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A Brief Review on Different Measures of Entropy

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ABSTRACT: Entropy shows up in numerous connections as a measure of discrete estates. Entropy measures are delivered which are based totally on Rayleigh and truncated Rayleigh distributions. It moreover gives the derived forms of the Shannon's Entropy Measures based totally on an exponential-power conclusion of the normal typical distribution. It also establishes a new generalized entropy measure and the indispensable homes of these measures are also reviewed. In this paper, a wonderful preservative and rapid affability of generalized hyperbolic measure of probabilistic entropy is presented. It additionally represents an effort of fuzzy idea in the variety of data principle which allowed chores to non-roman facts theory and discusses the generalized form of the Fisher's entropy type data measure. It also describes two types of complex fuzzy sets entropy measures, called type-A and type-B entropy measures and evaluates their rotational invariance claims.

Keywords: Information Theory, Entropy, Shannon's and Non-Shannon's Entropy Measures, Exponential Entropy, Complex Fuzzy Sets.

I. INTRODUCTION

The subject of 'Information Theory' expands the mathematical research of the troubles associated with communication, storage and transmission of messages. Information Theory has a measurable opening via Claude Shannon's 1948 paper i.e. 'A Mathematical Theory of Communication'. In that paper, he discussed that major problem of conversation system was emulating at one point either precisely or relatively information selected at any other point whereas information was correlated according to the sure physical or conceptual entities of the given machine of communication. The layout of the communication system should be like so that it can make use of for every feasible decision not simply be like which will surely be chosen as it was not recognized at the time of the graph of the verbal exchange system. The principal cause of a verbal exchange gadget was to elevate records from one point to any other point. One of the most sizeable consequences of Shannon's work was the thought of 'Entropy'. This thought was reformed the conversation theory. The origin of this time period was once brought in 1865 to describe a measure of the ailment of any machine and later it was once modified with the aid of Shannon in 1948 as a measure of the uncertainty. Shannon's forenamed that in the notion of records and entropy, there were in most cases three factors that were statistics supply which generated the message to be connected to the accepting period, channel that was the medium by which a signal travels from the supply toward destination and destination which received the message. The influential attempt of Shannon based totally on the work of Nyquist and Hartley intellectualized the tries in the direction of through to a systematic analytical idea of delivery and inaugurated the region of analysis that now recognized as Information Theory [13-15]. Shannon entropy estimated the conventional price of the records involved in an information often in system such as 'bits' and a message potential a genuine attention of the random variable. Uniformly, the Shannon's entropy was a measure of the ordinary facts complacent was lacking when one does now not be aware of the cost of the random variable. Entropy placed the authority for a considerable beneficiary of verbal exchange theory. Shannon's entropy perhaps studied in the process of one of the final meaningful crack concluded finished with the previous fifty years in the literature on probabilistic concern [10]. Non-Shannon's Entropies had a more superior vigorous vary than Shannon's Entropies over a range of diffusing actions and were consequently favourable in measuring solid density and consistency [16]. In the conversation theory, new generalized entropy measures were the generalizations of some infamous measures in the challenge of coding and statistics theory. Shannon measures can be attained from these generalized measures mutually [17]. Renyi's nominated a constant quantity conclusion of Shannon's measure in 1961 to enumerate affability to Shannon's measure [18, 19]. Later, Tsallis's hypothesized Shannon's measure in 1988 in the same direction [11]. Then, Shannon's entropy was the unique case about both the Renyi's entropy as properly as the Tsallis's entropy. Later on, Renyi, many one, two, three and four constant quantity conclusions had been supported with the help of the students within the subject of statistics concept. Havrda and Chavrat recommended another extension of Shannon's Entropy in 1967 to establish a new measure [8]. Later, Arimoto hypothesized Shannon's measure in 1971 and Awad elongated Shannon's measure in 1987 for the establishment of the new measures [7]. Awad modified Renyi's measure and corroborated Havrda and Chavrat's measure in 1987 to establish a new measure.
The view of fuzzy units was intended in 1965 by means of Zadeh with a component to deal with the troubles in which uncertainty appearing from an array of elemental doubt performed a central part [27, 28]. When the

