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Study of Rayleigh Waves in Transversely Isotropic Dual Phase Lag Thermo-elasticity under Magnetic Field

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ABSTRACT: The guiding equations of transversely isotropic Dual Phase Lag Thermo-elasticity (DPLT) along with the magnetic field were solved using the surface wave solution. Further, the frequency equation for Rayleigh waves is calculated by applying the particular solution using the boundary conditions at a surface which is thermally shielded as well as isothermal stress free. These conditions were also approximated to find the numerical solution for the non-dimensional velocity of Rayleigh waves. Later in this present work the Rayleigh wave's speed is represented graphically for the various types of thermo-elasticity i.e. Lord and shulman thermo-elasticity (LS-T), coupled thermo-elasticity (CT), Dual Phase Lag Thermo-elasticity (DPLT) and magnetic field. The effect of Dual Phase Lag (DPL), transverse isotropy and Magnetic field are also monitored for the speed of Rayleigh wave.

Keywords: Generalized thermo-elasticity, Magnetic Field, Rayleigh wave, frequency equation, Lord-shulman Theory (L-S-T), Dual –Phase-Lag (DPL).

I. INTRODUCTION

Biot [1] proposed theory of classical dynamical coupled thermo-elasticity (CDCT) , further sequentially to generalize the thermo-elastic theories the extended version of the classical-dynamical-coupled-thermo-elasticity (CDCT) was developed by Lord & Shulman (LS) [2] and Green & Lindsay (GL) [3] by introducing the concept of one relaxation-time and two relaxation time respectively and also considering the field equation to be hyperbolic these theories also relate heat to be a wave and on comparing these theories with those of Biot speed of heat propagation is predicted to be finite. Hetnarski and Ignaczak [4] also discussed the theories related with generalized thermo-elasticity. The concept of Dual-Phase-Lag-Thermo-elastic-Model (DPLTEM), which is the modified form of the classical thermo-elastic-model (CTEM), takes into account the phonons and electron interaction at the microscopic level was broaden by Tzou [5-7] ,by applying the approximation to modified fourier law instead of existing fourier law along with two major phase lag to the main two factors heat flux as well as the temperature gradient, this new model can be applied to explore the micro-structural outcomes on the behavior of heat transfer. Singh [8] studied the transmission of plane wave through DPLGT solid half space Plane wave speed is represented graphically in comparison to the angle of transmission for various thermoelasticity models namely DPL-model, coupled as well as LS. Reflection coefficient was also computed for isothermal as well as thermally insulated cases for these plane waves it is also shown graphically for the above mentioned theories. Deresiewicz [9], Sinha and Sinha [10], Othman and Song [11], Roy Choudhuri, [12] Singh [13, 14] and many others has studied the wave propagation through coupled and generalized thermo-elasticity and obtain remarkable results. The concept of Wave propagation has innumerous applications in fields like geophysics and environmental investigation, seismology, groundwater-related investigations, mineral and oil study etc. Abouelregal [15] by means of dual phase lag model (DPLM) the author studied Rayleigh-wave's in thermoelastic solid half-space. M.A. Ezzat *et al.,* [16] considered fractional ordered DPL heat conduction law to frame a new model which includes of two-temperature thermoelasticity along with magnetic field effect using Laplace transform. M. I. A. Othman *et. al* [17] measured the deformation under the effect of gravity of a rotating, generalized-thermoelastic medium by means of normal mode analysis method and also develops the analytic solution. Author also compared the results forecast by L-S Theory and DPLM with and without rotation and gravity. Fabrizio, M. and Lazzari, B. [18] used various approximations of Tzou's DPLT to obtain heat conductor model which are two in number having fading memory, during the process author obtained some similarity between the constraints of 2^{nd}law of thermodynamics and asymptotic stability for one of model whereas for second model the restrictions of thermo-dynamical model seems more deterring as compare to those of asymptotic stability. Without using laplace transformation El-Karamany, [19] shows the unique theorem along with reciprocal theorems for DPLT theory. The concept of D-P-L model is already explain extensively in the previous article Bharti et al., [20].

