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Transient Thermoelastic Problem of a Thin Rectangular Plate

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ABSTRACT: In this paper, an attempt has been made to investigate three dimensional transient thermoelastic problem of a thin rectangular plate due to partially distributed heat supply to determine temperature distribution, displacement and thermal stresses with the known boundary and initial conditions. The solutions are obtained by applying the Marchi-Fasulo transform and the Laplace transform techniques. The results are obtained in series form in terms of Bessel's functions. Some numerical results for the temperature change, the displacement, and the stress distributions are shown in figures.

Keywords: Thin rectangular plate, three dimensional transient thermoelastic problem, integral transform.

I. INTRODUCTION

The primary objective of the paper is to present analytical approach for finding solution to the plane problems in terms of stresses. And further to gain an effective solution and a better understanding of thermal stresses in thin rectangular plate due to partially distributed heat supply. The direct problem of thermoelasticity in a rectangular plate under thermal shock was studied by Tanigawa and Komatsubara (1997) [10], Vihak *et al.*, (1998) [12] and Adams and Bert (1999) [1]. The inverse steady state thermoelastic problem of a thin rectangular plate was studied by Durge and Khobragade (2003) [2] to determine the temperature displacement function and thermal stresses at the boundary by the application of finite Fourier sine transform technique. The steady-state thermal stresses in a circular plate subjected to an ax symmetric temperature distribution on the upper face with zero temperature on the lower face and the circular edge respectively was determined by Nowacki (1957) [6]. Quasi-static thermal stresses in a thin circular plate was determined by Roychoudhary [1973] [8] subjected to transient temperature along the circumference of circular upper face with lower face is at zero temperature and the fixed circular edge thermally insulated. Transient thermoelastic-plastic bending problems of a circular plate has discussed by Ishihara, Noda and Tanigawa (1997) [4]. Jha *et al* [2012] [17] studied Heat Treatment for 16 MnCr5 Material. A method of direct integration for determination of stresses and displacements within the frame-work of elasticity and thermoelasticity problems in terms of stresses proposed by Vihak *et al.,* (1995, 1998, 1999, 2000) [13-16].
To the author's knowledge, work on three dimensional inverse transient thermoelastic problem of a thin rectangular plate with given third kind boundary conditions has not been yet reported.
In the present paper, an attempt has been made to determine the temperature, displacement and thermal stress at any point of a thin rectangular plate occupying the region $D=\{(x,y,z)\in R^3:\ -a\le x\le a,\ -b\le y\le b,\ 0\le z\le h\}$ with known boundary conditions. Here the Marchi-Fasulo transform and the Laplace transform techniques have been used to find the solution of the problem.

II. STATEMENT OF THE PROBLEM

Consider a thin rectangular plate occupying the space $D=\{(x,y,z)\in R^3:\ -a\le x\le a,\ -b\le y\le b,\ 0\le z\le h\}$. The displacement components u_x, u_y, u_z in the x, y and z direction respectively are in the integral form as

$$u_x=\int_{-a}^{a}\left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial y^2}+\frac{\partial^2 U}{\partial z^2}-v\frac{\partial^2 U}{\partial x^2}\right)+\alpha T\right]dx \tag{1}$$

$$u_y=\int_{-b}^{b}\left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial z^2}+\frac{\partial^2 U}{\partial x^2}-v\frac{\partial^2 U}{\partial y^2}\right)+\alpha T\right]dy \tag{2}$$

$$u_z=\int_{-c}^{c}\left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}-v\frac{\partial^2 U}{\partial z^2}\right)+\alpha T\right]dz \tag{3}$$

where E, ν and α are the Young modulus, the Poisson ratio and the linear coefficient of thermal expansion of the material of the plate respectively, $U(x, y, z, t)$ is the Airy stress function which satisfies the differential equation

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)^2 U(x, y, z, t) = -\alpha\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)^2 T(x, y, z, t) \tag{4}$$