induction of fuzzy idea beyond using Zadeh, then strategy of indistinctness had altered concerning each and each classification of analysis [21]. Fuzziness was a factor about unpredictability consequences against the discount about nice contrast of the borderline of a set i.e. authentic was not at all really a unit of the set not either truly no longer a unit about it. An initial test into evaluate the fuzziness was constructed via Zadeh in 1968 that one installed with probabilistic shape imported the entropy linking likelihood and group function (i) a fuzzy tournament during the time that alloyed Shannon entropy [29]. De Luca and Termini developed postulates along that the fuzzy entropy measure must examine moreover decided a cordial of entropy of a fuzzy set placed near Shannon's function [22]. Yager described an entropy of a fuzzy set in details based on cost of divergence among fuzzy set along with its opposite located over criterion [30]. Pal and Pal investigated an academic Shannon's entropy in 1989 and preferred a unique measure of entropy based on exponential function to extent the fuzziness called exponential fuzzy entropy [31, 32]. Hwang and Yang described the entropy of a fuzzy set along merging the approaches belonging to Yager and Pal and Pal [33]. Some parametric conclusions of De Luca and Termini's entropy are examined over the means of Kapur, Hooda, Bhandari and Pal, Fan and Ma [34, 35]. These particular frameworks supplied assertive affability in purposes including their beliefs had been basically resolved taken away the facts itself [36, 37]. Complex Fuzzy Set (CFS) was a branch about well-known fuzzy units wherein the matter based on group characteristic was the entity layer of the complex plane [38-40]. In the preferred fuzzy units moreover other different developments of fuzzy units, the measures of entropy were of excessive significance [41-43]. In the view of the functions of complex fuzzy sets, a few measures also their approaches were delivered as complex fuzzy sets, for instance distance measures, linguistic variables, rotational invariance, parallelity and orthogonality associations [51-53]. On the other hand, as long as the entropy of complex fuzzy units, in expectation we had, no associated research work had been announced still [44, 45]. All over, complex fuzzy sets perhaps described through complex-valued membership functions were consisted of an amplitude term and a phase term [49, 50]. Although an amplitude term contained the ordinary idea of 'fuzziness', a phrase term was a fully innovative framework of group function, which can genuinely analyzed classical fuzzy units against complex fuzzy sets [54, 55]. In this paper, two types of complex entropy measures of complex fuzzy sets had been introduced, one of that built upon over the amplitude of the complex-valued membership functions, that was intently associated into entropy about usual fuzzy units although avoided the phase term, whereas the different relies upon about both the amplitude and phase terms [46-48]. Fisher's entropy measure was an indispensable one [59]. Poincare and Sobolev's Inequalities exhibited a fundamental issue in the authority of the generalized Fisher's entropy measure [60]. A developing effort by means of Shannon relevant entropy with information theory also awarded an advanced viewpoint into the software about statistics structures and among others [56-58]. The concepts of entropy and fuzzy entropy were broadly promoted during a form of applications in science, engineering and administration [24, 25].

II. LITERATURE REVIEW

This section deals with the following entropy measures:

(i) Shannon Entropy Measures: Let the term 'Entropy' be denoted by 'E'. In the ancient times of information theory, the most essential moment was the delivery of Shannon's measure of information that was known as entropy suggested with the aid of Von Neumann whose formulae turn out to be a portrait for information theory [1, 2]:

$$E = -\sum_{i=1}^{n} p_i \log_2 \quad (1)$$

Hartley's suggested Shannon's entropy as the extent of information in the message which was consisted of m symbols preferred from the alphabet of *n* symbols [3]:

$$E = m \log_2 n \quad (2)$$

This was the simple case in which the output of a source can be formed as a random variable X which can take values from a set of symbols say $\sum = \{1, 2, \dots, n\}$ with probabilities p_i, $i \in \sum$.

Shannon's Measure Entropy had the following foremost three fundamental postulates which established entropy especially [10]:

1. Measure $E(p_1, p_2, \dots, p_n)$ should be continuous $\forall\ p_i$.
2. Assuming that all p_i should be equal, $p_i = \frac{1}{n}$, at that point E will be a monotonic increasing function of n. For uniformly expected cases there was greater desire or anxiety, although there were extra feasible cases.
3. Assuming that a desire be damaged falling toward two alternating options, the initial E will be the weighted amount of the entity values of E.

Later, these postulates were simply modified through A.I. Khinchin in view of D. K. Fadeev [4, 5]:

1. The message attained dependable only on the probability distribution $p = (p_1, p_2, \dots, p_n)$, therefore, it will be denoted by E (p) or $E(p_1, p_2, \dots, p_n)$ and imagine more that $E(p_1, p_2, \dots, p_n)$ be a well-formed function about its variables $p_1, p_2, \dots, p_n$.
2. $E(p, 1-p)$ be a stable function $\forall\ (0 \le p \le 1)$.
3. $E\left(\frac{1}{2}, \frac{1}{2}\right) = 1$.
4. The affiliation dominances:

$$E(p_1, p_2, \dots, p_n) = E(p_1, p_2, \dots, p_n) + (p_1 + p_2) E\left(\frac{p_1}{(p_1+p_2)}, \frac{p_2}{(p_1+p_2)}\right).$$

Alfred Reyni hypothesized Shannon's Entropy and established the measure as [6, 9]:

$$E_\beta(Y) = \frac{1}{1-\beta} \ln \int_{H_y} [f(y)]^\beta \, dy; \beta > 0, \beta \neq 1;$$
$$H_y = \{y : f(y) \neq 0\} \quad (3)$$

where β be an additional parameter and Y be a definitely stable non-negative random variable acquiring probability density function f(y).