II. BASIC EQUATIONS

Following Tzou [5-7] the basic equations for anisotropic, thermo-elastic DPL body studied under the existence of magnetic field H can be illustrated as

i) Constitutive equations

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{1}$$

$$\sigma_{ij} = c_{ijkl} e_{kl} - \beta_{ij} T$$

ii) Equation of motion

$$\rho \ddot{u}_i = \sigma_{ji,j} + \rho F_i \quad (2)$$

iii) Modified Fourier law

$$-K_{ij}(T_j + \tau_\theta \dot{T}_j) = q_i + \tau_q \dot{q}_i \quad (3)$$

iv) Energy equation

$$-q_{i,i} = \rho T_o \dot{S} \quad (4)$$

V) Maxwell equation of stress electric and magnetic field

$$\bar{\sigma}_{ij} = \mu_e [H_i h_j + H_j h_i - (Hj)\delta_{ij}]$$
$$\nabla .h = 0, \nabla .E = 0; \nabla .h = j, \quad (5)$$
$$\nabla .E = \mu_e \frac{\partial h}{\partial t}$$

IF $\tau_\theta = \tau_q = 0$, then DPLT reduced to CT theory, also if τ_q is replaced by $\tau_\theta (\tau_\theta = 0)$ then DPLT reduced to the L-S generalized thermo-elasticity theory, further if we take **H**=**H**$_0$+**h** along distressed magnetic field (**h**) to be very minute then the multiplication (**hu)** along with its derivatives will be overlooked in linear approximation of the considered equations.

By taking Eqn. (1) only and neglecting all forces that are operating upon the body. Further we have,

$$\rho \frac{\partial^2 u_i}{\partial t^2} = (c_{ijkl} e_{kl} - \beta_{ij} T)_j + (JB)_i \quad (6)$$

Substituting (5) and (6) into Eqn. (4) we obtain

$$(1+\tau_\theta \frac{\partial}{\partial t}) K_{ij} T_{ij} = (1+\tau_q \frac{\partial}{\partial t})(\rho c_E \frac{\partial T}{\partial t} + T_0 \beta_{ij} \frac{\partial e_{ij}}{\partial t}) \quad (7)$$

Where T$_0$ is considered as uniform temperature of the body and can easily be obtained from the ratio $\left|\frac{T}{T_0}\right| << 1$

III. NOMENCLATURE

λ, μ	→	Lame's constants
e_{ij}	→	Strain tensor
σ_{ij}	→	Stress tensor
T	→	Time
T$_0$	→	Temperature(un-deformed and unstressed state)
T	→	Absolute temperature
$\vec{u}$	→	Displacement vector
F_i	→	Component of body force F
ρ	→	Density
δ_{ij}	→	Kronecker's delta
K	→	Coefficient of thermal conductivity
K	→	*D*iffusivity coefficient
C	→	Specific heat
c_E	→	Specific heat when strain is constant
Q	→	Measure of heat generation/unit volume
τ_0	→	Relaxation time
q_i	→	Heat flux vector
H	→	Magnetic Field
H$_0$	→	Magnetic Field constant vector
β_{ij}	→	Coefficient of thermal expansion
μ_e	→	Magnetic permeability
τ_q	→	Phase lag for Heat flux
τ_θ	→	Phase lag for Temperature gradient
c_{ij}	→	Incremental elastic co-efficient

IV. FORMULATION OF THE PROBLEM

We consider a homogenous system with a uniform temperature distribution under isotropic medium in terms of Cartesian type system i.e. (x, y, z) of equation. The pole of Cartesian type system is taken on the plane surface, where as the axis of Z is taken perpendicular to the medium $(z \geq 0)$. Also considering z = 0 as free and thermally insulated/isothermal surface. We restrict this paper to the plane strain which is parallel to the xz-plane with $\vec{u}(u_1, 0, u_3)$ & $H_0(0, H_0, 0)$ as displacement vector and magnetic field (constant) vector respectively.