Here $T(x, y, z, t)$ denotes the temperature of thin rectangular plate satisfying the following differential equation

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}=\frac{1}{k}\frac{\partial T}{\partial t} \tag{5}$$

and k is the thermal diffusivity of the material subject to initial conditions

$$T(x, y, z, t) = 0 \tag{6}$$

The boundary conditions are

$$\left[T(x, y, z, t)+k_1\frac{\partial T(x, y, z, t)}{\partial x}\right]_{x=a}=F_1(y, z, t) \tag{7}$$

$$\left[T(x, y, z, t)+k_2\frac{\partial T(x, y, z, t)}{\partial x}\right]_{x=-a}=F_2(y, z, t) \tag{8}$$

$$\left[T(x, y, z, t)+k_3\frac{\partial T(x, y, z, t)}{\partial y}\right]_{y=b}=F_3(x, z, t) \tag{9}$$

$$\left[T(x, y, z, t)+k_4\frac{\partial T(x, y, z, t)}{\partial y}\right]_{y=-b}=F_4(x, z, t) \tag{10}$$

$$\left[T(x, y, z, t)+c\frac{\partial T(x, y, z, t)}{\partial z}\right]_{z=0}=-\frac{Q}{\lambda}f(x, y, t) \tag{11}$$

$$\left[T(x, y, z, t)+c\frac{\partial T(x, y, z, t)}{\partial z}\right]_{z=h}=g(x, y, t) \qquad 12)$$

The stresses components in terms of $U(x, y, z, t)$ are given by

$$\sigma_{xx}=\left(\frac{\partial^2 U}{\partial y^2}+\frac{\partial^2 U}{\partial z^2}\right) \tag{13}$$

$$\sigma_{yy}=\left(\frac{\partial^2 U}{\partial z^2}+\frac{\partial^2 U}{\partial x^2}\right) \tag{14}$$

$$\sigma_{zz}=\left(\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}\right) \tag{15}$$

The equations (1) to (15) constitute the mathematical formulation of the problem under consideration.

III. SOLUTION OF THE PROBLEM

By applying the finite Marchi-Fasulo transform two times to equation (5) to (12), and then their inversion, we obtain

$$\begin{aligned} T(x, y, z, t) &= \frac{k}{c^2}\sum_{m,n=1}^{\infty}\left(\frac{P_m(x)}{\lambda_m}\right)\left(\frac{P_n(y)}{\mu_n}\right)\left[\phi_1(z)\psi_1(t)-\phi_2(z)\psi_2(t)\right] \\ &+\frac{2k\prod}{h^2}\sum_{l,m,n=1}^{\infty}\left(\frac{P_m(x)}{\lambda_m}\right)\left(\frac{P_n(y)}{\mu_n}\right)\left(\frac{l}{\cos l\prod}\right)\frac{1}{1+\left(\frac{cl\pi}{h}\right)^2}\left[\eta_1(z)\psi_3(t)-\eta_2(z)\psi_4(t)\right] \end{aligned} \tag{16}$$

where $\phi_1(z)=\dfrac{\sinh\left(\frac{z}{c}\right)-\cos\left(\frac{z}{c}\right)}{\sinh\left(\frac{h}{c}\right)}$

$$\phi_2(z) = \frac{\sinh\left(\frac{z-h}{c}\right) - \cos\left(\frac{z-h}{c}\right)}{\sinh\left(\frac{h}{c}\right)}$$

$$\eta_1(z) = \sin\left(\frac{l\pi}{h}\right) - \left(\frac{cl\pi}{h}\right)\cos\left(\frac{l\pi}{h}\right) z$$

$$\eta_2(z) = \left[\sin\left(\frac{l\pi}{h}\right) - \left(\frac{cl\pi}{h}\right)\cos\left(\frac{l\pi}{h}\right)\right](z-h)$$