Tsallis hypothesized Shannon's Entropy in the same direction and established the measure as [11]:

$$T_\beta(Y) = \frac{1}{\beta - 1}\ln \int_{H_y} [f(y)]^\beta\, dy; \beta > 0, \beta \neq 1;$$
$$H_y = \{y: f(y) \neq 0\} \quad (4)$$

Thus, Shannon's Entropy is the unique case about both the Renyi's entropy as properly as the Tsallis's entropy. Havrda and Chavrat recommended another extension of Shannon's Entropy and established the measure which is known entropy of degree β as [8]:

$$C^\beta(Y) = \frac{1}{2^{1-\beta} - 1}(\int_{H_y} [f(y)]^\beta \ln\frac{f(y)}{\varphi} dy); \beta > 0, \beta \neq 1;$$
$$H_y = \{y: f(y) \neq 0\};\ \text{where } \varphi = \text{Sup}_{x \in H_y} f(y) \quad (5)$$

These entropy measures were evolved by proposing one or more parameters in Shannon entropy $E_S(P)$.

Later, Sharma and Mittal further induced Havrda and Charvat's entropy and offered two-parametric measures called 'order-γ and type-ω' entropy as:

$$E^{(\gamma,\omega)}(P) = \frac{1}{(2^{1-\omega} - 1)}\left[\left(\sum_{i=1}^{n} p_i{}^\gamma\right)^{\frac{\omega-1}{\gamma-1}} - 1\right];$$
$$\gamma \neq 1,\ \omega \neq 1; \gamma,\ \omega > 0 \quad (6)$$

Sharma and Taneja also advised a two parametric generalization of Havrda and Charvat's entropy called 'type-(γ, ω) entropy' given by:

$$E^{(\gamma,\omega)}(P) = \frac{1}{(2^{1-\gamma} - 2^{1-\omega})}\sum_{j=1}^{n}(p_j{}^\gamma - p_j{}^\omega);$$
$$\gamma \neq \omega; \gamma, \omega > 0 \quad (7)$$

Arimoto established the measure of entropy as:

$$E(P) = \frac{1}{(2^{r-1}-1)}\left[\left(\sum_{i=1}^{n} p_i^{\frac{1}{r}}\right)^r - 1\right]; r \neq 1, r > 0 \quad (8)$$

Awad elongated Shannon's Entropy and established the measure as:

$$A(Y) = -\int_{H_y} f(y) \ln\frac{f(y)}{\varphi}\, dy \quad (9)$$

Awad et al. modified Renyi's Entropy and established the measure as:

$$A(Y)_\beta = \frac{1}{1-\beta}\ln \int_{H_y}\left[\frac{f(y)}{\varphi}\right]^{\beta-1} f(y)dy \quad (10)$$

Awad et al. corroborated Havrda and Chavrat's Entropy and established the measure as:

$$A^\beta(Y) = \frac{1}{2^{1-\beta} - 1}(\int_{H_y}\left[\frac{f(y)}{\varphi}\right]^{\beta-1} f(y)dy - 1);$$
$$\beta > 0, \beta \neq 1;$$
$$H_y = \{y: f(y) \neq 0\},\ \text{where } \varphi = \text{Sup}_{x \in H_y} f(y) \quad (11)$$

These exclusive entropy measures are used to determine the cost of entropy during the life time distribution is pretended new truncated Rayleigh over [0, t) rather based on thinking about Rayleigh distributions on [0, ∞)

(ii) Exponential Entropy Measures: Pal and Pal investigated an academic Shannon's entropy and preferred a unique measure of entropy based on exponential function as [31, 32]:

$$e^{E(P)} = \frac{1}{n(\sqrt{e}-1)}\sum_{j=1}^{n} p_j\,(e^{1-p_j} - 1) \quad (12)$$

Analogous to rationalizations of Shannon's Entropy including Renyi's entropy, Kvalseth [27] suggested and considered generalized exponential entropy of order-γ is inclined by:

$$E_\gamma(P) = \frac{\sum_{j=1}^{m} p(y_j)\left(e^{\left(1-p^\gamma(y_j)\right)} - 1\right)}{\gamma}; \gamma > 0 \quad (13)$$

(iii) Non-Shannon Entropy Measures: Let Non-Shannon's Entropy be denoted by E_{NS}.

Let the shaded plane circle graph $\{h_j,\ j = 0, 1, \ldots, m_{g-1}\}$ where m_g be the representation about definite shaded planes in the part about concern. In the case that when m is the entire representation of pixels in the domain, then the established circle graph about the region of concern be the set $\{H_j, j = 0, 1, \ldots, m_{g-1}\}$

where $H_j = \frac{h_j}{m}$.

