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Solving Bi-objective Interval Assignment Problem

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ABSTRACT: We proposed a minor-minimum method for BOIAP, where the coefficient of the objective functions has been considered as interval. This method gives the set of efficient/non-efficient solutions and best compromise solution for BOIAP. Using MATLAB software, a BOIAP is solved. For illustration, numerical example is given. This approach provides decision-makers with the necessary support when working with two objectives and different types of interval allocation problems.

Keywords: Bi-objective interval assignment problem, minor-minimum method, efficient solutions, non-efficient solutions and best compromise solution, MATLAB.

I. INTRODUCTION

One of the most important problems of optimization in production and service systems is the assignment problem (AP). Generally, AP consists of assigning a number of tasks (drivers, manager, teacher and bus) to an equal number of employees (buses, machine operator, classroom and distribution routes). Each task must be assigned to one and only one employee and one and only one task must be performed by each employee. Assignment cost is optimal if total costs are minimized or profit is maximized. The well-known Hungarian method (HM) developed by Kuhn [15] is recognized as the first practical method to solve the AP. Several articles on different methods for solving single objective AP is available in [5, 7, 10, 19-21]. Most of the studies, in real life circumstances, have considered only single objective function for the AP that is cost minimum or minimum time. An AP is called multi-objective AP if more than one objective is to be optimized in an AP such as the minimum cost, maximum profit and the minimum operation time. Bao *et al.*, investigated a new algorithm for the multi-objective AP [8]. Przybylski *et al.,* solved bi-objective AP by using two-phase method [18]. Bufardi obtained the efficiency of feasible solutions for MOAP [4]. Anuradha and Pandian solved bi-objective AP by obtaining all efficient solutions [3]. For finding an efficient solution to multi-objective AP, Adiche *et al.* applied a hybrid method [1]. Medvedeva and Medvedev solved multi-objective assignment problem by using dual uzawa approach [16]. In Ge *et al.*, (2012) used a new procedure for solving bi-criteria bottleneck AP [9]. Ahmed & Hammed proposed a bi-level multiobjective optimization model for solving the evacuation location AP [6]. Costs are not always in a crisp form, in real life problems. To deal with uncertain parameters in mathematical technique, inexact, fuzzy and interval programming technique have been proposed. Akilbasha *et al.*, have developed an advanced method for pharmaceutical sciences called the mid-width method for obtaining an optimal solution to fully interval integer TPs [2]. Kagade and Bajaj; Salehi solved multi-objective AP where the coefficient of cost of the objective functions are interval form by using fuzzy method and weighted min-max method [11, 12]. Khalifa and Al-Shabi (2018) studied MOAP under fuzzy environment by using an interactive approach [13]. The article focuses primarily on finding the set of all BOIAP solutions. Section II deals with the mathematical model of BOIAP and fundamental concepts. Minor-Minimum Method is discussed in Section III. Section IV provides the proposed algorithm supported with a numerical description and section V concludes the article.

II. MATHEMATICAL FORMULATION

We consider n buses in a company and the company has n drivers to process the buses. Each bus has to be associated with one and only one driver. A penalty c_{ij} and d_{ij} is the cost of execution and u_{ij} and v_{ij} is the deviation in route, time and so on, which is incurred when a bus j $(j=1,2,...,n)$ is processed by the driver i $(i=1,2,...,n)$. Let x_{ij} denote the assignment of j^{th} bus to i^{th} driver. Our aim is to determine the assignment of buses to drivers at minimum assignment cost and deviation in route.

Now, the mathematical model of the above BOIAP is given as follows.

$$\text{(G) Minimize } [z_1, z_2] = \sum_{i=1}^{m}\sum_{j=1}^{n}[c_{ij}, d_{ij}]\, x_{ij}$$

$$\text{Minimize } [z_3, z_4] = \sum_{i=1}^{m}\sum_{j=1}^{n}[u_{ij}, v_{ij}]\, x_{ij}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = 1, \quad i = 1,2,...,n \tag{1}$$

$$\sum_{i=1}^{n} x_{ij} = 1, \quad j = 1,2,...,n \tag{2}$$

$$x_{ij} = \begin{cases} 1, & \text{if } i\text{th driver is assigned to } j\text{th bus,} \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

We construct two interval assignment problems (IAP) from the given (G), namely, first objective IAP (G1) and second objective IAP (G2). We split (G1) and (G2) as lower bound IAP ($G1_L$), ($G2_L$) and upper bound IAP ($G1_U$), ($G2_U$) which is shown given below.

