



## Redundancy Optimization of Flow Network Using Importance Measure

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**ABSTRACT:** Capacity-related reliability (CRR) is the probability that a flow network (FN) has at least a flow capacity of  $W_{\min}$  between a pair of nodes. Redundancy optimization of networks as connectivity measure has extensively been studied. However, redundancy optimization of flow network is a rarely studied problem to the best of our knowledge. In this paper CRR of flow network is optimized using prioritization of link under capacity constraint. A new importance measure has been proposed. The method is demonstrated with the help of bridge flow network.

**Keywords:** Capacity-related reliability; importance measure; flow networks; redundancy

### I. INTRODUCTION

System reliability analysis often assumes that the system is represented by probabilistic graph, and the system is functioning if there is a path from input node to output node. Thus the reliability is considered as a measure of connectivity only and reliability analysis has been primarily concerned with the enumeration of paths or cuts in graph. But in many physical systems such as telecommunication network, power transmission system or oil/water pipeline system etc. there are numbers associated with every branch representing flow through the branch; consequently, reliability of a network is not necessarily characterized by only connectivity but a network is good if and only if a specific amount of flow is transmitted from the input node to the output node. Based on this criterion, we consider reliability of flow network whose branches are subjected to failure. Generally every flow network has a certain required capacity of information to be transmitted, and hence a minimum flow capacity requirement,  $W_{\min}$ , from input node to the output node of the network be available in order to keep successful operation of network.

#### A. Capacity Related Reliability

Redundancy optimization has extensively been studied in literature [1-5]. Capacity-related reliability (CRR) computes the probability that a flow network has at least a flow capacity of  $W_{\min}$  between a pair of nodes. Several methods are proposed for CRR computation. [6-15]. The aim of the present study is to redundancy optimization of flow systems.

### II. PROPOSED METHOD

In this paper, prioritization of component (link) of flow network is done using a newly defined importance measure. It considers both capacity and reliability of component/link. Component having highest priority is considered as critical component of flow network. This also gives the guidance for improving the reliability/making maintenance strategy etc. to improve CRR in optimum cost.

#### A. Assumption

- The flow network is modeled by a graph where in an order pair is assigned as the weight of each link. The members of the ordered pair are reliability and capacity.
- Each link can have only two stages success and fail. If the nodes are not perfect, Performance Indices can be obtained but with increased complexity in computation, if node failures are accounted for.

#### B. Methodology used

*Component Importance Factor(CIF):* Component importance factor is defined as ratio of reciprocal of component reliability multiplied with reciprocal of normalized capacity of component considered. Given as

$$CIF(x_i) \propto \frac{1}{r_i}$$

$$CIF(x_i) \propto \frac{1}{\left(\frac{c_i}{W_{min}}\right)}$$

$$CIF(x_i) = \frac{W_{min}}{r_i \times c_i} \dots \dots \dots (1)$$

Where,  $r_i$  is component reliability and  $c_i$  capacity of  $i^{th}$  component (link) and  $i = 1, 2, 3, \dots, n$ .  $W_{min}$  is minimum required flow capacity.

**C. Steps for Algorithm:**

Step I: Compute CIF for all the component / links.

Step II: Select critical link such that:

$$CIF(l_i) = \max\{CIF(l_i)\}$$

for  $i = 1, 2, 3, \dots, n$  and add a redundancy to this stage.

Step III: Check for the constraints:

- a. If no constraints are violated, then add one redundant component to subsystem  $i'$ , replace  $x_i'$  with  $x_i' + 1$ , and go to step I.
- b. If at least one constraint is exactly satisfied and other are not violated, then add one redundant component to subsystem  $i'$  replace  $x_i'$  with  $x_i' + 1$ .
- c. If at least one constraint is violated, then drop this component. The  $X^*(X_1, X_2, X_3, X_4, X_5)$  is the optimal solution. Stop the procedure.

Step IV: Revised table of  $X^*$  in term of modified reliability and capacity is generated.

Step V: This table is used to evaluate the CRR\* expression using one of the existing technique.

**D. CRR Evaluation**

Capacity related

$$CRR^*(system) = \sum_{i=1}^m P(T_i) - \sum_i \sum_j P(T_i, T_j) + \sum_i \sum_j \sum_k P(T_i, T_j, T_k) \dots$$

**III. ILLUSTRATION WITH BRIDGE NETWORK**

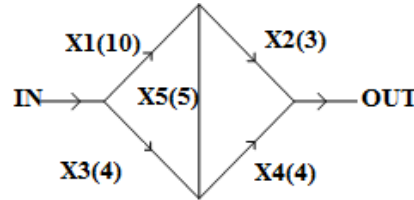
Here we consider a bridge network (shown in fig 1) having five components (link) whose reliability, capacity and cost data is given in table 1. Here it is assumed that at least 7 unit flow capacity is required from source to terminal node for proper functioning.

Maximize  $CRR_s(X)$  using  $CIF$ .

**Table 1: THE SUBSYSTEM DATA ARE.**

Component	X <sub>1</sub>	X <sub>2</sub>	X <sub>3</sub>	X <sub>4</sub>	X <sub>5</sub>
Reliability	0.70	0.85	0.75	0.8	0.9
Capacity	10	3	4	4	5
Cost	2	3	2	3	1

Consider the network shown in Fig 1:



**Fig. 1. Bridge Network.**

There is one linear constraint,  $\sum C_i X_i = 20$ , where  $i = 1$  to 5

Assume Minimum capacity of the network  $W_{min} = 7$  units. These 7 units of capacity have to flow through the network. Minimal forward paths are  $T_1\{x_1, x_2\}$ ,  $T_2\{x_3, x_4\}$ ,  $T_3\{x_1, x_4, x_5\}$ ,  $T_4\{x_2, x_3, x_5\}$  and there flow capacity before the optimization are tabulated in Table 2.