$$\psi_1(t) = \int_0^t \bar{\bar{f}}(m,n,t')\, e^{k\left(\frac{1-c^2 q}{c^2}\right)t'} dt'$$

$$\psi_2(t) = \int_0^t \bar{\bar{g}}(m,n,t')\, e^{k\left(\frac{1-c^2 q}{c^2}\right)t'} dt'$$

$$\psi_3(t) = \int_0^t \bar{\bar{f}}(m,n,t')\, e^{-k\left(q^2+\left(\frac{l\pi}{h}\right)^2\right)t'} dt'$$

$$\psi_4(t) = \int_0^t \bar{\bar{g}}(m,n,t')\, e^{-k\left(q^2+\left(\frac{l\pi}{h}\right)^2\right)t'} dt'$$

Here $\bar{\bar{f}}(m,n,t)$ and $\bar{\bar{g}}(m,n,t)$ denote the Marchi-Fasulo transform of $\bar{f}(m,y,t)$ and $\bar{g}(m,y,t)$ respectively. $\bar{f}(m,y,t)$, $\bar{g}(m,y,t)$ denote the finite Marchi-Fasulo transform of $f(x,y,t)$ and $g(x,y,t)$ respectively.

$$\bar{\bar{f}}(m,n,t) = \int_{-b}^{b} \bar{f}(m,y,t) P_n(y)\,dy \; ; \; \bar{\bar{g}}(m,n,t) = \int_{-b}^{b} \bar{g}(m,y,t) P_n(y)\,dy ,$$

And $\lambda_n = \int_{-b}^{b} P_n^{\,2}(y)\,dy$

$$P_n(y) = Q_n \cos(a_n y) - w_n \sin(a_n y) ,$$

$$Q_n = a_n(\alpha_3 + \alpha_4)\cos(a_n b) - (\beta_3 - \beta_4)\sin(a_n b) ,$$

$$w_n = a_n(\beta_3 + \beta_4)\cos(a_n b) - (\alpha_4 - \alpha_3)\sin(a_n b)$$

Equation (16) is the desired solution of the given problem with $\beta_3 = \beta_4 = 1$, $\alpha_3 = k_3$, $\alpha_4 = k_4$.

IV. DETERMINATION OF THE AIRY STRESS FUNCTION

Substituting the values of T(x, y, z, t) from equation (16) in equation (4), one obtains

$$U(x,y,z,t) = \frac{\alpha E k}{c^2} \sum_{m,n=1}^{\infty} \left(\frac{P_m(x)}{\lambda_m}\right)\left(\frac{P_n(y)}{\mu_n}\right)\left[\frac{\phi_1(z)\psi_1(t) - \phi_2(z)\psi_2(t)}{a_m^{\,2} + a_n^{\,2} - 1/c^2}\right]$$

$$+ \frac{2\alpha E k\pi}{h^2} \sum_{l,m,n=1}^{\infty} \left(\frac{P_m(x)}{\lambda_m}\right)\left(\frac{P_n(y)}{\mu_n}\right)\left(\frac{l}{\cos l\pi}\right)\left[\frac{\eta_1(z)\psi_3(t) - \eta_2(z)\psi_4(t)}{a_m^{\,2} + a_n^{\,2} + (l\pi/h)^2}\right] \tag{17}$$

V. DETERMINATIONS OF DISPLACEMENT COMPONENTS

Substituting the values of (17) in the equation (1) to (3), one obtains

$$u_x = \frac{\alpha k}{c^2} \sum_{m,n=1}^{\infty} \left(\frac{(k_1+k_2)\sin 2a_m a}{\lambda_m}\right)\left(\frac{P_n(y)}{\mu_n}\right)(1+\upsilon)a_m^{\,2}\left[\frac{\phi_1(z)\psi_1(t) - \phi_2(z)\psi_2(t)}{a_m^{\,2} + a_n^{\,2} - 1/c^2}\right]$$