(G1) Minimize $\left[z_1, z_2\right] = \sum_{i=1}^{m}\sum_{j=1}^{n}\left[c_{ij}, d_{ij}\right] x_{ij}$

Subject to (1), (2) and (3) are satisfied.

($G1_L$) Minimize $z_1 = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} x_{ij}$

Subject to (1), (2) and (3) are satisfied.

($G1_U$) Minimize $z_2 = \sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij} x_{ij}$

Subject to (1), (2) and (3) are satisfied

(G2) Minimize $\left[z_3, z_4\right] = \sum_{i=1}^{m}\sum_{j=1}^{n}\left[u_{ij}, v_{ij}\right] x_{ij}$

Subject to (1), (2) and (3) are satisfied.

($G2_L$) Minimize $z_3 = \sum_{i=1}^{m}\sum_{j=1}^{n} u_{ij} x_{ij}$

Subject to (1), (2) and (3) are satisfied

($G2_U$) Minimize $z_4 = \sum_{i=1}^{m}\sum_{j=1}^{n} v_{ij} x_{ij}$

Subject to (1), (2) and (3) are satisfied.

The fundamental concepts of the arithmetic operators, partial ordering of closed bounded intervals, feasible and optimal solutions of the interval can be obtained in [14, 17].

Definition 1: A set $X^0 = \{x_{ij}^0, i = 1,2,...,m, j = 1,2,...,n\}$ is said to be feasible to the problem (G) if X^0 satisfies the conditions (1) to (3).

Definition 2: A feasible solution X^0 is said to be an efficient solution to the problem (G) if there exists no other feasible X of (G) such that $\left[Z_1(X), Z_2(X)\right] \le \left[Z_1(X^0), Z_2(X^0)\right]$ and $\left[Z_3(X), Z_4(X)\right] \le \left[Z_3(X^0), Z_4(X^0)\right]$ (or) $\left[Z_3(X), Z_4(X)\right] < \left[Z_3(X^0), Z_4(X^0)\right]$ and $\left[Z_1(X), Z_2(X)\right] < \left[Z_1(X^0), Z_2(X^0)\right]$. Otherwise, it is called non-efficient solution to the problem (G).

The minor-minimum method proceeds as follows:

III. MINOR-MINIMUM METHOD

Step 1: Construct (G1) and (G2) from the given (G).

Step 2: From the (G1), construct the ($G1_U$) and ($G1_L$) and obtain an optimal solution to the ($G1_U$) and ($G1_L$) by the HM.

Step 3: Construct ($G2_U$) and ($G2_L$) from the given (G2) and obtain an optimal solution to the ($G2_U$) and ($G2_L$) by the HM.

Step 4: Take the optimal solution of ($G1_U$) and ($G1_L$) in the ($G2_U$) and ($G2_L$) as a feasible solution which is efficient/non-efficient solution to (G).

Step 5: Find the minor of the highest assignment cost, say in the i^{th} row, from ($G2_U$) and ($G2_L$) and find an optimal solution by HM. Then go to the next highest cost cell of the i^{th} row to obtain an optimum solution. Repeat this procedure to obtain efficient/non-efficient solutions until all the highest cost cells of the ith row are considered.

Step 6: Now we begin with the optimal solution of ($G2_U$) and ($G2_L$) as a feasible solution of ($G1_U$) and ($G1_L$) which is efficient/non-efficient solution to (G).

Step 7: Repeat Step 5 for the ($G1_U$) and ($G1_L$).

Step 8: Combine all the efficient/non-efficient solutions of (G) found using the optimal solutions of (G1) and (G2). A set of efficient/non-efficient solutions to the (G) can be obtained from this.

IV. NUMERICAL EXAMPLE

A company has to work out the assignment of three different buses on three different drivers. Assume that there are two objectives in consideration: (i) the minimization of the total allocation costs that are used in the allocation (ii) the minimization of the total deviation in route that is used in the allocation. Because the allocation plan has been prepared in advance, we are generally unable to get this information precisely. For this condition, the usual way to obtain the interval data is through experience evaluation. The corresponding interval data is shown in Table 1.

Table 1.

