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# Effects of Heat Transfer with MHD Flow of Micropolar Fluid between Two Moving Concentric Vertical Porous Cylinders in the Presence of Heat Source

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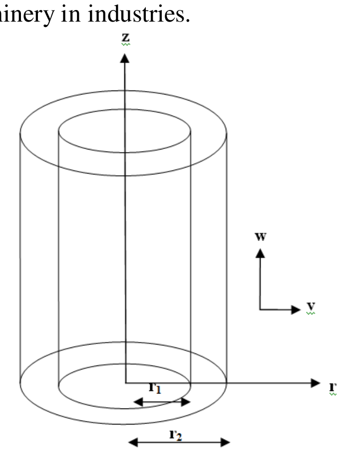
**ABSTRACT: The steady laminar magneto-hydrodynamic boundary layer flow past a concentric moving porous annulus immersed in a micropolar fluid with heat transfer has been studied. The governing partial differential equations are first converted into set of ordinary differential equations using suitable transformations and then solved by Runge- Kutta Felberg method with shooting technique. Results show that micro rotation velocity component increases with increasing cross flow Reynolds number. The micro inertia parameter and also with coupling number but it decreases with increasing values of α. It is observed that temperature increases with increasing values of heat source parameter but decreases with increasing Prandtl number.**

**Keywords:** Boundary layer flow; micropolar fluid; porous annulus; Runge-Kutta method.

## I. INTRODUCTION

The boundary layer theory has been successfully applied to non-Newtonian fluids models and has received much attention during the last few decades. One of the important non- Newtonian fluids is the micropolar fluid introduced by Eringen [2, 3]. Micropolar fluid takes the microscopic effects arising from the local structure and micro motions of the fluid elements. Many industrially related fluids like lubricants, biological fluids, paints, suspension fluids etc. belong to the category of micro-polar fluids. Extensive reviews of the theory and applications of micro polar fluids have been done by Lukaszewicz [4] and Eringen [1]. The micropolar fluid theory requires an additional transport equation representing the principle of conservation of local angular momentum, with usual transport equations of conservation of mass and momentum. In 1967 Terrill [10] studied the steady laminar flow through an annulus with porous walls. In 1969 Shrestha [8] presented the steady laminar flow of a non- Newtonian fluid through an annulus with porous walls of different permeability. Srinivasacharya and Shiferaw [9] in 2008 gave numerical solution to the MHD flow of micro polar fluid between two concentric porous cylinders. In 2011 Murthy *et al* [5] studied the steady incompressible electrically conducting micropolar fluid flow between two concentric rotating cylinders with porous lining in the presence of a radial magnetic field and presented the velocity profiles graphically. In 2015 Nagaraju *et al* [6] have carried out the numerical solutions for the flow and temperature fields in an annulus with a porous lining between two rotating cylinders in the presence of radial magnetic field and obtained the effects of various control parameters on the velocity profiles and temperature. The study of the heat transfer flow in a porous annulus in which there is constant suction at the wall has many important applications in the design of various types of machinery in industries.




In present study the effect of heat transfer in steady laminar electrically conducting micro polar fluid flow through porous moving cylindrical annulus with a heat source have been studied. The governing equations are solved using Runge- Kutta Felberg method with shooting technique and the effects of different parameters on velocity, temperature and micro rotation components are depicted graphically.

## II. MATHEMATICAL FORMULATION

The steady laminar flow of micropolar fluid through an annulus with porous walls, magnetic field, and heat transfer, with heat source has been made. The radii of concentric cylinders have been assumed $r_1$ and $r_2$ ($r_2 > r_1$). In the cylindrical coordinate system (r, θ, z) the z-axis is the common axis for both the cylinders and r axis is along the radial direction. The fluid is injected through the inner and outer cylinder with arbitrary velocity $v_1$ and $v_2$ respectively. Also the inner and outer cylinders of annulus are moving with arbitrary velocities $w_1$ and $w_2$. The velocity components along radial and axial direction are u(r, z) and w(r, z) respectively, also N(r, z) is the micro rotation velocity component. According to these assumptions the governing fluid flow equations are:

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\partial w}{\partial z}=0 \qquad (1)$$

$$\rho\left(u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial P}{\partial r}-(\mu+\kappa)\left(-\frac{\partial^2 u}{\partial r^2}-\frac{1}{r}\frac{\partial u}{\partial r}+\frac{u}{r^2}-\frac{\partial^2 w}{\partial r\partial z}-\frac{\partial^2 u}{\partial z^2}\right)-\kappa\frac{\partial N}{\partial z}-\sigma B_0^2 u \qquad (2)$$

$$\rho\left(u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}\right)=-\frac{\partial P}{\partial z}+(\mu+\kappa)\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}-\frac{1}{r}\frac{\partial^2 u}{\partial r\partial z}+\frac{\partial^2 w}{\partial z^2}\right)-\kappa\left(\frac{\partial N}{\partial r}+\frac{N}{r}\right)-\sigma B_0^2 w \qquad (3)$$

$$\rho j\left(u\frac{\partial N}{\partial r}+w\frac{\partial N}{\partial z}\right)=-2\kappa N-\kappa\left(\frac{\partial w}{\partial r}-u\frac{\partial u}{\partial z}\right)-\gamma\left(\frac{\partial^2 N}{\partial r^2}+\frac{1}{r}\frac{\partial N}{\partial r}-\frac{N}{r^2}+\frac{\partial^2 N}{\partial z^2}\right) \qquad (4)$$

$$u\frac{\partial T}{\partial r}=\frac{\alpha}{r}\left(r\frac{\partial^2 T}{\partial r^2}+\frac{\partial T}{\partial r}-\frac{\partial^2 T}{\partial r\partial z}+\frac{\partial^2 T}{\partial z^2}\right)+\frac{Q(T-T_2)}{\rho C_p} \qquad (5)$$

here P is the fluid pressure, μ is dynamic viscosity of fluid, j is micro inertia parameter, α is thermal diffusivity of fluid, κ is vortex viscosity, ρ is fluid density, γ is spin-gradient viscosity, σ is electrical conductivity of fluid and Q is heat source.

The boundary conditions for these set of equations are:

$u(r, z)= -v_1$, $w(r, z)= w_1$, $N(r, z)=0$, $T(r, z)= T_1$ at $r = r_1$

$u(r, z)= -v_2$, $w(r, z)= w_1$, $N(r, z)=0$, $T(r, z)= T_2$ at $r = r_2$

The velocity components as given by Shrestha in [8] are

$$u(\lambda,z)=-\frac{v_2 f(\lambda)}{\sqrt{\lambda}}$$

$$w(\lambda,z)=-\left(U_0-\frac{2v_2 z}{r_2}\right)f'(\lambda)$$

$$N(\lambda,z)=\frac{1}{r_2}\left(U_0-\frac{2v_2 z}{r_2}\right)g(\lambda),$$

$$\theta=\frac{T-T_2}{T_1-T_2},\ S=\frac{Qr_2^2}{\rho C_p v},$$

here $U_0$ is the arbitrary constant, g(λ) and f(λ) are unknown functions and $\lambda=\left(\frac{r}{r_2}\right)^2$. θ is non dimensional temperature and S is non dimensional heat source parameter. After using above transformation the equations from (2) – (5) are converted into following set of ordinary differential equations:

$$Re(f'f''-ff''')-\frac{4}{1-M}(\lambda f''''+2f''')-\frac{M}{2(1-M)\sqrt{\lambda}}\left(-\frac{g}{\lambda}+4g'+4\lambda g''\right)+Ha^2 f''=0 \qquad (6)$$