Using Eqns. (6) and (7) we get the equations defining motion and that defining heat conduction.

$$c_{11}\frac{\partial^2 u_1}{\partial x^2} + (c_{13}+c_{44})\frac{\partial^2 u_3}{\partial x \partial z} + c_{44}\frac{\partial^2 u_1}{\partial z^2} - \beta_1 \frac{\partial T}{\partial x}$$
$$= \rho \frac{\partial^2 u_1}{\partial t^2} - \mu_e H_0^2 [\frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_3}{\partial x \partial z}] \quad (8)$$

$$c_{44}\frac{\partial^2 u_3}{\partial x^2} + (c_{13}+c_{44})\frac{\partial^2 u_1}{\partial x \partial z} +$$
$$+ c_{44}\frac{\partial^2 u_3}{\partial z^2} - \beta_3 \frac{\partial T}{\partial z} = \rho \frac{\partial^2 u_3}{\partial t^2} - \quad (9)$$
$$- \mu_e H_0^2 [\frac{\partial^2 u_1}{\partial x \partial z} + \frac{\partial^2 u_3}{\partial z^2}]$$

$$\left(1+\tau_\theta \frac{\partial}{\partial t}\right)\left[K_{11}\frac{\partial^2 T}{\partial x^2} + K_{33}\frac{\partial^2 T}{\partial z^2}\right] =$$
$$= \left(1+\tau_q \frac{\partial}{\partial t}\right)\left[\rho c_E \frac{\partial T}{\partial t} + T_0 \beta_1 \frac{\partial^2 u_1}{\partial t \partial x} + T_0 \beta_3 \frac{\partial^2 u_3}{\partial t \partial z}\right] \quad (10)$$

V. SURFACE WAVE SOLUTION

We determine the function (u_1, u_3, T) in half space along x-axis as,

$$(u_1, u_3, T) = \{\phi_1(z), \phi_3(z), \varphi(z)\} e^{ik(x-ct)} \quad (11)$$

Using Eqn. (11) into Eqn. (8)-(10), we get homogenous system of 3-equations in terms of $(\phi_1, \phi_3, \varphi)$ which are as follows.

$$[k^2(\rho c^2 - c_{11} - \mu_e H_0^2) + D^2 c_{44}]\phi_1 +$$
$$ik(c_{13} + \mu_e H_0^2 + c_{44}) D\phi_3 - \beta_1 ik \varphi c_{44} = 0 \quad (12)$$

$$ik(c_{13} + \mu_e H_0^2 + c_{44}) D\phi_1 +$$
$$[k^2(\rho c^2 - c_{44}) + (c_{11} - \mu_e H_0^2) D^2]\phi_3 \quad (13)$$
$$- \beta_3 D \varphi c_{44} = 0$$

$$\beta_1 ik^3 \varepsilon\rho c^2 \phi_1 + \beta_3 \rho c^2 \varepsilon k^2 D\phi_3 + [k^2(\rho c^2 - K_1^*) + K_3^* D^2]\varphi = 0 \quad (14)$$

where

$$\varepsilon = \frac{\beta_1^2 T_0}{\rho^2 c_E c^2},\ \tau^* = \frac{\tau_q + \frac{i}{w}}{1 - iw\tau_\theta},$$

$$K_1^* = \frac{K_1}{c_E \tau^*},\ K_3^* = \frac{K_3}{c_E \tau^*},$$

$$\beta_{11} = \beta_1, \beta_{33} = \beta_3, \overline{\beta} = \frac{\beta_3}{\beta_1}$$

For the non trivial solution of (12)-(14) we get

$$D^6 - AD^4 + BD^4 - C = 0 \quad (15)$$

Where

$$A = -k^2[(\rho c^2 - c_{11} - \mu_e H_0^2) + \frac{(\rho c^2 - c_{44})}{c_{33} + \mu_e H_0^2} + \frac{(\rho c^2 - K_1^*)}{K_3^*} + \frac{(c_{13} + \mu_e H_0^2 + c_{44})^2}{c_{33} + \mu_e H_0^2} + \frac{\overline{\beta}^2 \varepsilon\rho c^2}{(c_{33} + \mu_e H_0^2)K_3^*}]$$

$$B = k^4[\frac{(\rho c^2 - c_{11} - \mu_e H_0^2)(\rho c^2 - c_{44})}{c_{33} + \mu_e H_0^2} + \frac{(\rho c^2 - c_{11} - \mu_e H_0^2)(\rho c^2 - K_1^*)}{K_3^*} + \frac{(\rho c^2 - c_{44})(\rho c^2 - K_1^*)}{(c_{33} + \mu_e H_0^2 1)K_3^*} + \frac{(c_{13} + \mu_e H_0^2 + c_{44})^2(\rho c^2 - K_1^*)}{(c_{33} + \mu_e H_0^2)K_3^*} + \frac{(\rho c^2 - c_{11} - \mu_e H_0^2)\overline{\beta}^2 \varepsilon\rho c^2}{(c_{33} + \mu_e H_0^2)K_3^*} + \frac{2(c_{13} + \mu_e H_0^2 + c_{44})\overline{\beta}}{(c_{33} + \mu_e H_0^2)K_3^*} - \frac{\varepsilon\rho c^2}{K_3^*}]$$