	Buses		
	B_1	**B_2**	**B_3**
D_1	[1, 3] [3, 5]	[5, 9] [2, 4]	[4, 8] [1, 5]
Drivers, D_2	[7,10] [4, 6]	[2, 6] [7, 10]	[3, 5] [9, 11]
D_3	[7, 11] [4, 8]	[3, 5] [3, 6]	[5, 7] [1, 2]

Now, using Step 1 the (G1) and (G2) to the given (G) is given in Table 2.

Table 2.

	G1			**G2**		
	B_1	**B_2**	**B_3**	**B_1**	**B_2**	**B_3**
D_1	[1, 3]	[5, 9]	[4, 8]	[3, 5]	[2, 4]	[1, 5]
D_2	[7, 10]	[2, 6]	[3, 5]	[4, 6]	[7, 10]	[9, 11]
D_3	[7, 11]	[3, 5]	[5, 7]	[4, 8]	[3, 6]	[1, 2]

Now, using Step 2 the ($G1_L$) and ($G1_U$) to the given (G1) is shown in Table 3.

Table 3.

	$G1_L$			**$G1_U$**		
	B_1	**B_2**	**B_3**	**B_1**	**B_2**	**B_3**
D_1	1	5	4	3	9	8
D_2	7	2	3	10	6	5
D_3	7	3	5	11	5	7

Now, the optimal allotment of ($G1_L$) and ($G1_U$) by HM is $D_1 \rightarrow B_1$, $D_2 \rightarrow B_3$ and $D_3 \rightarrow B_2$ and the optimal assignment costs are 7 and 13. Therefore, the (G1) assignment cost is [7, 13].

Now, using Step 3 the ($G2_L$) and ($G2_U$) to the given (G2) is shown in Table 4.

Table 4.

	$G2_L$			**$G2_U$**		
	B_1	**B_2**	**B_3**	**B_1**	**B_2**	**B_3**
D_1	3	2	1	5	4	5
D_2	4	7	9	6	10	11
D_3	4	3	1	8	6	2

Now, the optimal allotment of ($G2_L$) and ($G2_U$) by HM is $D_1 \rightarrow B_2$, $D_2 \rightarrow B_1$ and $D_3 \rightarrow B_3$ and the optimal assignment costs are 7 and 12. Therefore, the (G2) assignment cost is [7, 12].

In the below Table 5 by using Step 4, consider the optimal solution ($G1_L$) and ($G1_U$) in the ($G2_L$) and ($G2_U$) as a feasible solution.

Table 5.

	$G2_L$			$G2_U$		
	B_1	B_2	B_3	B_1	B_2	B_3
D_1	**3**	2	1	**5**	4	5
D_2	4	7	**9**	6	10	**11**
D_3	4	**3**	1	8	**6**	2

Therefore, the assignment cost of (G2) is [15, 22] and assignment cost of (G1) is [7, 13] for the allotment is $D_1 \to B_1$, $D_2 \to B_3$ and $D_3 \to B_2$.
Now, using Step 5, we obtain the set of all solutions S_1 of the (G) obtained from (G1) to (G2) is given in Table 6.

Table 6.

S. No.	Assignment	Efficient/ Non-efficient solutions
1.	$D_1 \to B_1$,$D_2 \to B_3$and$D_3 \to B_2$	[7, 13], [15, 22])
2.	$D_1 \to B_1$,$D_2 \to B_2$and$D_3 \to B_3$	([8, 16], [11, 17])
3.	$D_1 \to B_3$,$D_2 \to B_1$and$D_3 \to B_2$	([15, 26], [8, 12])

Similarly, by Step 6 and 7, we find the set of all solutions S_2 of the (G) obtained from (G2) to (G1) is given in Table 7.

Table 7.

