids, given below

$$\frac{\partial}{\partial x} \left[ \frac{\psi(N, \Lambda, h)}{\mu} \frac{\partial P}{\partial x} \right] + \frac{\partial}{\partial z} \left[ \frac{\psi(N, \Lambda, h)}{\mu} \frac{\partial P}{\partial z} \right] = 6U \frac{\partial h}{\partial x}$$

$$\psi(N, \Lambda, h) = h^2 + 12\Lambda^2 h - 6N\Lambda h^2 \coth\left(\frac{Nh}{2\Lambda}\right) \quad (8)$$

$$\text{Where } \Lambda = \left(\frac{c_a + c_d}{4\mu}\right)^{1/2}, N = \left(\frac{\mu_r}{\mu + \mu_r}\right)^{1/2} \quad (9)$$

In Eq. (8),  $h$  is the film thickness in the clearance space of the lobe with the journal in a state of steady state and is expressed as

$$h_i = \frac{1}{\delta} - X_j \cos \theta - Y_j \sin \theta + \left(\frac{1}{\delta} - 1\right) \cos(\theta - \theta_0^1) \quad (10)$$

$(X_j, Y_j)$  are the steady state journal center coordinates and  $\theta_0$  is the angle of lobe line of centers.  $\delta$  is the preload in the bearing, the ratio between minor Clearance when journal and bearing geometric centers are coincident to conventional radial clearance.

$N$  is a dimensionless parameter, the coupling-number, it relates the coupling of the linear and angular momentum equations. It can be shown that  $0 < N < 1$ . Large  $N$  means, the individuality of the substructure becomes significant. The parameter  $\Lambda$  represents the interaction between the micropolar fluid and the film-gap and is called the characteristic length. As  $\Lambda$  approaches zero the effect of microstructure becomes less important. When it vanishes, Eqn. (8) reduces to the classical form of the Reynolds equation for a Newtonian fluid.

Introducing the following dimensionless quantifies:

$$L_m = \frac{C}{A}, \bar{\theta} = \frac{x}{R}, \bar{h} = \frac{h}{C_m}, Z = \frac{z}{L}, P = \frac{PC_m^2}{\mu UR} \quad (11)$$

Eq. (8) can be written in non-dimensional form as

$$\frac{\partial}{\partial \bar{\theta}} \left[ \frac{\psi(N, \Lambda, \bar{h})}{\mu} \frac{\partial P}{\partial \bar{\theta}} \right] + \left(\frac{R}{L}\right) \frac{\partial}{\partial Z} \left[ \frac{\psi(N, \Lambda, \bar{h})}{\mu} \frac{\partial P}{\partial Z} \right] = 6U \frac{\partial \bar{h}}{\partial \bar{\theta}}$$

$$\psi(N, \Lambda, \bar{h}) = \bar{h}^3 + \frac{12\bar{h}}{L_m^2} - \frac{6N\bar{h}^2}{L_m} \coth\left(\frac{N\bar{h}L_m}{2}\right) \quad (12)$$

#### B. Load Carrying Capacity

The hydrodynamic dimensionless forces developed in the bearing can be evaluated by integrating the dimensionless fluid pressure over the entire bearing area:

$$\bar{F}_x = \int_0^{L/R} \int_0^{\theta_2} \bar{p} \cos(\theta + \phi) d\theta d\bar{z} \quad (13)$$

$$\bar{F}_z = \int_0^{L/R} \int_0^{\theta_2} \bar{p} \sin(\theta + \phi) d\theta d\bar{z} \quad (14)$$

Resultant load is given as:

$$\bar{F} = \sqrt{\bar{F}_x^2 + \bar{F}_z^2} \quad (15)$$

#### C. Stiffness Characteristics

The non-dimensional coefficients of stiffness are given as:

$$\begin{bmatrix} \bar{K}_{xx} & \bar{K}_{xy} \\ \bar{K}_{yx} & \bar{K}_{yy} \end{bmatrix} = \begin{bmatrix} -\frac{\delta \bar{F}_x}{\delta \bar{x}_j} & -\frac{\delta \bar{F}_x}{\delta \bar{y}_j} \\ -\frac{\delta \bar{F}_y}{\delta \bar{x}_j} & -\frac{\delta \bar{F}_y}{\delta \bar{y}_j} \end{bmatrix} \quad (16)$$