Renyi's established Non-Shannon's measures as:

$$R(E_{NS}) = \frac{1}{1-\gamma}\log_2\left(\sum_{j=0}^{m_g-1} H_j^\gamma\right);\ \gamma \neq 1, \gamma > 0 \quad (14)$$

Havrda and Charvat provided Non-Shannon's measures as:

$$HC(E_{NS}) = \frac{1}{1-\beta}\left(\sum_{J=0}^{m_g-1} H_j^\beta - 1\right); \beta \neq 1, \beta > 0 \quad (15)$$

Kapur awarded Non-Shannon's measures as:

$$K^{(\gamma,\omega)}(E_{NS}) = \frac{1}{\omega-\gamma}\log_2\frac{\sum_{j=0}^{m_g-1} H_j^\gamma}{\sum_{j=0}^{m_g-1} H_j^\omega}; \gamma \neq \omega; \omega, \gamma > 0 \quad (16)$$

These measures were suitable criterion for normal and abnormal mammogram images analysis.

(iv) New Generalized Entropy Measure: Illustrate a new generalized entropy measure as [17]:

$$E_\gamma^\omega(P) = \frac{1}{\omega(1-\gamma)}\sum_{j=1}^{m} p_j^{\frac{\gamma}{\omega}}; 0 < \gamma < 1;$$
$$\omega \geq 1, p_j \geq 0\ \forall\, j = 1,2,\ldots,m \text{ and } \sum_{j=1}^{m} p_j = 1 \quad (17)$$

When ω = 1, then (17) moderated to entropy as:

$$E_\gamma(P) = \frac{1}{1-\gamma}\sum_{j=1}^{m} p_j^\gamma,\ 0 < \gamma < 1 \quad (18)$$

If ω = 1 and γ→ 1, then (17) diminished to Shannon's entropy as:

$$E(P) = -\sum_{j=1}^{n} p_j \log p_j \quad (19)$$

Properties of New Generalized Entropy Measures:

1. $E_\gamma^\omega(P)$ be a non-negative measure.

As from (17),

$$E_\gamma^\omega(P) = \frac{1}{\omega(1-\gamma)}\sum_{j=1}^{m} p_j^{\frac{\gamma}{\omega}}, \text{where}, 0 < \gamma < 1, \omega \geq 1$$

For given values of γ and ω,

$$\sum_{j=1}^{m} p_j^{\frac{\gamma}{\omega}} \geq 1 \text{ also } 0 < \gamma < 1, \omega \geq 1 \text{ and } \frac{1}{\omega(1-\gamma)} > 0$$

Therefore this terminate that,

$$\frac{1}{\omega(1-\gamma)}\sum_{j=1}^{m} p_j^{\frac{\gamma}{\omega}} \geq 0$$

2. $E_\gamma^\omega(P)$ be a symmetric function on every p_j ∀ j = 1, 2,...,m.

Note that $E_\gamma^\omega(P)$ be a symmetric function on every

$$p_j, j = 1,2,\ldots,m$$
$$i.e.\ E_\gamma^\omega(P_1, P_2, \ldots, P_{m-1}, P_m) = E_\gamma^\omega(P_m, P_1, P_2, \ldots, P_{m-1})$$

3. $E_\gamma^\omega(P)$ be a maximum function when ω=1 and γ→ 1 and all the events have equal probabilities.

When $p_j = \frac{1}{m}\ \forall\, j = 1,2,\ldots,m$ and $\omega = 1$ and $\gamma \to 1$ then, $E_\gamma^\omega(P) = \log m$ is the maximum entropy.

4. For γ→ 1 and ω=1, $E_\gamma^\omega(P)$ be a concave downward function for $P_1, P_2, \ldots, P_{m-1}, P_m$.

From (17), $\frac{1}{\omega(1-\gamma)}\sum_{j=1}^{m} p_j^{\frac{\gamma}{\omega}}, 0 < \gamma < 1, \omega \geq 1$

If γ→ 1 and ω=1, then the first derivative of (17)

With respect to p_j is inclined by:

$$\left[\frac{d}{dp_j} E_\gamma^\omega(P)\right]_{\omega=1,\gamma\to 1} = -1 - \log p_j$$

And the second derivative is inclined by:

$$\left[\frac{d^2}{dp_j{}^2} E_\gamma^\omega(P)\right]_{\omega=1,\gamma\to 1} = -\left(\frac{1}{p_j}\right) < 0$$
$$\forall\, p_j \in [0,1] \text{ and } j = 1,2,\ldots,m$$

As the second derivative of $E_\gamma^\omega(P)$ with respect to p_j is negative on given interval $p_j \in [0,1]$, as $j = 1, 2, \ldots, m$ as γ→ 1 and ω=1.

Therefore, $E_\gamma^\omega(P)$ be a concave downward function for $P_1, P_2, \ldots, P_{m-1}, P_m$.