$$a_j Re(-fg'+f'g)+\frac{2M}{1-M}\left(g-\sqrt{\lambda}f''\right)+\frac{(2-M)M}{m^2(1-M)}\left(-\frac{g}{\lambda}+4g'+4\lambda g''\right)=0 \qquad (7)$$

$$4(\lambda\theta''+\theta')+Re\,Pr\,f\,\theta'+S\,\theta\,Pr=0 \qquad (8)$$

The transformed boundary conditions are:

| | | | | | |
|---|---|---|---|---|---|
| f=α, | f '=$\beta_1$, | g=0, | θ=1 | at $\lambda=\lambda_0$ | |
| f=1, | f '=$\beta_2$, | g=0, | θ=0 | at λ=1 | (9) |

Where,

$$\beta_1=\frac{w_1}{-U_0+\frac{2v_2 z}{r_2}},\qquad \beta_2=\frac{w_2}{-U_0+\frac{2v_2 z}{r_2}}$$

$$\alpha=\frac{v_1}{v_2}\sqrt{\lambda_0},\quad \lambda_0=\left(\frac{r_1}{r_2}\right)^2,\qquad M=\frac{\kappa}{\mu+\kappa}$$

$$a_j=\frac{j}{r_2^2},\qquad m^2=\frac{r_2^2\kappa(2\mu+\kappa)}{\gamma(\mu+\kappa)}$$

$$Ha=B_0 r_2\sqrt{\frac{\sigma}{\mu}},\qquad Re=\frac{\rho v_2 r_2}{\mu}$$

M is known as coupling number, Ha is Hartmann number, Re is the cross flow Reynolds number, $m^2$ is the micropolar parameter and $a_j$ is the micro inertia parameter.

## III. NUMERICAL COMPUATION

The system of boundary value problem (6)-(8) with boundary conditions (9) has been solved with the help of Runge-Kutta Felberg method with shooting technique, by taking $0.25 \le \lambda \le 1$.

In this method firstly system of equations (6)-(8) with boundary conditions are reduced to the first order system by introducing new variables,

$$y_1=f, y_2=f', y_3=f'', y_4=f'''$$
$$y_5=g, y_6=g'$$
$$y_7=\theta, y_8=\theta'$$

$$\frac{d}{d\lambda}\begin{pmatrix}y_1\\y_2\\y_3\\y_4\\y_5\\y_6\\y_7\\y_8\end{pmatrix}=\begin{pmatrix}y_2\\y_3\\y_4\\f''''\\y_6\\g''\\y_8\\\theta''\end{pmatrix} \qquad \dots\dots\dots\ (10)$$

The converted boundary conditions are

$y_1=\alpha$, $y_2=\beta 1$, $y_5=0$, $y_7=1$ at $\lambda=0.25$

$y_1=1$, $y_2=\beta 2$, $y_5=0$, $y_7=0$ at $\lambda=1$

Here the missing initial conditions i.e. $y_3, y_4, y_6, y_8$ at $\lambda=0$ are chosen in such a way so that the target values at $\lambda=1$ are satisfied by the boundary conditions.

Then first order system of equation (10) with converted boundary values are solved with shooting method in the MATLAB software.

## V. RESULTS AND DISCUSSION

In the study effects of various parameters on radial velocity (f '), axial velocity (f), micro rotation velocity (g) components and temperature (θ) have been studied keeping other parameters constant (α=0.4, $\beta_1$=0.1, $\beta_2$=0.1, M=0.5, Ha=5, S=0.5, m=3, Pr=0.7, $a_j$=0.001, Re=5).