The first subscript denotes the direction of force and the second, the direction of displacement. For example  $\bar{K}_{xx} = -\frac{\delta \bar{F}_x}{\delta \bar{y}_j}$  corresponds to a stiffness produced by a fluid force in the  $x$  direction due to a journal static displacement in the  $y$  direction. By definition, this coefficient is evaluated by finding the forces along the  $x$ -axis at the equilibrium position and with the journal centre displaced along  $y$ -axis for velocities set to zero. The negative sign in the definition ensures that a positive magnitude stiffness coefficient corresponds to a restorative force. The coefficients  $K_{xx}$  and  $K_{yy}$  are known as the direct stiffness terms. While  $K_{xy}$  and  $K_{yx}$  are referred as cross-coupled.

D. Damping Characteristics

$$\begin{bmatrix} \bar{C}_{xx} & \bar{C}_{xy} \\ \bar{C}_{yx} & \bar{C}_{yy} \end{bmatrix} = \begin{bmatrix} -\frac{\delta \bar{F}_x}{\delta \dot{x}_j} & -\frac{\delta \bar{F}_x}{\delta \dot{y}_j} \\ -\frac{\delta \bar{F}_y}{\delta \dot{x}_j} & -\frac{\delta \bar{F}_y}{\delta \dot{y}_j} \end{bmatrix} \quad (17)$$

The first subscript denotes the direction of force and the second, the direction of velocity. For example  $\bar{C}_{xx} = -\frac{\delta \bar{F}_x}{\delta \dot{x}_j}$  corresponds to a damping produced

by a fluid force in the  $y$  direction due to a journal velocity in the  $x$  direction. By definition, this coefficient is evaluated by finding the forces along the  $x$ -axis at the equilibrium position and with the journal centre velocity along  $y$ -axis for displacement set to zero. The negative sign in the definition ensures that a positive magnitude damping coefficient corresponds to a restorative force. The coefficients  $\bar{C}_{xx}$  and  $\bar{C}_{yy}$  are known as the direct damping terms, while  $\bar{C}_{xy}$  and  $\bar{C}_{yx}$  are referred as cross-coupled damping.

III. RESULTS AND DISCUSSIONS

A. Dynamic Characteristic

The dynamic characteristics i.e. stiffness and damping coefficients, representing the stability of the journal are dependent on the steady state film pressure. This film pressure is depends upon the micropolar parameters  $L_m$ ,  $N^2$ , and eccentricity ratio. To study the lubricating effectiveness of fluids that exhibit micropolarity, a parametric study was performed to show their effects in the respective non-dimensional form of stiffness and damping coefficients.

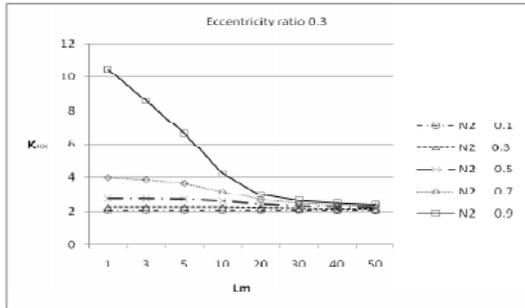


Fig. 1.

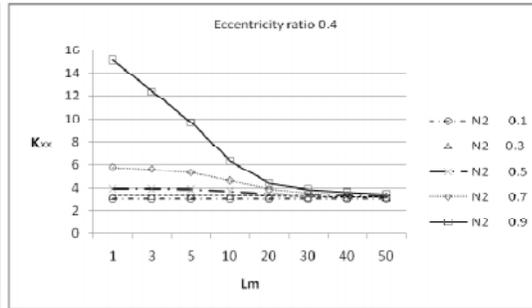


Fig. 2.

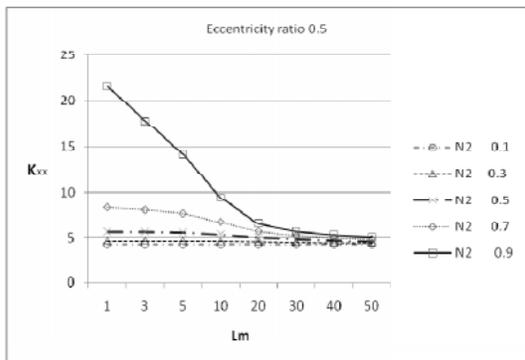


Fig. 3.