$$C = -k^6[\frac{(\rho c^2 - c_{11} - \mu_e H_0^2)(\rho c^2 - c_{44})(\rho c^2 - k_1^*)}{(c_{33} + \mu_e H_0^2)K_3^*} - \frac{\varepsilon\rho c^2(\rho c^2 - c_{44})}{(c_{33} + \mu_e H_0^2)K_3^*}]$$

The common solutions of Eqns. (12)-(14) are as follows

$$\phi_1(z) = [\sum_{i=1}^{3} A_i e^{-m_i z} + \sum_{i=1}^{3} A_i^* e^{m_i z}]e^{ik(x-ct)} \quad (16)$$

$$\phi_3(z) = [\sum_{i=1}^{3} B_i e^{-m_i z} + \sum_{i=1}^{3} B_i^* e^{m_i z}]e^{ik(x-ct)} \quad (17)$$

$$\varphi(z) = [\sum_{i=1}^{3} C_i e^{-m_i z} + \sum_{i=1}^{3} C_i^* e^{m_i z}]e^{ik(x-ct)} \quad (18)$$

where $A_i, B_i, C_i, A_i^*, B_i^*, C_i^*$ constants and m_i are the roots satisfying the equation

$$m^6 - Am^4 + Bm^2 - C = 0 \quad (19)$$

and the above equation is cubic in m^2

In case of surface waves when all roots (m_1, m_2, m_3) are complex we assume Re (mi)>0, i = 1, 2, 3 and we use only those roots which satisfies following conditions

$$\phi_1, \phi_3, \varphi \to 0, \quad z \to \infty$$

Now using this condition (16)-(18) converts to the solution for the half space as z>0

$$\phi_1(z) = [\sum_{i=1}^{3} A_i e^{-m_i z}]e^{ik(x-ct)} \quad (20)$$

$$\phi_3(z) = [\sum_{i=1}^{3} F_i A_i e^{-m_i z}]e^{ik(x-ct)} \quad (21)$$

$$\varphi(z) = [\sum_{i=1}^{3} F_i^* A_i e^{-m_i z}]e^{ik(x-ct)} \quad (22)$$

$$F_i = -i\frac{m_i}{k}\left[\frac{\left(\begin{array}{l}\overline{\beta}(\rho c^2 - c_{11} - \mu_e H_0^2 + \frac{m_i^2}{k^2}c_{44}) \\ + (c_{13} + \mu_e H_0^2 + c_{44})\end{array}\right)}{\left(\begin{array}{l}\overline{\beta}\frac{m_i^2}{k^2}(c_{13} + \mu_e H_0^2 + c_{44}) - \\ \{\rho c^2 - c_{44} + (c_{33} + \mu_e H_0^2)\frac{m_i^2}{k^2}\}\end{array}\right)}\right]$$

i=1, 2, 3

$$F_i^* = -i\frac{\varepsilon c_{44} m_i}{\beta_1}\left[\frac{\left(\begin{array}{l}\overline{\beta}(\rho c^2 - c_{11} - \mu_e H_0^2 + \frac{m_i^2}{k^2}c_{44}) + \\ (c_{13} + \mu_e H_0^2 + c_{44})\end{array}\right)}{\left(\begin{array}{l}\overline{\beta}\varepsilon c_{44} + (c_{13} + \mu_e H_0^2 + c_{44}) \\ (1 - \frac{K_1^*}{\rho c^2} + \frac{K_3^*}{\rho c^2}\frac{m_i^2}{k^2})\end{array}\right)}\right]$$

VI. CONDITIONS ON THE BOUNDARY

The essential boundary conditions for the free surface z = 0 are the vanishing of tangential stress, normal stress and heat flux or the temperature potential:

$$\sigma_{zz} + \overline{\sigma}_{zz} = 0 \quad (23)$$

$$\sigma_{xz} + \overline{\sigma}_{xz} = 0 \quad (24)$$

$$\frac{\partial T}{\partial z} + hT = 0 \quad (25)$$

Where $h \to 0$ for the surface defined as insulated thermally,

$h \to \infty$ For the surface which is isothermal.