Figure 1 gives the effects of cross flow Reynolds number (Re) on micro rotation velocity component and shows that micro rotation velocity decreases with decreasing values of cross flow Reynolds number. It has been studied that there is no significant change on axial and radial velocity component with cross flow Reynolds number. In figure 2 it can be observed that temperature increases with decreasing values of cross flow Reynolds number. Figures 3 and 4 represent the effects of coupling number (M) and Hartmann number (Ha) on micro rotation velocity component. From these figures it can be seen that micro rotation velocity component increases with coupling number but decreases with Hartmann number. Figure 5 depicts the effects of micro inertia parameter ($a_j$) on micro rotation velocity component and shows that micro rotation velocity increases with increasing values of $a_j$. In figure 6 it can be seen that temperature decreases with increasing values of Prandlt number (Pr). From figures (8) – (10) the effects of α on axial velocity, radial velocity, micro rotation velocity and on temperature have been studied. These figures show that axial velocity increases, but radial velocity, micro rotation velocity and temperature decreases with increasing values of α. At last figure 11 present the effects of heat source parameter (S) on temperature and it has been observed that temperature increases with increasing values of heat source parameter.

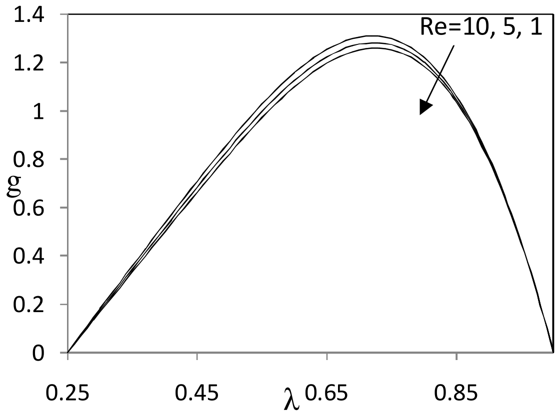



**Fig. 1.** Effects of Re on micro rotation velocity component (α=0.4, $\beta_1$=0.1, $\beta_2$=0.1, M=0.5, Ha=5, S=0.5, m=3, Pr=0.7, $a_j$=0.001)

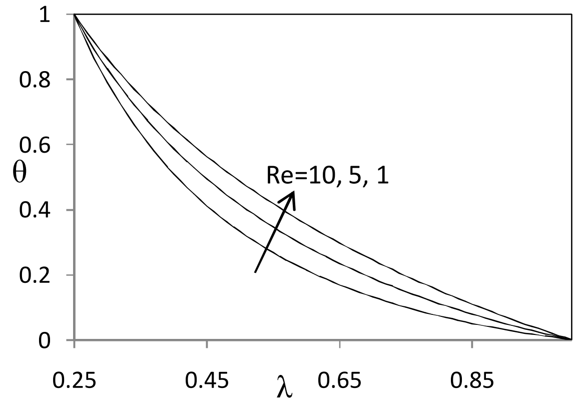



**Fig. 2.** Effects of Re on temperature (α=0.4, β1=0.1, β2=0.1, M=0.5, Ha=5, S=0.5, m=3, Pr=0.7, aj=0.001)

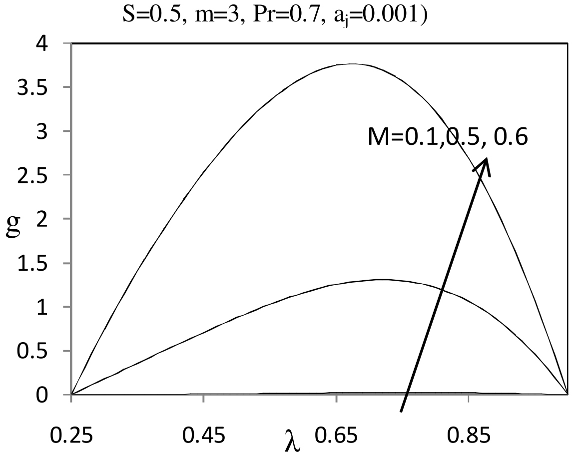



**Fig. 3.** Effects of M on micro rotation velocity component (α=0.4, β1=0.1, β2=0.1, Re=5, Ha=5, S=0.5, m=3, Pr=0.7, aj=0.001)