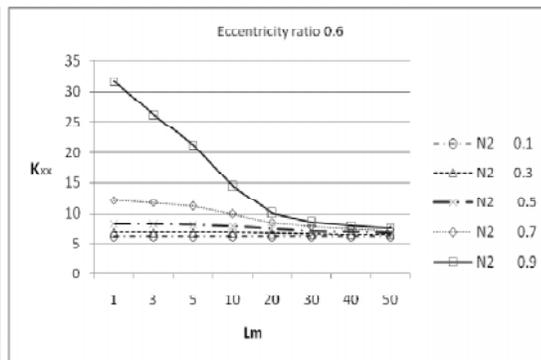


Fig. 4.

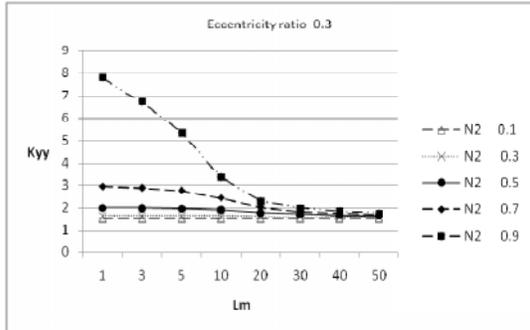


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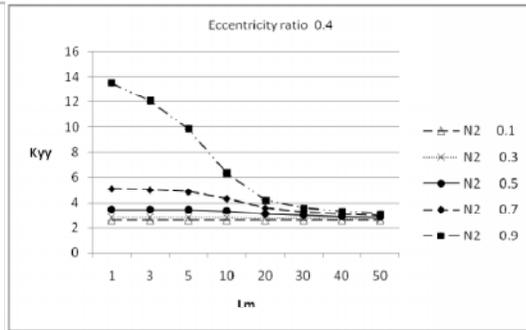


Fig. 6.

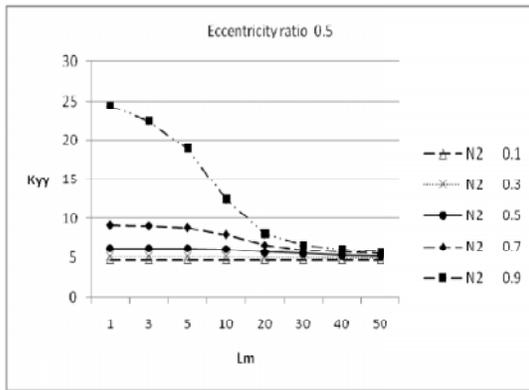


Fig. 7.

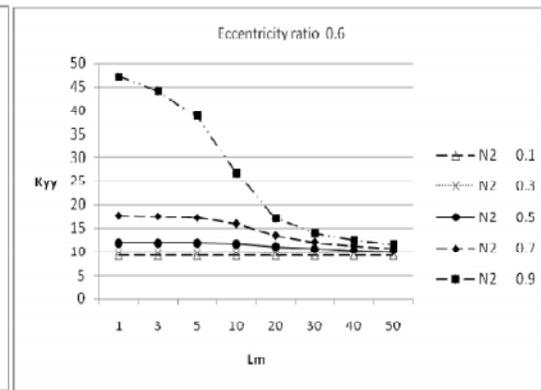


Fig. 8.

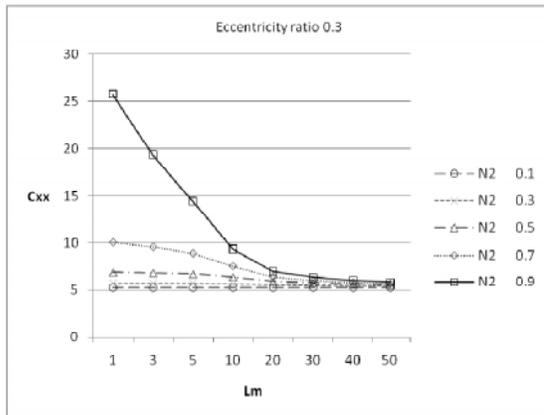


Fig. 9.