Using Eqns. (20)-(22) in the boundary condition (23)-(25), we attain the system of equations homogeneous in nature.

$$\sum_{i=1}^{3}\left[\begin{array}{l}\left(c_{33}-\mu_e H_0^2\right) m_i F_i - \\ ik\left(c_{13}-\mu_e H_0^2\right)+\beta_3 F_i^*\end{array}\right] A_i = 0 \qquad (26)$$

$$\sum_{i=1}^{3}\left(m_i - ikF_i\right) A_i = 0 \qquad (27)$$

$$\sum_{i=1}^{3} F_i^*\left(m_i - h\right) A_i = 0 \qquad (28)$$

Non Trivial solution of Eqn. (26)-(28) yield

$$\begin{aligned}
&(m_1-h)F_1^*\left[\left(\begin{array}{l}\left(c_{33}-\mu_e H_0^2\right)m_2F_2- \\ ik\left(c_{13}-\mu_e H_0^2\right)+\beta_3F_2^*\end{array}\right)\left(m_3-ikF_3\right)-\left(\begin{array}{l}\left(c_{33}-\mu_e H_0^2\right)m_3F_3- \\ ik\left(c_{13}-\mu_e H_0^2\right)+\beta_3F_3^*\end{array}\right)\left(m_2-ikF_2\right)\right]+\\
&(m_2-h)F_2^*\left[\left(\begin{array}{l}\left(c_{33}-\mu_e H_0^2\right)m_3F_3- \\ ik\left(c_{13}-\mu_e H_0^2\right)+\beta_3F_3^*\end{array}\right)\left(m_1-ikF_1\right)-\left(\begin{array}{l}\left(c_{33}-\mu_e H_0^2\right)m_1F_1- \\ ik\left(c_{13}-\mu_e H_0^2\right)+\beta_3F_1^*\end{array}\right)\left(m_3-ikF_3\right)\right]+\\
&(m_3-h)F_3^*\left[\left(\begin{array}{l}\left(c_{33}-\mu_e H_0^2\right)m_1F_1- \\ ik\left(c_{13}-\mu_e H_0^2\right)+\beta_3F_1^*\end{array}\right)\left(m_2-ikF_2\right)-\left(\begin{array}{l}\left(c_{33}-\mu_e H_0^2\right)m_2F_2- \\ ik\left(c_{13}-\mu_e H_0^2\right)+\beta_3F_2^*\end{array}\right)\left(m_1-ikF_1\right)\right]=0
\end{aligned} \qquad (29)$$

Equation (29) is the Rayleigh wave's frequency equation under the impact of magnetic field for transversely isotropic /isothermal Dual Phase Lag thermo-elasticity.

VII. PARTICULAR CASES

(i) Thermally Insulated Surface: For a surface to be thermally insulated , we put $h \rightarrow 0$ in the frequency equation (29) and we get the frequency equation as follows

$$\begin{aligned}
&m_1F_1^*\left[\left(\begin{array}{l}\left(c_{33}-\mu_e H_0^2\right)m_2F_2- \\ ik\left(c_{13}-\mu_e H_0^2\right)+\beta_3F_2^*\end{array}\right)\left(m_3-ikF_3\right)-\left(\begin{array}{l}\left(c_{33}-\mu_e H_0^2\right)m_3F_3- \\ ik\left(c_{13}-\mu_e H_0^2\right)+\beta_3F_3^*\end{array}\right)\left(m_2-ikF_2\right)\right]+\\
&m_2F_2^*\left[\left(\begin{array}{l}\left(c_{33}-\mu_e H_0^2\right)m_3F_3- \\ ik\left(c_{13}-\mu_e H_0^2\right)+\beta_3F_3^*\end{array}\right)\left(m_1-ikF_1\right)-\left(\begin{array}{l}\left(c_{33}-\mu_e H_0^2\right)m_1F_1- \\ ik\left(c_{13}-\mu_e H_0^2\right)+\beta_3F_1^*\end{array}\right)\left(m_3-ikF_3\right)\right]+\\
&m_3F_3^*\left[\left(\begin{array}{l}\left(c_{33}-\mu_e H_0^2\right)m_1F_1- \\ ik\left(c_{13}-\mu_e H_0^2\right)+\beta_3F_1^*\end{array}\right)\left(m_2-ikF_2\right)-\left(\begin{array}{l}\left(c_{33}-\mu_e H_0^2\right)m_2F_2- \\ ik\left(c_{13}-\mu_e H_0^2\right)+\beta_3F_2^*\end{array}\right)\left(m_1-ikF_1\right)\right]=0
\end{aligned} \qquad (30)$$