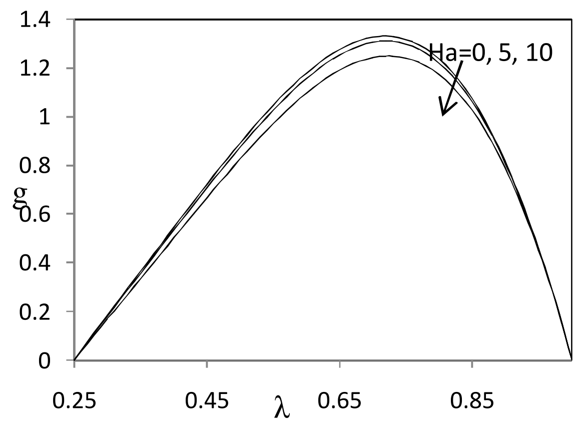



**Fig. 4.** Effects of Ha on micro rotation velocity component (α=0.4, β1=0.1, β2=0.1, Re=5, M=0.5, S=0.5, m=3, Pr=0.7, aj=0.001)

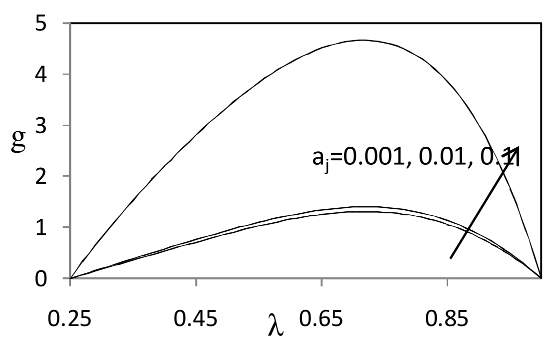



**Fig. 5.** Effects of aj on micro rotation velocity component (α=0.4, β1=0.1, β2=0.1, Re=5, Ha=5, S=0.5, m=3, Pr=0.7, aj=0.001)

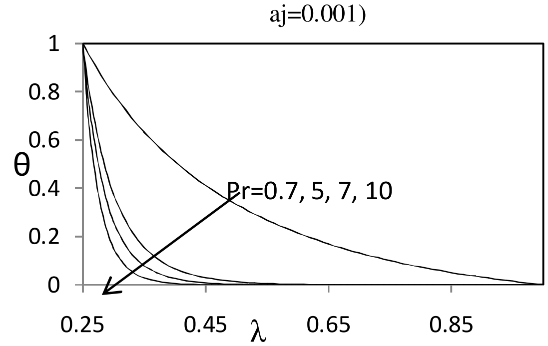



**Fig. 6.** Effects of Pr on temperature (α=0.4, β1=0.1, β2=0.1, Re=5, Ha=5, S=0.5, m=3, M=0.5, aj=0.001)

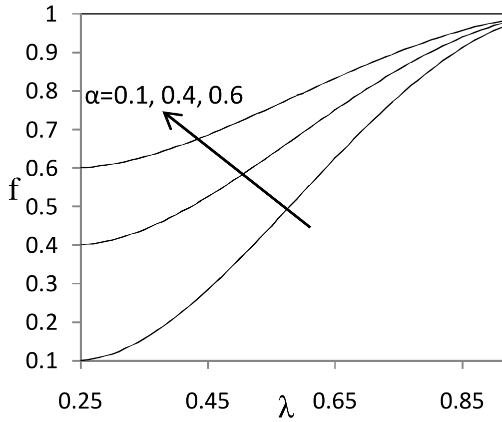



**Fig. 7.** Effects of α on axial velocity component (M=0.5, $\beta_1$=0.1, $\beta_2$=0.1, Re=5, Ha=5, S=0.5, m=3, Pr=0.7, $a_j$=0.001)