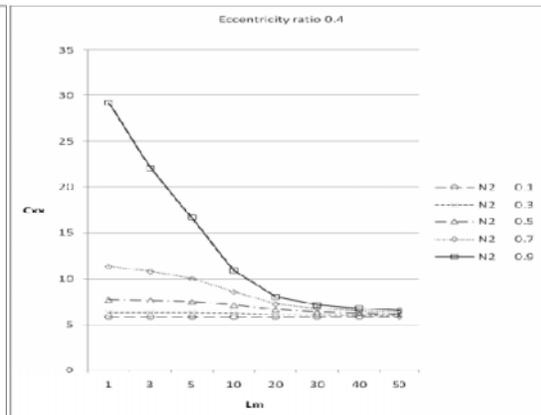


Fig. 10.

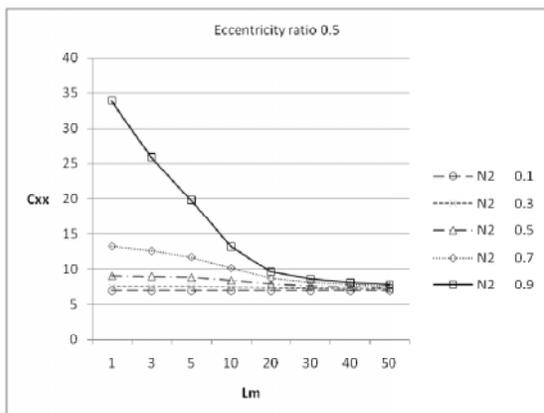


Fig. 11.

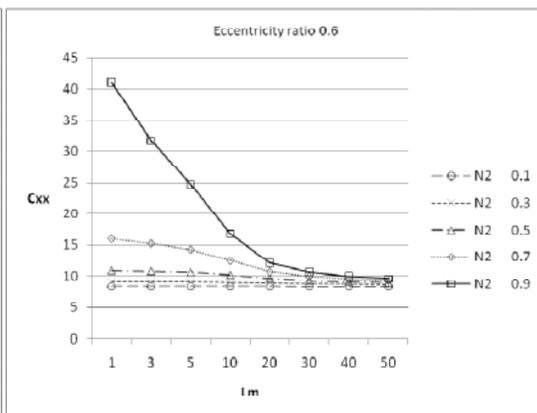


Fig. 12.

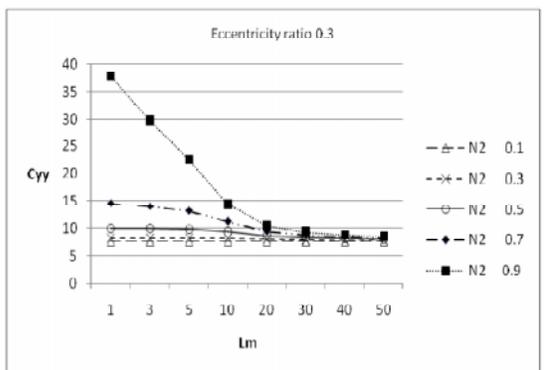


Fig. 13.

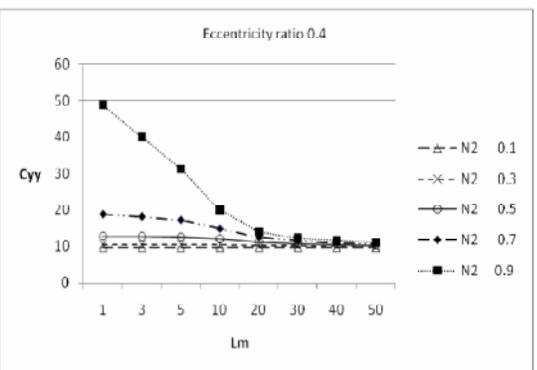


Fig. 14.