(v) Generalized Hyperbolic Entropy Measure: The normal information comfort of a case against intent about 'm' cases, the entropy of the set A be explained as [18]:

$$E(P) = \sum_{j=1}^{m} p_j Inf(A_j) = -\sum_{j=1}^{m} p_j log p_j \quad (20)$$

Bhatia and Singh offered a constant quantity hyperbolic entropy measure as [20]:

$$E_\gamma(P) = -\frac{1}{Sinh(\gamma)} \sum_{j=1}^{m} p_j \operatorname{Sinh}(\gamma log p_j); \gamma > 0 \quad (21)$$

$$\lim_{\gamma \to 0} E_\gamma\ (P) = E(P) \quad (22)$$

After De Luca and Termini, several established forms of this fuzzy entropy were determined. Bhatia et al. intended a constant quantity hyperbolic entropy measure as [23]:

$$E_\beta(B) = \frac{-1}{Sinh(\beta)} \Big[\sum_{j=1}^{m} \delta_B(y_j) Sinh\big(\beta log \delta_B(y_j)\big) + \sum_{j=1}^{m} \big(1 - \delta_B(y_j)\big) Sinh\Big(\beta \log\big(1 - \delta_B(y_j)\big)\Big)\Big] \quad (23)$$

$$lim_{\beta \to 0} E_\beta\ (B) = E(B) \quad (24)$$

The real estimate β is correlated for non-ranginess about the entity. For conceptual appliance, measure designed in (21) is known as hyperbolic entropy and expressed as:

$$E_{hyp}(P) = \frac{-1}{Sinh(\gamma)} \sum_{j=1}^{m} p_j\, Sinh(\gamma \log p_j), \gamma > 0 \quad (25)$$

The hyperbolic entropy advised in (25) is related by Renyi entropy and Havrda-Charvat entropy [19, 26]. In order to compare, all of three entropies have been assigned.

(vi) Fuzzy Entropy Measure: If $Y = (y_1, \ldots, y_m)$ be a distinct universe of communication. A fuzzy set B at Y be constituted through a group function $\delta_B(y): Y \to [0,1]$ and the amount $\delta_B(y)$ about B on $y \in Y$ views during the extent about group of y in B.

If $\Delta_m = \{P = (p_1, \ldots, p_m): p_j \geq 0, \sum_{j=1}^{m} p_j = 1\}, m \geq 2$ be a set about m-entire probability distributions.

Considering several probability distributions as:

$$p_j = (p_1, \ldots, p_m) \in \Delta_m$$

Let Shannon's Entropy be explained just as:

$$E(P) = -\sum_{j=1}^{m} p(y_j) \log p(y_j) \quad (26)$$

First try through evaluate the concern combined along a fuzzy tournament in the situation about discrete probabilistic structure seems to have been built through Zadeh that fact described the entropy of a fuzzy set B including appreciate to (Y,P) as:

$$E(Y,P) = -\sum_{j=1}^{m} \delta_B(y_j) p(y_j) log p(y_j) \quad (27)$$

A degree about fuzziness in a fuzzy set ought to have at least the successive postulates:

1. (Incisiveness): $E(B)$ is minimum in the case that B is a crusty set i.e. $\delta_B(y_j) = 0\ or\ 1\ \forall\ j$.
2. (Maximality): $E(B)$ is maximum in the case that i.e. $\delta_B(y_j) = \frac{1}{2} \forall\ j$.
3. (Immovability): $E(B^*) \leq E(B)\ where\ B^*$ be an edged form of B.
   [A fuzzy set B^* be known as an edged form about fuzzy set B if the successive conditions be appeased: $\delta_{B^*}(y) \leq \delta_B(y), \text{if } \delta_B(y) \leq \frac{1}{2}\ \forall\ j$ and
   $\delta_{B^*}(y) \geq \delta_B(y), if\ \delta_B(y) \geq \frac{1}{2}\ \forall\ j$]
4. (Similarity): $E(B) = E(B^c)$
   where B^c is the complement set of B.

In view of $\delta_B(y_j)$ and $(1 - \delta_B(y_j))$ gives the identical diploma of fuzziness, accordingly De Luca and Termini described fuzzy entropy about a fuzzy set B analogous to (26) just as:

$$E(B) = \frac{-1}{m} \sum_{j=1}^{m} \left\{ \begin{matrix} \delta_B(y_j) log \delta_B(y_j) + \\ \big(1 - \delta_B(y_j)\big) \log\big(1 - \delta_B(y_j)\big) \end{matrix} \right\} \quad (28)$$

where B be a fuzzy set also $\delta_B(y_j)$ be group amount about y_j in B.