(ii) Isothermal Surface: For a surface to be isothermal, we put $h \rightarrow \infty$ in the frequency Eqn. (29) and we get the following frequency equation

$$\begin{aligned}
&F_1^*\left[\left(\begin{array}{l}\left(c_{33}-\mu_e H_0^2\right)m_2F_2- \\ ik\left(c_{13}-\mu_e H_0^2\right)+\beta_3F_2^*\end{array}\right)\left(m_3-ikF_3\right)-\left(\begin{array}{l}\left(c_{33}-\mu_e H_0^2\right)m_3F_3- \\ ik\left(c_{13}-\mu_e H_0^2\right)+\beta_3F_3^*\end{array}\right)\left(m_2-ikF_2\right)\right]+\\
&F_2^*\left[\left(\begin{array}{l}\left(c_{33}-\mu_e H_0^2\right)m_3F_3- \\ ik\left(c_{13}-\mu_e H_0^2\right)+\beta_3F_3^*\end{array}\right)\left(m_1-ikF_1\right)-\left(\begin{array}{l}\left(c_{33}-\mu_e H_0^2\right)m_1F_1- \\ ik\left(c_{13}-\mu_e H_0^2\right)+\beta_3F_1^*\end{array}\right)\left(m_3-ikF_3\right)\right]+\\
&F_3^*\left[\left(\begin{array}{l}\left(c_{33}-\mu_e H_0^2\right)m_1F_1- \\ ik\left(c_{13}-\mu_e H_0^2\right)+\beta_3F_1^*\end{array}\right)\left(m_2-ikF_2\right)-\left(\begin{array}{l}\left(c_{33}-\mu_e H_0^2\right)m_2F_2- \\ ik\left(c_{13}-\mu_e H_0^2\right)+\beta_3F_2^*\end{array}\right)\left(m_1-ikF_1\right)\right]=0
\end{aligned} \qquad (31)$$

Isotropic Thermoelasticity: In case of isotropic thermo-elasticity,

$$H_0 = 0, \quad c_{11} = c_{33} = \lambda + 2\mu,$$
$$c_{13} = \lambda, \quad c_{44} = \mu, \quad \beta_1 = \beta_3 = \beta,$$
$$K_1 = K_3 = K$$

if we put in the Eqn. (28) we get the required frequency of Rayleigh wave in case of isotropic thermo-elasticity.

and if we put $\beta = 0$, $K = 0, \varepsilon = 0$ in the case isotropic thermo-elasticity, then we get the Rayleigh wave's equation in an isotropic as well as elastic solid half space as

$$\left(2-\frac{c^2}{c_2^2}\right)^2 = 4\sqrt{\left(1-\frac{c^2}{c_1^2}\right)\left(1-\frac{c^2}{c_2^2}\right)} \tag{32}$$

VIII. NUMERICAL RESULT AND DISCUSSION

For the majority of materials, ε is very small at a normal temperature. For $\varepsilon \ll 1$, Eqn. (15) has the approximated roots as

$$\frac{m_1^2}{k^2} \cong \frac{c_{11} - \rho c^2 + \mu_e H_0^2}{c_{44}} \tag{33}$$

$$\frac{m_2^2}{k^2} \cong \frac{c_{44} - \rho c^2}{c_{44} + \mu_e H_0^2} \tag{34}$$

$$\frac{m_3^2}{k^2} \cong -\frac{K_1 - \rho c^2 c_E T^*}{K_3} \tag{35}$$

We numerically calculated the Rayleigh wave speed for non -dimensional quantities; we considered only the thermally insulated surface. Therefore, Eqn. (31) is approximated with the help of (33-35) and solved to obtain the speed of propagation for particular range of non dimensional constants. Following Chadwick and Seet [21], we consider the physical quantity of Zinc