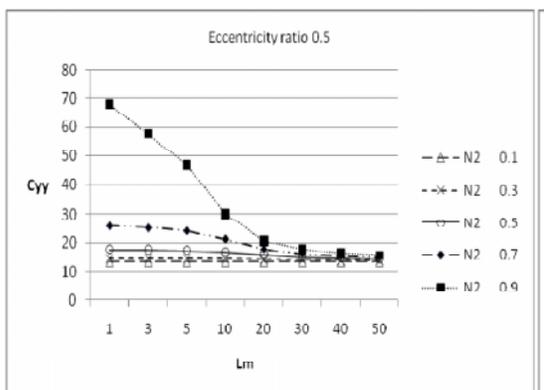


Fig. 15.

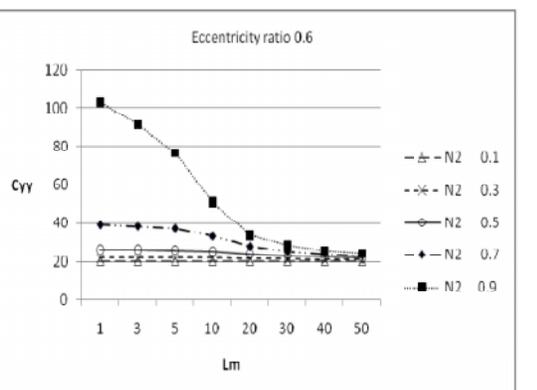


Fig. 16.

**IV. CONCLUSIONS**

The effect of micropolar lubricant on stiffness and damping coefficients of a rigid circular bearing has been done. The results have been obtained by writing

a program in MATLAB, considering the effect of coupling number and characteristics length.

It can be concluded that with micropolar fluids the stiffness and damping coefficients of a journal bearing increases considerably at higher coupling number and lower characteristic length.

**Nomenclature**

C conventional radial clearance (m)  
 $C_m$  minor clearance when journal and bearing geometric centers are coincident (m)  
 $C_o, C_a, C_d$  coefficient of angular viscosities  
 $F_f$  friction force (N)  
 $F_f$  - non-dimensional friction force ( $C/\mu$  URL)F  
F - friction coefficient  
h-film thickness (m)  
 $L_m$ -non-dimensional characteristic length of micropolar fluid,  $L_m = C/A$   
N -coupling number,  $N = (\mu_r/\mu + \mu_r)^{1/2}$   
O -bearing center  
 $O_j$ -journal center  
P -fluid film Pressure ( $N/m^2$ )  
P -non-dimensional fluid film pressure ( $C^2/\mu UR$ )P  
Q-side leakage flow ( $m^3/s$ )  
Q-non-dimensional side leakage flow ( $6L/CUR^2$ )Q  
R -journal radius (m)  
U -velocity of journal (m/s)  
 $v_x, v_y, v_z$  component of lubricant velocity in the x, y and z directions (m/s)  
X,Y,Z Cartesian axes with origin at bearing geometric center  
 $X_i, Y_i$  -coordinates of journal center  
W -resultant of load (N)  
W -non-dimensional load ( $C^2/\mu ULR^2$ )W  
 $W_x, W_y$  -non-dimensional load, components in X and Y direction  
 $w_x, w_y, w_z$  microrotation velocity components about the axes (1/s)  
 $\xi$  eccentricity ratio  
 $\theta$ -Angular coordinate measured from x-axis  
 $\theta_0^i$  -Angle of lobe line of centers  
 $\theta_1^i, \theta_2^i$ -Angles at leading and cavitating edge of the lobe  
 $\delta$ -Preload of the bearing ( $C'n/C$ )

$\phi$ -Attitude angle  
 $\mu$ -Viscosity of the Newtonian fluid ( $N\ s/m^2$ )  
 $\mu_r$  -microrotation viscosity ( $N\ s/m^2$ )  
 $\Lambda$ -Characteristic length of micropolar fluid (m),  $\Lambda = (c_a + c_d/4\mu)^{1/2}$   
 $\rho$ -Lubricant density ( $kg/m^3$ )  
 $\lambda$ -Aspect ratio, bearing length to diameter ratio  
 $\lambda_o$ -second viscosity coefficient ( $N\ s/m^2$ )  
 $\omega$ -Angular speed of the journal (1/s)  
I-subscript and superscript for lobe designation  
--Superscript for non-dimensional quantities

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