$$+ \frac{2\alpha k\pi}{h^2} \sum_{l,m,n=1}^{\infty} \left(\left(\frac{(k_1+k_2)\sin 2a_m a}{\lambda_m}\right)\right)\left(\frac{P_n(y)}{\mu_n}\right)\left(\frac{l}{\cos l\pi}\right)\left(\frac{(1+\upsilon)a_m^{\,2}}{a_m^{\,2} + a_n^{\,2} + (l\pi/h)^2}\right)\left[\frac{\eta_1(z)\psi_3(t) - \eta_2(z)\psi_4(t)}{1 + (cl\pi/h)^2}\right] \tag{18}$$

$$u_y = \frac{\alpha\, k}{c^2} \sum_{m,n=1}^{\infty} \left(\frac{(k_3+k_4)\sin 2a_n b}{\mu_n} \right) \left(\frac{P_m(x)}{\lambda_m} \right) (1+\upsilon) a_n^{\,2} \left[\frac{\phi_1(z)\psi_1(t) - \phi_2(z)\psi_2(t)}{a_m^{\,2} + a_n^{\,2} - 1/c^2} \right]$$

$$+ \frac{2\alpha k\pi}{h^2} \sum_{l,m,n=1}^{\infty} \left(\left(\frac{(k_3+k_4)\sin 2a_n b}{\mu_n} \right) \right) \left(\frac{P_m(x)}{\lambda_m} \right) \left(\frac{l}{\cos l\pi} \right) \left(\frac{(1+\upsilon) a_n^{\,2}}{a_m^{\,2} + a_n^{\,2} + (l\pi/h)^2} \right) \left[\frac{\eta_1(z)\psi_3(t) - \eta_2(z)\psi_4(t)}{1 + (cl\pi/h)^2} \right] \tag{19}$$

$$u_z = \frac{\alpha\, k}{c^2} \sum_{m,n=1}^{\infty} \left(\frac{P_m(x)}{\lambda_m} \right) \left(\frac{P_n(y)}{\mu_n} \right) \left(-\frac{(1+\upsilon)}{c^2} \right) \left[\frac{\phi_1'(h)\psi_1(t) - \phi_2'(h)\psi_2(t)}{a_m^{\,2} + a_n^{\,2} - 1/c^2} \right]$$

$$+ \frac{2\alpha k\pi}{h^2} \sum_{l,m,n=1}^{\infty} \left(\frac{P_n(y)}{\mu_n} \right) \left(\frac{P_m(x)}{\lambda_m} \right) \left(\frac{l}{\cos l\pi} \right) \left(\frac{(1+\upsilon)(l\pi/h)^2}{a_m^{\,2} + a_n^{\,2} + (l\pi/h)^2} \right) \left[\frac{\eta_1'(h)\psi_3(t) - \eta_2'(h)\psi_4(t)}{1 + (cl\pi/h)^2} \right] \tag{20}$$

where

$$\phi_1'(h) = \frac{\cosh\left(\frac{h}{c}\right) - \sin\left(\frac{h}{c}\right) - 1}{\frac{1}{c}\sinh\left(\frac{h}{c}\right)}$$

$$\phi_2'(h) = \frac{1 - \cosh\left(\frac{h}{c}\right) - \sin\left(\frac{h}{c}\right)}{\frac{1}{c}\sinh\left(\frac{h}{c}\right)}$$

$$\eta_1'(h) = \frac{-\cos\left(\frac{l\pi}{h}\right) - \left(\frac{cl\pi}{h}\right)\sin\left(\frac{l\pi}{h}\right)h + 1}{\left(\frac{l\pi}{h}\right)}$$

$$\eta_1'(h) = \frac{-\cos\left(\frac{l\pi}{h}\right) - \left(\frac{cl\pi}{h}\right)\sin\left(\frac{l\pi}{h}\right)h + 1}{\left(\frac{l\pi}{h}\right)}$$