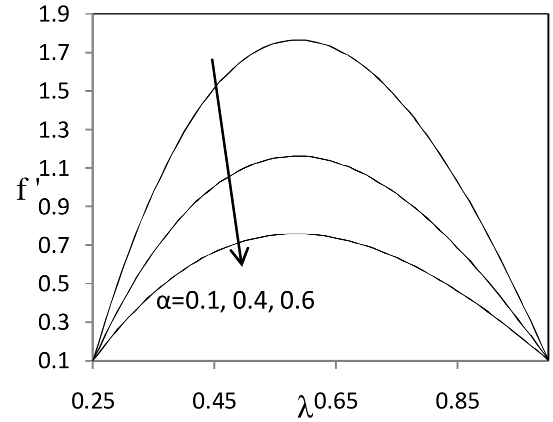



**Fig. 8.** Effects of α on radial velocity component (M=0.5, $\beta_1$=0.1, $\beta_2$=0.1, Re=5, Ha=5, S=0.5, m=3, Pr=0.7, $a_j$=0.001)

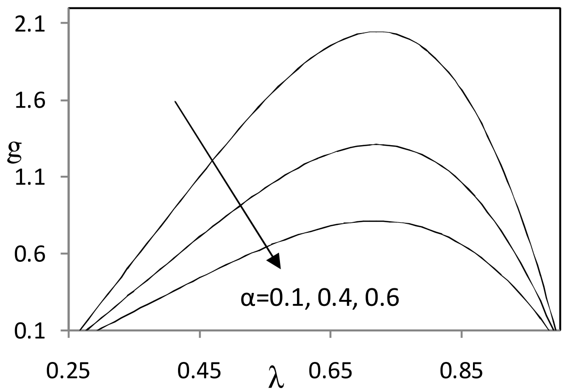



**Fig. 9.** Effects of α on micro rotation velocity component (M=0.5, $\beta_1$=0.1, $\beta_2$=0.1, Re=5, Ha=5, S=0.5, m=3, Pr=0.7, $a_j$=0.001)

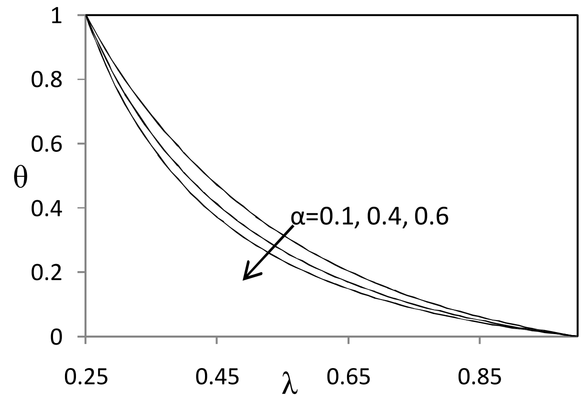



**Fig. 10.** Effects of α on temperature (M=0.5, $\beta_1$=0.1, $\beta_2$=0.1, Re=5, Ha=5, S=0.5, m=3, Pr=0.7, $a_j$=0.001)

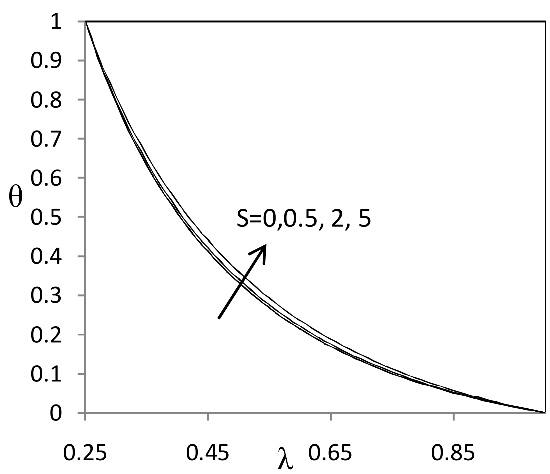



**Fig. 11.** Effects of S on temperature (M=0.5, $\beta_1$=0.1, $\beta_2$=0.1, Re=5, Ha=5, α=0.5, m=3, Pr=0.7, $a_j$=0.001)

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