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Convergence of CUIA Iteration in Real Hilbert Space

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ABSTRACT: This paper is based on CUIA iteration which is used for finding the fixed point of quasi contractive operators. Many researchers have introduced various iterative processes for self mappings but only a few researchers have introduced iterative processes for non self mappings and in both cases, they verified these results for the rate of convergence. This paper extends the result of Chauhan *et al.,* (Honam Mathematical J 39, 2017, No.1, pp 1-25) of self mapping to non-self mapping by proving the strong convergence result.
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Keywords: M-Lipschtiz contraction, CUIA, Pseudo contractive mapping.

I. INTRODUCTION

Fixed point theory is a tool to give the solutions of the equation $G(t) = t \;\forall\; t \in T$ where T is non-empty set and G be a self-map of T and $t \in T$. The most important result that is used in the area of fixed point theory is a Banach's contraction principle. It states that if $\exists$ a contraction mapping $G: T \to T$ and complete metric space T then the mapping $G: T \to T$ will have a unique fixed point $t \in T$.There are two methods to solve the equation $G(t) = t$. One is direct method and other is iterative method.
There are so many reasons, due to which direct methods fail to find the solution of the equation $G(t) = t$, therefore, we use iterative methods to find the solution of equation $G(t) = t$. Iterative method is a method in which we repeat the iteration again and again to find the solution of the equation. In the iterative fixed point procedure, the output varies like some of results are completely significant while the others are not.
Mann [2] introduced new iterative method to find the solution of a fixed point equation for non-expansive mapping where Picard's iterative method [1] fails to find the solution of fixed point equation for non-expansive mapping. Later, Ishikawa [4] introduced new iterative method for obtaining the convergence of a Lipschitzian pseudo-contractive operator while Mann's iterative method fails to apply on this mapping.
Choudhury [6, 10] applied Mann's iteration in Hilbert spaces for finding the solutions of fixed points for different mappings. Atsushiba [5] worked on asymptotically non- expansive mapping to find the rate of convergence with help of Mann iteration. There are more researchers [8-17], [19-20] who also worked on iteration process in different spaces in different types of mappings.

We get different results on iterative approximation from following direction:

1. The different result in spaces from where we are defining the operators.
2. The different result in contractiveness conditions that is associated to these operators.
3. The different result for estimating the parameters in different iterative procedure.

Many authors have given their iterative process for finding the fixed point. The commonly used fixed point iteration methods are the Picard iteration, the Mann iteration, the Krasnoselski iteration, the Ishikawa iteration, Noor iteration, CR iteration etc. The authors of these iteration worked on various types of mappings like expansive, non expansive, quasi–contractive, pseudo contractive mapping etc. These iterative schemes also applicable in Physics, coding theory, statistical physics.
In 1890, Picard [1] worked on iterative process for finding the fixed point and proved some convergence results by defining the iteration as follow:

$$i_{n+1} = G(i_n)$$

In 1953, Mann [2] gave the iteration as follow:

$$i_{n+1} = (1-\epsilon_n)i_n + \epsilon_n G(i_n)$$

where $\{\epsilon_n\}$ be a positive real numbers sequence in [0,1].

In 1955, Krasnoselski [3] improved the Mann iteration by introducing a constant 'λ' instead of the sequence ϵ_n and defined modified Krasnoselski iteration as:

$$i_{n+1} = (1-\lambda)i_n + \lambda\, G(i_n).$$

where λ lies in closed interval [0, 1].

In 1978, Ishikawa [4] defined new iteration for finding the fixed point and define by:

$$i_{n+1} = (1-\epsilon_n)i_n + \epsilon_n G[(1-\zeta_n)i_n + \zeta_n G(i_n)]$$

where $\{\epsilon_n\}$ and $\{\zeta_n\}$ are the positive real numbers sequence in [0,1].

After this, in 2000, Noor [7] gave their iterative process with help of iteration of Mann, Agarwal, and Thianwan by defining:

$$i_{n+1} = (1-\epsilon_n)i_n + \epsilon_n G(i_n)$$

$$j_n = (1-\zeta_n)i_n + \zeta_n G(k_n)$$

$$k_n = (1-\eta_n)i_n + \eta_n G(i_n)$$

where $\{\epsilon_n\}$, $\{\zeta_n\}$, $\{\eta_n\}$ are positive real numbers sequence in [0, 1].