VI. DETERMINATION OF THE STRESS FUNCTION

Substituting the values of (17) in the equation (13) to (15), one obtains

$$\sigma_{xx} = \frac{\alpha E k}{c^2} \sum_{m,n=1}^{\infty} \left(\frac{P_m(x)}{\lambda_m} \right) \left(\frac{P_n(y)}{\mu_n} \right) \left[\frac{1}{c^2} - a_n^{\,2} \right] \left[\frac{\phi_1(z)\psi_1(t) - \phi_2(z)\psi_2(t)}{a_m^{\,2} + a_n^{\,2} - \frac{1}{c^2}} \right]$$

$$+ \frac{2\alpha E k\pi}{h^2} \sum_{l,m,n=1}^{\infty} \left(\frac{P_m(x)}{\lambda_m} \right) \left(\frac{P_n(y)}{\mu_n} \right) \left(\frac{l}{\cos l\pi} \right) \left(\frac{-a_n^{\,2} - (l\pi/h)^2}{a_m^{\,2} + a_n^{\,2} + (l\pi/h)^2} \right) \tag{21}$$

$$\sigma_{yy} = \frac{\alpha E k}{c^2} \sum_{m,n=1}^{\infty} \left(\frac{P_m(x)}{\lambda_m} \right) \left(\frac{P_n(y)}{\mu_n} \right) \left[\frac{1}{c^2} - a_m^{\,2} \right] \left[\frac{\phi_1(z)\psi_1(t) - \phi_2(z)\psi_2(t)}{a_m^{\,2} + a_n^{\,2} - \frac{1}{c^2}} \right]$$

$$+ \frac{2\alpha E k\pi}{h^2} \sum_{l,m,n=1}^{\infty} \left(\frac{P_m(x)}{\lambda_m} \right) \left(\frac{P_n(y)}{\lambda_n} \right) \left(\frac{l}{\cos l\pi} \right) \left(\frac{-\left(\frac{l\pi}{h}\right)^2 - a_m^{\,2}}{a_m^{\,2} + a_n^{\,2} + \left(\frac{l\pi}{h}\right)^2} \right) \left[\frac{\eta_1(z)\psi_3(t) - \eta_2(z)\psi_4(t)}{1 + (cl\pi/h)^2} \right] \tag{22}$$

$$\sigma_{zz} = \frac{\alpha E k}{c^2} \sum_{m,n=1}^{\infty} \left(\frac{P_m(x)}{\lambda_m} \right) \left(\frac{P_n(y)}{\mu_n} \right) \left[-a_m^{\,2} - a_n^{\,2} \right] \left[\frac{\phi_1(z)\psi_1(t) - \phi_2(z)\psi_2(t)}{a_m^{\,2} + a_n^{\,2} - 1/c^2} \right]$$

$$+ \frac{2\alpha E k\pi}{h^2} \sum_{l,m,n=1}^{\infty} \left(\frac{P_n(y)}{\mu_n} \right) \left(\frac{P_m(x)}{\lambda_m} \right) \left(\frac{l}{\cos l\pi} \right) \left(\frac{-a_m^{\,2} - a_n^{\,2}}{a_m^{\,2} + a_n^{\,2} + \left(\frac{l\pi}{h}\right)^2} \right) \left[\frac{\eta_1(z)\psi_3(t) - \eta_2(z)\psi_4(t)}{1 + (cl\pi/h)^2} \right] \tag{23}$$

VII. SPECIAL CASE AND NUMERICAL RESULT

Setting, $f(x,y,t)=\left(1-e^{-t}\right)(x+a)^2(x-a)^2(y+b)^2(y-b)^2$,

$g(x,y,t)=\left(1-e^{-t}\right)(x+a)^2(x-a)^2(y+b)^2(y-b)^2e^h$,

$\delta=\frac{8(k_1+k_2)}{h^2}$, $a=0.75$, $k=0.86$, $b=0.75$, $h=0.25$, $t=1$

in the equation (3.1), we obtain

$$T(x,y,z,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{l=1}^{\infty}(-1)^{\left(l+\frac{1}{2}\right)}\left(l+\frac{1}{2}\right)\left(\frac{P_m(y)}{\lambda_m}\right)\left(\frac{P_n(x)}{\mu_n}\right)[\phi(z)e-\psi(z)] \times\left[\frac{a_n^2\cos^2(a_n)-\cos(a_n)\sin(a_n)}{a_n^2}\right] \tag{24}$$