**TABLE 2: MINIMAL FORWARD PATHS AND RELATED FLOW CAPACITY.**

Path	Capacity
$T_1\{x_1, x_2\}$	$W_{p_1} = 3$
$T_2\{x_3, x_4\}$	$W_{p_2} = 4$
$T_3\{x_1, x_4, x_5\}$	$W_{p_3} = 4$
$T_4\{x_2, x_3, x_5\}$	$W_{p_4} = 3$

No path fulfils flow constrain. Hence all are considered failure paths.

Table 3 shows the data of component importance factor for all components using equation (1), on the bases of which the component having highest critical factor will be select for the redundancy allocation.

The table 4 gives the idea about the change in total number of component at each link and the total cost of network after each iteration of redundancy.

**TABLE 3: COMPONENT IMPORTANCE FACTOR (CIF).**

Component Importance Factor (CIF) of Five component					Selected component
X <sub>1</sub>	X <sub>2</sub>	X <sub>3</sub>	X <sub>4</sub>	X <sub>5</sub>	
1	2.75*	2.33	2.19	1.56	X <sub>2</sub>
1	1.19	2.33*	2.19	1.56	X <sub>3</sub>
1	1.19	0.933	2.19*	1.56	X <sub>4</sub>
1	1.19	0.933	0.912	1.56*	X <sub>5</sub>

**TABLE 4: COMPONENT REDUNDANCY ALLOCATION AND TOTAL COST OF NETWORK.**

Component redundancy allocation					Total Cost of network
X <sub>1</sub>	X <sub>2</sub>	X <sub>3</sub>	X <sub>4</sub>	X <sub>5</sub>	
1	1	1	1	1	11
1	2	1	1	1	14
1	2	2	1	1	16
1	2	2	2	1	19
1	2	2	2	2	20

Table 5 shows the revised reliability and  $\Delta r_i$ , is increment in component reliability after adding redundant component.

**TABLE 5: REVISED RELIABILITY.**

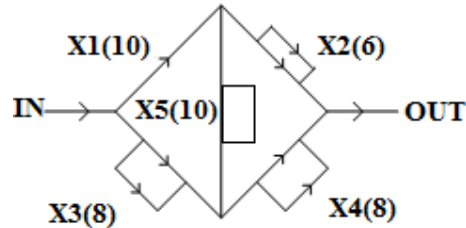
Component	X <sub>1</sub>	X <sub>2</sub>	X <sub>3</sub>	X <sub>4</sub>	X <sub>5</sub>	$\Delta r_i$
Change in the component reliability after redundant component	0.70	0.85	0.75	0.8	0.9	--
	0.70	0.97	0.75	0.8	0.9	0.1275
	0.70	0.978	0.938	0.8	0.9	1.875
	0.70	0.978	0.938	0.9	0.9	0.16
	0.70	0.978	0.938	0.9	0.99	0.099

Table 6 constructed after the redundancy allocation. The revised reliability and capacity of each link are computed after redundancy optimization and shown in the following table.

**TABLE 6: DATA OF COMPONENTS AFTER REDUNDANCY OPTIMIZATION.**

Component	X <sub>1</sub>	X <sub>2</sub>	X <sub>3</sub>	X <sub>4</sub>	X <sub>5</sub>
Reliability	0.70	0.9775	0.9375	0.96	0.999
Capacity	10	6	8	8	10

Here X<sub>3</sub> link is selected hence the total flow capacity of X<sub>3</sub> will be double that is 8 unit. Table 6: Data of components after redundancy optimization:



**Fig. 2. Optimal Bridge Network Structure.**

**TABLE 7: MINIMAL FORWARD PATHS AND RELATED FLOW CAPACITY AFTER OPTIMIZATION.**

Path	Capacity
$T_1 \{x_1, x_2\}$	$W_{T_1} = 6$
$T_2 \{x_3, x_4\}$	$W_{T_2} = 8$
$T_3 \{x_1, x_4, x_3\}$	$W_{T_3} = 8$
$T_4 \{x_2, x_3, x_5\}$	$W_{T_4} = 6$

Table 7 contains minimal forward paths and related flow capacity after optimization. From table it is clear that path  $T_2 \{x_3, x_4\}$  and  $T_3 \{x_1, x_4, x_3\}$  are the success paths, which allow the minimum required flow units ( $W_{min} = 7$ ). Paths  $T_1 \{x_1, x_2\}$  and  $T_4 \{x_2, x_3, x_5\}$  are failed paths, do not allow minimum required flow units ( $W_{min} = 7$ ). The intersection of these two failed paths results in a success flow path i.e.  $T_1 \{x_1, x_2\} \cap T_4 \{x_2, x_3, x_5\} = T_4 \{x_1, x_2, x_3, x_5\}$ . It can transmit flow of 10 units ( $> W_{min} = 7$ ). There are in total three success paths (allowing  $W_{min} = 7$ ). CRR evaluation of the flow network after redundancy optimization using equation (2) is found to be 0.94 shown in . CRR evaluation of the flow network before redundancy optimization using same equation is found to be 0.482.

**IV. CONCLUSION**

CRR for the given flow bridge network, without optimization criterion, (Fig. 1) for  $W_{min} = 7$  is 0.482.

Redundancy optimization yields  $X^*$  as (1, 2, 2, 2, 2). CRR\* is computed and it is found to be 0.94. There is 48.72 % of increase in system CRR with  $W_{\min} = 7$ . CRR is more practical, hence more useful measure as computed to connectivity measure.

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