In 2012, CR [18] introduced new iterative process for finding the fixed point defining by:

$$i_{n+1} = (1-\epsilon_n)j_n + \epsilon_n G(k_n)$$

$$j_n = (1-\zeta_n)G(j_n) + \zeta_n G(k_n)$$

$$k_n = (1-\eta_n)i_n + \eta_n G(i_n)$$

where $\{\epsilon_n\}$, $\{\zeta_n\}$, $\{\eta_n\}$ are positive real numbers sequence in [0,1]. In 2017, Chauhan *et al.*, [21] introduced some changes in existing iterations and established a new iteration by

$$i_{n+1} = (1-\epsilon_n)j_n + \epsilon_n G(k_n)$$

$$j_n = (1-\zeta_n)G(l_n) + \zeta_n G(k_n)$$

$$k_n = (1-\eta_n)G(i_n) + \eta_n G(l_n) \tag{1}$$

$$l_n = (1-\theta_n)i_n + \theta_n G(i_n)$$

where $\{\epsilon_n\}$, $\{\zeta_n\}$, $\{\eta_n\}$ and $\{\theta_n\}$ are positive real numbers sequence in [0, 1]

Definition 1.1 [22] Let K is a real Hilbert space with norm $\|\,.\,\|$andB is a non-empty subset of K. A mapping $G: B \to K$ is known as M-Lipschtiz if $\exists\ M \geq 0$ such that

$$\|G(i) - G(j)\| \leq M\,\|i - j\| \qquad \forall\ i, \quad j \in K$$

Definition 1.2 [22] A mapping $G: B \to K$ is known as pseudo contractive mapping if it satisfies the condition:

$$\|G(i) - G(j)\|^2 \leq \|i - j\|^2 + \|i - j - [G(i) - G(j)]\|^2 \qquad \forall\ i, j \in K$$

Lemma 1.1 [21] Let $\{\epsilon_n\}$ is a non-negative real numbers sequence which satisfied the following relation:

$$\epsilon_{n+1} \leq (1-\epsilon_n)b_n + \epsilon_n\theta_n \qquad n \geq 0$$

where $\{\varepsilon_n\} \subset [0,1]$ and $\theta_n \subset R$ satisfy the conditions $\sum_{n=0}^{\infty}\epsilon_n = \infty$ and $\lim\, max_{n\to\infty}\theta_n \leq 0$, then $\lim_{n\to\infty} b_n = 0$

Lemma 1.2 [22] Let us assume that B is an closed convex sub-set of space K and $G: B \to B$ is an mapping which is continuous pseudo- contractive mapping. Then

(1) S (G) will be closed convex sub-set of B.
(2) I-T will be zero for demi-closed set.

Lemma 1.3 [22] Let K is a real Hilbert space, then $\forall\ p,\ q \in K$ and $\epsilon \in [0,1]$, the resulting equality holds:

$$\|\epsilon\, i + (1-\epsilon)j\|^2 = \epsilon\,\|\,i\,\|^2 + (1-\epsilon)\|\,j\,\|^2 - \epsilon\,(1-\epsilon)\|i - j\|^2$$

Chauhan *et al.*, [21] estimated a fixed point in real and complex Banach space for self mapping and prove the strong convergence result and finally a comparison is made up on CUIA-iteration. The motive of this paper is to estimate a fixed point in the real Hilbert space for non-self mapping by using Chauhan *et al.*, [21] iteration. This proves the strong convergence result for CUIA iteration in the Hilbert space by using the Lemma 1.1-1.3.
Here in this paper, we prove a strong convergence theorem for non-self mappings using CUIA iteration. This is an extension of CUIA result.

II. MAIN RESULT

Theorem 2.1: Let B is a non- empty closed convex sub-set of space K and $G: B \to K$ be an M-Lipchitz non-self inward mappings such that $S\,(G) \neq \emptyset$ with $M \in [0,1)$ for any $i_0 \in B$. Let us define a sequence $\{i_n\}_{n=0}^{\infty}$ with the help of CUIA iteration. Let us define $\{\epsilon_n\}$, $\{\zeta_n\}$, $\{\eta_n\}$ and $\{\theta_n\}$ of real numbers that lie between [0,1] which satisfy the condition $\sum_{n=0}^{\infty}\epsilon_n = \infty$. Then the sequence $\{i_n\}_{n=0}^{\infty}$ converges strongly to fixed point of G.