VIII. MATERIAL PROPERTIES

The numerical calculation has been carried out for an Aluminum (Pure) rectangular plate with the material properties as,

Density ρ	= 169 lb/ft3
Specific heat	= 0.208 Btu/ lb0F
Thermal conductivity k	= 117 Btu/(hr.ft0F)
Thermal diffusivity α	= 3.33 ft2/hr
Poisson ratio ν	= 0.35
Coefficient of linear thermal expansion α_t	= 12.84 $12.84\times10^{-6}\,1/F$ 1/F
Lame constant μ	= 26.67
Young's Modulus of elasticity E	=70GPa

IX. GRAPHICAL ANALYSES

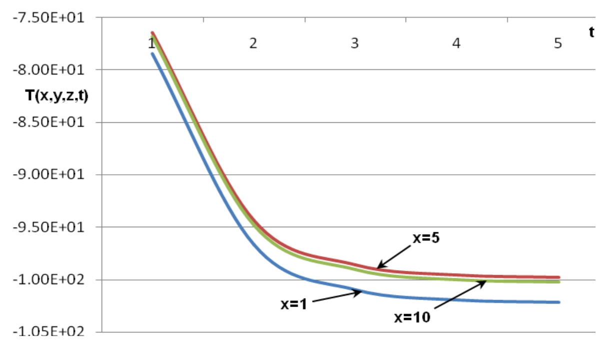

Fig. 1. T(x, y, z, t) versus t for different values of x.

From the plotted graph between obtained temperatures versus t for different values of x, it is observed that as t vary from 1 to 3 seconds temperature decreases gradually and after time t=3 it becomes stable for different values of x, or we can say that as x increases, the temperature gradually decreases due to partially distributed heat supply.

The following graphs give the characteristic of stresses versus different values of t.

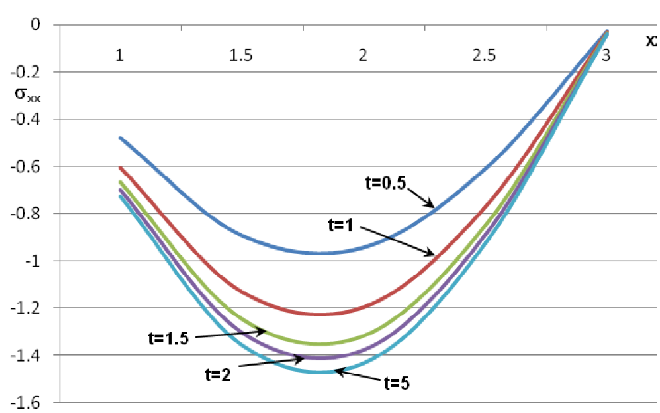

Fig. 2. σ_{xx} versus x for different values of t.