Later on Bhandari and Pal invented an analysis about fuzzy sets also provided a few recent measures of fuzzy entropy analogous to (3) they had recommended the consecutive measure as:

$$E_\beta(B) = \frac{1}{(1-\beta)} \sum_{j=1}^{m} \log\left\{\delta_B^\beta(y_j) + \big(1 - \delta_B(y_j)\big)^\beta\right\}$$

where $\beta \neq 1, \beta > 0$ (29)

Pal and Pal described exponential fuzzy entropy considering a fuzzy set analogous to (12) just as [31, 32]:

$$E(B) = \frac{1}{m(\sqrt{e} - 1)} \sum_{j=1}^{m} \left[\begin{matrix} \delta_B(y_j) e^{(1 - \delta_B(y_j))} \\ + \big(1 - \delta_B(y_j)\big) e^{\delta_B(y_j)} - 1 \end{matrix} \right] \quad (30)$$

(vii) Exponential Fuzzy Entropy: Assigned the exponential fuzzy entropy of order-β analogous to (13) just as [27]:

$$E_\beta(B) = \frac{1}{m\big(e^{(1-0.5^\beta)} - 1\big)} \sum_{j=1}^{m} \left[\begin{matrix} \delta_B(y_j) e^{(1 - \delta_B^\beta(y_j))} \\ + \big(1 - \delta_B(y_j)\big) e^{\left(1 - (1 - \delta_B(y_j))^\beta\right)} - 1 \end{matrix} \right]$$

$where\ \beta > 0$ (31)

Note that this exponential fuzzy entropy of order-β diminished into Pal and Pal exponential entropy also De-Luca and Termini logarithmic entropy about the definite amount of β just as displaces:

On the assumption when β=1 in (31), it diminished to

$$E(B) = \frac{1}{m(\sqrt{e} - 1)} \sum_{j=1}^{m} \left[\begin{matrix} \delta_B(y_j) e^{(1 - \delta_B(y_j))} \\ + \big(1 - \delta_B(y_j)\big) e^{\delta_B(y_j)} - 1 \end{matrix} \right] \quad (32)$$

where (32) is known as Pal and Pal exponential entropy.

In case if β→ 0, then (31) allows

$$\lim_{\beta \to 0} E_\beta(B) = E(B) = \frac{1}{m} \sum_{j=1}^{m} \left\{ \begin{matrix} \delta_B(y_j) log \delta_B(y_j) \\ + \big(1 - \delta_B(y_j)\big) \log\big(1 - \delta_B(y_j)\big) \end{matrix} \right\} \quad (33)$$

where (33) is known as De-Luca and Termini logarithmic entropy.

(viii) Complex Fuzzy Entropy Measures: If $Z = \{z_1, \ldots, z_m\}$ be a domain of communication, a complex fuzzy set C at Z can be expressed in the process of the set of ordered pairs $C = \{(z, \vartheta_c(z)) |\ z \in Z\}$ where membership function $\vartheta_c(z)$ is of the form $\varphi_C(z).e^{i\theta_c(z)}, i = \sqrt{-1}$, the amplitude term $\varphi_C(z)$ and the phase term $\theta_c(z)$ are both real-valued also $\varphi_C(z) \in [0,1]$ where $e^{i\theta_c(z)}$ be an alternate function that iteration law be 2π. So, $\theta_c(z) \in [0, 2\pi)$.

If A_1 and A_2 be two complex fuzzy sets. If a mapping $e: CFS(Z) \to [0,1]$ be known as a type-A entropy over $CFS(Z)$ in case that e amuses the successive adages:

(a1). $e(A_1) = 0$ in the case if $|\vartheta_{A_1}(z)| = 0\ or$ $|\vartheta_{A_1}(z)| = 1\ \forall\ z \in Z$.

(a2). $e(A_1) = 1\ if\ |\vartheta_{A_1}(z)| = 0.5\ \forall\ z \in Z$.

(a3). $e(A_1) \leq e(A_2)$ $if\ |\vartheta_{A_1}(z)| \leq |\vartheta_{A_2}(z)|$ when $|\vartheta_{A_2}(z)| \leq 0.5$ and $|\vartheta_{A_1}(z)| \geq |\vartheta_{A_2}(z)|$ when $|\vartheta_{A_2}(z)| \geq 0.5$

(a4). $e(A_1)=e(-A_1)$

where $-A_1$ is a complex fuzzy complement of A_1.

Consider two specific functions as:

$$e_1(C)=\frac{-1}{m}\sum_{j=1}^{m}\left[\begin{array}{c}\vartheta_c(z_j)\log\vartheta_c(z_j)\\+\left(1-\vartheta_c(z_j)\right)\log\left(1-\vartheta_c(z_j)\right)\end{array}\right] \quad (34)$$

$$e_2(C)=\frac{4}{m}\sum_{j=1}^{m}(\vartheta_c(z_j)\left(1-\vartheta_c(z_j)\right) \quad (35)$$

Two type-A entropy formulae respectively analogous to (34) and (35) are recommended as follows:

$$e_3(C)=\frac{-1}{m}\sum_{j=1}^{m}\left[\begin{array}{c}|\vartheta_c(z_j)|\log|\vartheta_c(z_j)|+\left(1-|\vartheta_c(z_j)|\right)\\ \log\left(1-|\vartheta_c(z_j)|\right)\end{array}\right] \quad (36)$$

$$e_4(C)=\frac{4}{m}\sum_{j=1}^{m}|\vartheta_c(z_j)|.\ |1-\vartheta_c(z_j)| \quad (37)$$

when $\theta_c(z)=0\ \forall\ z$ then, $e_3(C)=e_1(C)$ and $e_4(C)=e_2(C)$.