Proof: Let $m\ \in\ S\,(G)$ is a fixed point of mapping

Let CUIA iteration defines by (1)

$$\|i_{n+1} - m\|^2 = \|(1-\epsilon_n)j_n + \epsilon_n G(j_n) - m\|^2$$

$$= \|(1-\epsilon_n)j_n + \epsilon_n G(j_n) - m + \epsilon_n m - \epsilon_n m\|^2$$

$$= \|(1-\epsilon_n)[j_n - m] + \epsilon_n[G(j_n) - m]\|^2$$

$$= \|\epsilon_n(G(j_n) - m) + (1-\epsilon_n)(j_n - m)\|^2$$

Now, by lemma (1.3)

$$\leq \epsilon_n\|[G(j_n) - m]\|^2 + (1-\epsilon_n)\|j_n - m\|^2 - \epsilon_n(1-\epsilon_n)\|G(j_n) - j_n\|^2$$

By definition (1.1)

$$\leq \epsilon_n M^2\|j_n - m\|^2 + (1-\epsilon_n)\|j_n - m\|^2 - \epsilon_n(1-\epsilon_n)\|G(j_n) - j_n\|^2$$

$$\leq [\epsilon_n M^2 + (1-\epsilon_n)]\|j_n - m\|^2 - \epsilon_n(1-\epsilon_n)\|G(j_n) - j_n\|^2 \tag{2}$$

Now $\|G(j_n) - j_n\|^2 = \|G\,(j_n) - j_n - m + \ m\|^2$
$= \|(G(j_n) - m) - (j_n - m)\|^2$

$$\leq \|(G(j_n) - m)\|^2 + \|(j_n - m)\|^2$$

$$\leq M^2\|(j_n - m)\|^2 + \|(j_n - m)\|^2$$

$$\leq (1 + M^2)\|(j_n - m)\|^2 \tag{3}$$

Put values of Eqn. (3) in (2) equation

Then Eqn. (2) will be turns into

$$\|i_{n+1} - m\|^2 \le [\epsilon_n M + (1-\epsilon_n) - \epsilon_n(1-\epsilon_n)(1 + M^2)]\|(\epsilon_n - m)\|^2$$

$$\le [1 - 2\epsilon_n + (1+M^2)\epsilon_n^2]\|(j_n - m)\|^2 \quad (4)$$

Now we will find value of $\|(j_n - m)\|^2$

So further,

$$\|j_n - m\|^2 = \|(1-\zeta_n)G(l_n) + \zeta_n G(k_n) - m\|^2$$

$$= \|(1-\zeta_n)G(l_n) + \zeta_n G(k_n) + \zeta_n m - \zeta_n m - m\|^2$$

$$= \|(1-\zeta_n)(G(l_n) - m) + \zeta_n(G(k_n) - m)\|^2$$

$$= \|\zeta_n(G(k_n) - m) + (1-\zeta_n)(G(l_n) - m)\|^2$$

$$\le \zeta_n\|[G(k_n) - m]\|^2 + (1-\zeta_n)\|G(l_n) - m]\|^2 - \zeta_n(1-\zeta_n)\|G(k_n) - G(l_n)\|^2$$

$$\le \zeta_n M^2\|k_n - m\|^2 + (1-\zeta_n)M^2\|l_n - m\|^2 - \zeta_n(1 - \zeta_n)M^2\|k_n - l_n\|^2 \quad (5)$$

Now, we will find the value of $\|k_n - m\|^2, \|l_n - m\|^2, \|k_n - s_n\|^2$

So,

$$\|k_n - m\|^2 = \|(1-\eta_n)G(i_n) + \eta_n G(l_n) - m\|^2$$

$$= \|(1-\eta_n)G(i_n) + \eta_n G(l_n) + \eta_n m - \eta_n m - m\|^2$$

$$= \|(1-\eta_n)(G(i_n) - m) + \eta_n(G(l_n) - m)\|^2$$

$$\le \eta_n\|[G(l_n) - m]\|^2 + (1-\eta_n)\|[G(i_n) - m]\|^2 - \eta_n(1-\eta_n)\|G(l_n) - G(i_n)\|^2$$