S. No.	Assignment	Efficient/Non-efficient solutions
1.	$D_1 \to B_2$, $D_2 \to B_1$ and $D_3 \to B_3$	([17, 26], [7, 12])
2.	$D_1 \to B_3$, $D_2 \to B_1$ and $D_3 \to B_2$	([14, 23], [8, 17])
3.	$D_1 \to B_1$,$D_2 \to B_2$and $D_3 \to B_3$ and $D_1 \to B_1$,$D_2 \to B_3$ and $D_3 \to B_2$	([8, 13], [11, 22])
4.	$D_1 \to B_1$,$D_2 \to B_2$ and $D_3 \to B_3$	([8, 16], [11, 17])
5.	$D_1 \to B_1$,$D_2 \to B_3$ and $D_3 \to B_2$ and $D_1 \to B_1$, $D_2 \to B_2$ and $D_3 \to B_3$	([7, 16], [15, 17])
6.	$D_1 \to B_2$, $D_2 \to B_3$ and $D_3 \to B_1$	([13,25], [15, 23]

Now, the set of all solutions S of the (G) obtained from (G1) to (G2) and from (G2) to (G1) is given in Table 8.

Table 8.

S.No.	Assignment	$S=S_1 \cup S_2$
1.	$D_1 \to B_1$, $D_2 \to B_3$ and $D_3 \to B_2$	([7,13], [15, 22])
2.	$D_1 \to B_1$, $D_2 \to B_2$ and $D_3 \to B_3$	([8,16], [11, 17])
3.	$D_1 \to B_1$, $D_2 \to B_3$ and $D_3 \to B_2$ and $D_1 \to B_1$, $D_2 \to B_2$ and $D_3 \to B_3$	([7,16], [15, 17])
4.	$D_1 \to B_1$, $D_2 \to B_2$ and $D_3 \to B_3$ and $D_1 \to B_1$, $D_2 \to B_3$ and $D_3 \to B_2$	([8,13], [11, 22])
5.	$D_1 \to B_3$, $D_2 \to B_1$ and $D_3 \to B_2$	([14,23], [8, 17])
6.	$D_1 \to B_3$, $D_2 \to B_1$ and $D_3 \to B_2$	([15,26], [8, 12])
7.	$D_1 \to B_2$, $D_2 \to B_1$ and $D_3 \to B_3$	([17,26], [7, 12])
8.	$D_1 \to B_2$, $D_2 \to B_3$ and $D_3 \to B_1$	([13,25], [15,23])

The above table shows the set of all solutions for (G) using MMM obtained through manual calculation and further verified using MATLAB 2018 Intel Core i5 processor.
By using [2], we obtain the mid value of an interval which is shown in Table 9.

Table 9.

	Interval Value	Mid Value
Ideal Solution	([7, 13], [7,12])	(10, 9.5)
Efficient Solution	([7,13], [15, 22]) ([7,16], [15,17]) ([8, 16], [11,17]) ([14, 23], [8,17]) ([15, 26], [8,12]) ([17, 26], [7,12])	(10, 18.5) (11.5, 16) (12,14) (18.5, 12.5) (20.5,10) (21.5, 9.5)
Non-efficient Solution	([13, 25], [15, 23] ([8, 13], [11, 22])	(19, 19) (10.5, 16.5)
Best Compromise Solution	([8, 16], [11, 17])	(12,14)

From the Fig. 1, we see that the proposed method can find the ideal solution and set of efficient/non-efficient solutions. Kagade and Bajaj [11] applied the fuzzy programming approach using linear membership function and hyperbolic membership function for this example and obtained the best compromise solution as [[12,16],[14,17]]. Using our proposed technique, the best compromise solution to this example is obtained as [[8,16], [11,17]] which is better than the solution in [11].

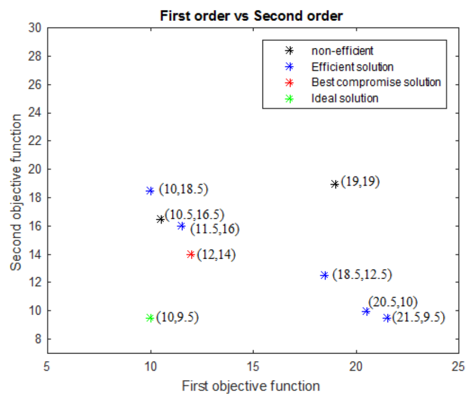

Fig. 1. The solutions obtained from the MMM.

V. CONCLUSION

To obtain a set of all solutions for a bi-objective interval assignment problem (BOIAP), the minor-minimum method (MMM) is proposed. The solutions obtained will help decision-makers assess economic behavior and make suitable administrative decisions while facing various logistical problems with two criteria.

VI. FUTURE SCOPE

The proposed method can also further be extended to different types of assignment problems like as bi-objective fuzzy assignment problem and bi-objective interval solid assignment problem.

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