These mappings e_3 and e_4 described by formulae (36) and (37) respectively are type-A entropy measure as complex fuzzy sets.

If $C\in CFS(Z)$ with membership function $\vartheta_c(z)=\partial_C(z).e^{i\emptyset_C(z)}$. The revolution about C through θ radians, indicated by $Rot_\theta(C)$ is defined as [55]:

$$Rot_\theta(\vartheta_c(z))=\partial_C(z).e^{i(\emptyset_C(z)+\theta)}.$$

Let the entropy of CFSs $e:CFS(Z)\to[0,1]$ be rotationally invariable in the case:

If $e(Rot_\theta(C))=e(C)$ for any θ and $CFS(C)\in CFS(Z)$.

The mappings e_3 and e_4 are rotationally invariant.

As $|\vartheta_c(z_j)|=|\vartheta_c(z_j).e^{i\theta}|$ from this, $e_4(Rot_\theta(C))=e_4(C)$.

If B_1 and B_2 be two complex fuzzy sets. Let a mapping $e:CFS(Z)\to[0,1]$ be termed as type-B entropy about $CFS(Z)$ supposing that e amuses the successive adages:

(b1). $e(B_1)=0$ in the case if $\varphi_{B_1}(z)=0$ *or* $\varphi_{B_1}(z)=1$ and $\theta_{B_1}(z)=0\ \forall\ z\in Z$.

(b2). $e(B_1)=1$ *if* $\varphi_{B_1}(z)=0.5$ and $\theta_{B_1}(z)=\pi\ \forall\ z\in Z$.

(b3). $e(B_1)\le e(B_2)$ *if* $\varphi_{B_1}(z)\le\varphi_{B_2}(z)$ and $\theta_{B_1}(z)\ge\theta_{B_2}(z)$ for $\varphi_{B_2}(z)\le\frac{\theta_{B_2}(z)}{2\pi}$ *or* $\varphi_{B_1}(z)\ge\varphi_{B_2}(z)$ and $\theta_{B_1}(z)\le\theta_{B_2}(z)$ *for* $\varphi_{B_2}(z)\ge\frac{\theta_{B_2}(z)}{2\pi}$

(b4). $e(B_1)=e(-B_1)$

where $-B_1$ is a complex fuzzy complement of B_1.

Two type-B entropy formulae respectively analogous to (34) and (35) are recommended as follows:

$$e_5(C)=\frac{-1}{2m}\sum_{j=1}^{m}\begin{array}{c}[\varphi_c(z_j)\log\varphi_c(z_j)+\\ \left(1-\varphi_c(z_j)\right)\log\left(1-\varphi_c(z_j)\right)\end{array}+\frac{\theta_C(z)}{2\pi}log\frac{\theta_C(z)}{2\pi}+\left(1-\frac{\theta_C(z)}{2\pi}\right)\log\left(1-\frac{\theta_C(z)}{2\pi}\right)] \quad (38)$$

$$e_6(C)=\frac{2}{m}\sum_{j=1}^{m}\left[\begin{array}{c}\varphi_c(z_j)\left(1-\varphi_c(z_j)\right)+\left(\frac{\theta_C(z)}{2\pi}\right)\\ (1-\frac{\theta_C(z)}{2\pi})\end{array}\right] \quad (39)$$

These mappings e_5 and e_6 described by formulae (38) and (39) commonly are type-B entropy measure about complex fuzzy sets and are not rotationally invariant.

(ix) Fisher's Entropy Type Information Measure: Let U be a multivariate random variable for parameter vector $\emptyset=\{\emptyset_1,\ldots,\emptyset_q\}\in\mathbb{R}^q$ and probability density function $g(U)=g_u(u;\emptyset),u\in\mathbb{R}^q$ [12].

The Parametric Form Fisher's Information Matrix $I_F(U;\emptyset)$ represented as the covariance of $\nabla_\emptyset logg_u(u;\emptyset)$ (where $\nabla_\emptyset$ *be a* gradient about the parameters $\emptyset_j, j=1,\ldots,q$) be parametric form information measure indicated as:

$$I_F(U;\emptyset)=Cov[\nabla_\emptyset logg_u(u;\emptyset)]=E_\emptyset[\nabla_\emptyset logg_u(\nabla_\emptyset logg_u)^T]=E_\emptyset[\|\nabla_\emptyset logg_u\|^2]$$

(where $\|.\|$ be the usual $\mathbb{N}^2(\mathbb{R}^q)$ norm although $E_\emptyset[.]$ be the normal value operant enforced to random variables about the parameter $\emptyset$.