$$\le \eta_n M^2\|l_n - m\|^2 + (1-\eta_n)M^2\|i_n - m\|^2 -$$

$$\eta_n(1-\eta_n)M^2\|l_n - i_n\|^2 \quad (6)$$

Now, we will find $\|l_n - m\|^2, \|l_n - i_n\|^2$

Therefore,

$$\|l_n - m\|^2 = \|(1-\theta_n)i_n + \theta_n G(i_n) - m\|^2$$

$$= \|(1-\theta_n)i_n + \theta_n G(i_n) + \theta_n m - \theta_n m - m\|^2$$

$$= \|(1-\theta_n)(i_n - m) + \theta_n(G(\theta_n) - m)\|^2$$

$$= \|\theta_n(G(i_n) - m) + (1-\theta_n)(i_n - m)\|^2$$

$$\le \theta_n\|G(i_n) - m\|^2 + (1-\theta_n)\|i_n - m\|^2 - \theta_n(1-\theta_n)\|G(i_n) - i_n\|^2$$

$$\le \theta_n M^2\|i_n - m\|^2 + (1-\theta_n)\|i_n - m\|^2 - \theta_n(1-\theta_n)\|G(i_n) - i_n\|^2$$

$$\le [\theta_n M^2 + (1-\theta_n)]\|i_n - m\|^2 - \theta_n(1-\theta_n)\|G(i_n) - i_n\|^2 \quad (7)$$

Thus,

$$\|G(i_n) - i_n\|^2 = \|G(i_n) - m + m - i_n\|^2$$

$$= \|(G(i_n) - m) - (i_n - m)\|^2$$

$$\le \|(G(i_n) - m)\|^2 + \|(i_n - m)\|^2$$

$$\le M^2\|(i_n - m)\|^2 + \|(i_n - m)\|^2$$

$$\le (1 + M^2)\|(i_n - m)\|^2$$

By putting this value in Eqn. (7) the equation will reduce

$$\|l_n - m\|^2 \le [\theta_n M^2 + (1-\theta_n) - \theta_n(1-\theta_n)(1 + M^2)]\|i_n - m\|^2$$

$$\le [1 - 2\theta_n + \theta_n^2(1 + M^2)]\|i_n - m\|^2 \quad (8)$$

Now

$$\|l_n - i_n\|^2 = \|l_n - i_n - m + m\|^2$$

$$= \|(l_n - m) - (i_n - m)\|^2$$

$$\le \|l_n - m\|^2 + \|i_n - m\|^2$$

$$\le [1 - 2\theta_n + \theta_n^2(1 + M^2)]\|i_n - m\|^2 + \|i_n - m\|^2$$

By Eqn. (8)

$$\le [1 - 2\theta_n + \theta_n^2(1 + M^2) + 1]\|i_n - m\|^2$$

$$\le [2 - 2\theta_n + \theta_n^2(1 + M^2)]\|i_n - m\|^2 \quad (9)$$

Now, put Eqns. (8) and (9) in (6) we get

$$\|k_n - m\|^2 \le \eta_n M^2[1 - 2\theta_n + \theta_n^2(1 + M^2)]\|i_n - m\|^2 + (1-\eta_n)M^2\|[i_n - m\|^2 - \eta_n(1-\eta_n)M^2[2 - 2\theta_n + \theta_n^2(1 + M^2)]\|i_n - m\|^2$$

$$\le \eta_n M^2\big[1 - 2\eta_n + \theta_n^2(1 + M^2) + (1-\eta_n)M^2 - \eta_n(1-\eta_n)M^2[2 - 2\theta_n + \theta_n^2(1 + M^2)]\big]\|i_n - m\|^2$$

$$\|k_n - m\|^2 \le M^2[1 - 2\eta_n(1 + \eta_n) - \theta_n\eta_n(2\eta_n - \theta_n\eta_n - \theta_n\eta_n M^2)]\|i_n - m\|^2 \quad (10)$$

Now we will find $\|k_n - l_n\|^2$

$$\|k_n - l_n\|^2 = \|k_n - l_n - m + m\|^2$$

$$= \|(k_n - m) - (l_n - m)\|^2$$

$$\le \|(k_n - m)\|^2 + \|(l_n - m)\|^2$$

$$\le M^2[1 - 2\eta_n(1 + \eta_n) - \theta_n\eta_n(2\eta_n - \theta_n\eta_n - \theta_n\eta_n M^2)]\|i_n - m\|^2 + [1 - 2\theta_n + \theta_n^2(1 + M^2)]\|i_n - m\|^2$$