$c_{11} = 1.628\times10^{11}\text{Nm}^{-2}$, $c_{33} = 1.562\times10^{11}\text{Nm}^{-2}$,
$c_{13} = 0.508\times10^{11}\text{Nm}^{-2}, c_{44} = 0.385\times10^{11}\text{Nm}^{-2}$,
$\beta_1 = 5.75\times10^{11}\text{Nm}^{-2}\deg^{-1}$, $\beta_3 = 5.17\times10^{11}\text{Nm}^{-2}\deg^{-1}$,
$K_1 = 1.24\times10^{2}\text{Wm}^{-1}\deg^{-1}, K_3 = 1.34\times10^{2}\text{Wm}^{-1}\deg^{-1}$
$C_E = 3.9\times10^{2}\text{JKg}^{-1}\deg^{-1}, \rho = 7.14\times10^{3}\text{Kgm}^{-3}$
$T_0 = 296\text{K}$, $\tau_q = 0.005\text{s}$, $\tau_\theta = 0.0005\text{s}$.

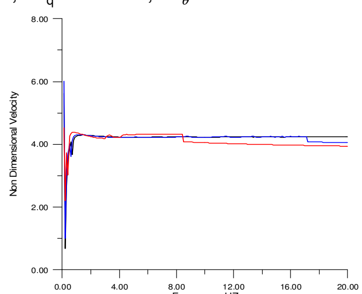

Fig. 1. : Dependence of non dimension velocity on the frequency of Rayleigh wave in the presence of Magnetic effect in DPL Theory.

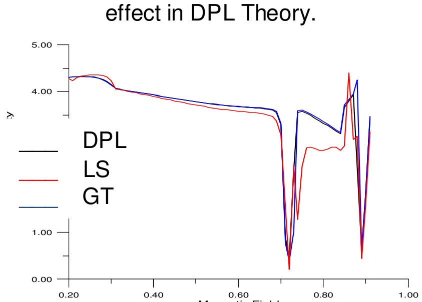

Fig. 2. Dependence of non dimension velocity under Magnetic field on the frequency of Rayleigh wave in DPL, LS and GT Theory.

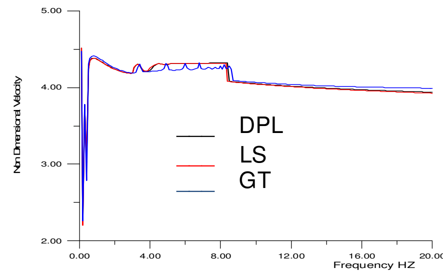

Fig. 3. Dependence of non dimension velocity on the Magnetic field in LS and GT Theory .In DPL Theory.

Fig. 1 states that in DPL Theory, the non dimension velocity of Rayleigh wave $\frac{\rho c^2}{c_{44}}$ is calculated for the frequency range [0, 20] HZ for various values of H such as 0, 0.1 and 0.2. While comparing the three curves appearing in figure 1, we observe the dependency of Rayleigh wave velocity on the frequency of the wave in presence of the considered value of magnetic effect.

Fig. 2 states that the non dimension velocity of propagation in case of Rayleigh wave $\frac{\rho c^2}{c_{44}}$ for the range of frequency to be [0, 20] HZ under magnetic effect is evaluated for the D-P-L theory, LS Theory and GT Theory and shows the dependence of magnetic field.

Fig. 3 illustrate that the non dimension velocity of Rayleigh wave $\frac{\rho c^2}{c_{44}}$ is calculated for the magnetic field range [0.2, 1] calculated for the DPL theory, LS Theory and GT Theory.

IX. CONCLUSION

The solution of surface wave for the governing equation of transversely isotropic dual phase lag (DPL) with magnetic field is obtained. By taking into account the boundary condition, general solution reduced to particular solution in half space. This particular solution satisfies the boundary conditions and hence we obtained the frequency equation of the Rayleigh wave for a stress free thermally insulated /isothermal .For the numerical calculation Rayleigh wave's frequency equation is then approximated for little thermal coupling and then solved numerically for a particular material. The non dimensional velocity of transmission is also plotted against the frequency, non-dimensional elastic and thermal constants in presence of magnetic field gives remarkable results. The numerical results indicate the effects of dual-phase-lag and transverse isotropy on the non-dimensional velocity of propagation

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