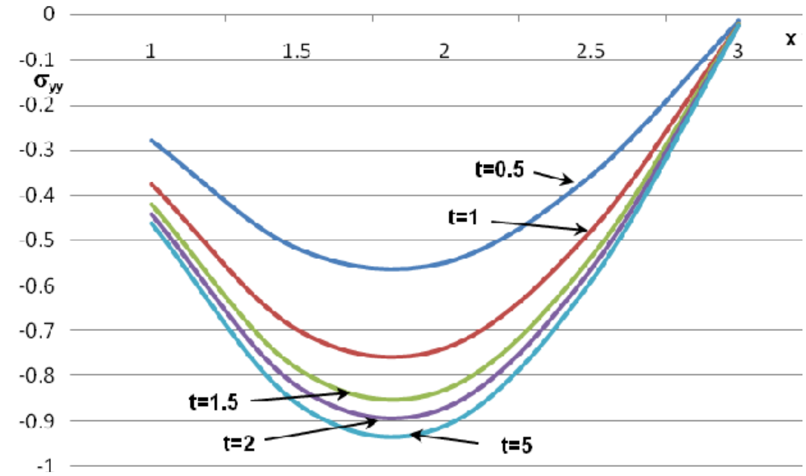

Fig. 3: σ_{yy} versus x for different values of t.

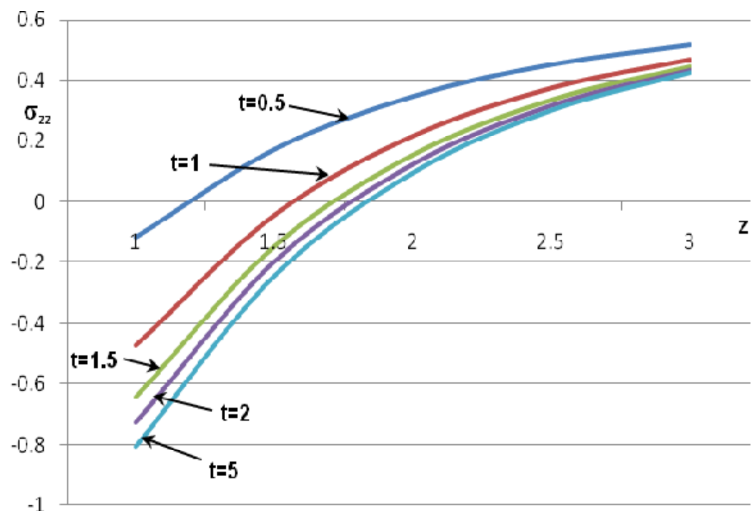

Fig. 4. σ_{zz} versus z for different values of t.

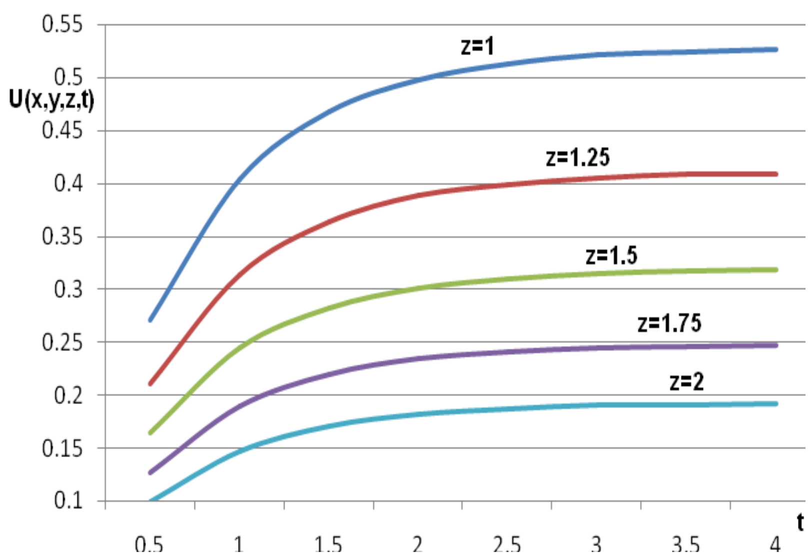

Fig. 5. U(x, y, z, t) versus t for different values of z.

X. CONCLUSION

The temperature, displacements, and thermal stresses at any point of the plate have been obtained, when the boundary conditions are known with the aid of the finite Marchi-Fasulo transform technique. The expressions are represented graphically. The results are obtained in the form of infinite series. It is observed that as x increases, the temperature gradually decreases. Any particular case of special interest can be derived by assigning suitable values to the parameters and functions in the expressions.

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