By Eqns. (10) and (8)

$$\le M^2[1 - 2\eta_n(1 + \eta_n) - \theta_n\eta_n(2\eta_n - \theta_n\eta_n - \theta_n\eta_n M^2)] + [1 - 2\theta_n + \theta_n^2(1 + M^2)]\|i_n - m\|^2 \quad (11)$$

Put values of Eqns. (8), (10) and (11) in (5) equation

$$\|j_n - m\|^2 \le \zeta_n M^2 M^2[1 - 2\eta_n(1+\eta_n) - \theta_n\eta_n(2\eta_n - \theta_n\eta_n - \theta_n\eta_n M^2)]\|i_n - m\|^2 + (1-\zeta_n)M^2[1 - 2\theta_n + \theta_n^2(1+M^2)]\|i_n - m\|^2 - \zeta_n(1-\zeta_n)M^2(M^2[1 - 2\eta_n(1+\eta_n) - \theta_n\eta_n(2\eta_n - \theta_n\eta_n - \theta_n\eta_n M^2) + [1 - 2\theta_n + \theta_n^2(1+M^2)]])\|i_n - m\|^2$$

$$\le [\zeta_n M^4[1 - 2\eta_n(1+\eta_n) - \theta_n\eta_n(2\eta_n - \theta_n\eta_n - \theta_n\eta_n M^2)] + (1-\zeta_n)M^2[1 - 2\theta_n + \theta_n^2(1+M^2)] - \zeta_n(1-\zeta_n)M^2(M^2[1 - 2\eta_n(1+\eta_n) - \theta_n\eta_n(2\eta_n - \theta_n\eta_n - \theta_n\eta_n M^2)] + [1 - 2\theta_n + \theta_n^2(1+M^2)])]]\|i_n - m\|^2$$

By solving this constant we get,

$$\|j_n - m\|^2 \le [(1-\zeta_n)^2 M^2[1 - 2\zeta_n + \zeta_n^2(1+M^2)] + \zeta_n^2 M^4[1 - 2\eta_n(1+\eta_n) - \theta_n\eta_n(2\eta_n - \theta_n\eta_n - \theta_n\eta_n M^2)]]\|i_n - m\|^2 \qquad (12)$$

Substitute Eqn. (12) in (4) we get

$$\|i_{n+1} - m\|^2 \le [1 - 2\epsilon_n + (1+M^2)\epsilon_n^2][(1-\zeta_n)^2 M^2[1 - 2\theta_n + [1 - 2\theta_n + \theta_n^2(1+M^2)] + \zeta_n^2 M^4[1 - 2\eta_n(1+\eta_n) - \theta_n\eta_n(2\eta_n - \theta_n\eta_n - \theta_n\eta_n M^2)]]\|i_n - m\|^2$$

$$\le [1 - 2\epsilon_n + (1+M^2)\epsilon_n^2]M^2[[(1-\zeta_n)^2[1 - 2\theta_n + \theta_n^2(1+M^2)] + \zeta_n^2 M^2[1 - 2\eta_n(1+\eta_n) - \theta_n\eta_n(2\eta_n - \theta_n\gamma_n - \theta_n\eta_n M^2)]]]\|i_n - m\|^2$$

$$\le M^{n+1}\prod_{i=0}^{n}[1 - 2\epsilon_i + (1+M^2)\epsilon_i^2]\|i_0 - m\|^2$$

As $0 \le M < 1$ and $\sum_{n=0}^{\infty}\epsilon_n = \infty$

$$\Rightarrow \lim_{n\to\infty} M^{n+1}\prod_{i=0}^{n}(1 - 2\epsilon_i + (1+M^2)\epsilon_i^2) = 0$$

$$\Rightarrow \lim_{n\to\infty}\|i_{n+1} - m\|^2 = 0$$

$\Rightarrow \{i_n\}_{n=0}^{\infty}$ is converging at fixed point of S.

III. CONCLUSION

From this paper, we conclude that if we take this iteration on different spaces and we extend the iteration from self mapping to non-self mappings then it will converge to fixed point.

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