Let the Fisher's Entropy Type Information Measure $I_F(U)$ about a random variable U including probability density function g on $\mathbb{R}^q$ be termed as the covariance of random variable $\nabla\log g(U)$ *i.e.*

$$I_F(U)=\int_{\mathbb{R}^q}g(u)\|\nabla\log g(u)\|^2du=\int_{\mathbb{R}^q}g(u)^{-1}\|\nabla g(u)\|^2du=\int_{\mathbb{R}^q}\nabla g(u).\nabla logg(u)du=4\int_{\mathbb{R}^q}\left\|\nabla\sqrt{g(u)}\right\|^2du \quad (40)$$

Mostly, the group of the entropy form information measures I(U) about a random variable U including probability density function g, be termed through a co-function of U, i.e. $F(U)=\|\nabla\log g(u)\|$ *as*:

$$I(U)=I(U:f,k)=f\left(E\left[k\left(F(U)\right)\right]\right)$$

where f and k be real − valued functions.

Particular Case: When $f=i.d.$ and $k(U)=U^2$, then the entropy form Fisher's information measure of U is attained just as in (40) specially,

$$I_F(U)=E[\|\nabla\log g(u)\|^2] \quad (41)$$

Let Vajda's Parametric Form Information Measure $I_V(U;\emptyset,\beta)$, which is literally a generalization of $I_F(U;\emptyset)$ defined as [61, 62] :

$$I_V(U;\emptyset,\beta)=E_\emptyset\left[\|\nabla_\emptyset\log g(u)\|^\beta\right],\beta\ge1 \quad (42)$$

In addition, Vajda's Entropy Form Information Measure $I_V(U)$ concluded Fisher's entropy form information $I_F(U)$, expressed just as:

$$I_V(U;\beta)=E\left[\|\nabla\log g(u)\|^\beta\right],\beta>1 \quad (43)$$

It is indicated on the point of the established Fisher's entropy form information measure or $\beta-GFI$.

Let the second-GFI be abbreviated through the general I specially $I_2(U)=I(U)$ which is expressed as:

$$I_\beta(U)=\int_{\mathbb{R}^q}\|\nabla logg(u)\|^\beta g(u)du=\int_{\mathbb{R}^q}\|\nabla g(u)\|^\beta g^{1-\beta}(u)du=\beta^\beta\int_{\mathbb{R}^q}\left\|\nabla g^{1/\beta}(u)\right\|^\beta du \quad (44)$$

The Blackman-Stam inequality is influenced rigidly through the $\beta-GFI$ measure [63-65].

III. DISCUSSION

In this review paper, we have contributed a brief discourse almost a course of diverse entropy measures. Shannon's entropy is the measure as it is particular which fulfils clearly self-evident sayings of the degree of data. We have too compared diverse entropy measures for the Rayleigh conveyed life time demonstrate in terms of relative data misfortune. For exponentially disseminated lifetime demonstrate, Awad's entropy measures are found to be superior from misfortune of data point of see. Be that as it may, in our case, it is essential that Awad's entropy measures and their amplified adaptations are no better than that of Shannon, Renyi or Tsallis and Havrda and Charvat entropy measures are moderately superior than one another depending on diverse settings or circumstances.

An aspire is formed to establish the modern highlights description positioned on Non-Shannon's entropies (Renyi, Havrda and Charvat and Kapur) as analyzing typical also anomalous mammogram pictures. Test comes about have illustrated that Havrda and Charvat entropy established highlight as analyzing ordinary also unusual mammogram pictures works. We have introduced an advanced generalized entropy measure also analogous to this entropy, the important properties of the new entropy measure have also been discussed. We have also shown that the measures defined in this communication are the generalizations of a few recognized measures in the subject of coding and information theory. This paper also offers an advanced entropy measure which is termed as exponential fuzzy entropy of order-β in the context of fuzzy set theory. A few assets about this measure are also considered. This measure theorizes Pal and Pal exponential entropy and De-Luca and Termini logarithmic entropy. Presentation about parametric-β contributes unique affability also broad appliance of exponential fuzzy entropy through distinct positions.

A great additivity of established hyperbolic entropy measure be examined also observed a rapid adaptability about established hyperbolic entropy degree that related with a few academic established entropy measures for the view of parameter β. This parameter β within an established entropy measures be exceptionally critical in distinction to the point of appliance. Two, three or four parameter entropy measures give higher adaptability of appliance.

We have given a brief introduction about a lesson of established Fisher's entropy sort data measures along with entropy measures which amplify regular Shannon entropy like β-Shannon entropy and Renyi entropy. These expansions are based on an additional framework. This additional framework of the established Fisher's entropy sort data or about the established entropy alters 'assured' data measures into superior calms. These generalizations require encourage examination utilizing genuine information in other areas. Subsequently, these measures are connected on an exponential-power conclusion about the general typical dissemination also talked about. A ponder about an assertive frame of involvement has been too examined for such irregular factors. We have moreover presented two representations of entropy measures about complex fuzzy sets: type-A and type-B. With them, type-A builds upon the adequacy of acceptance capacities also may be decrease into its conventional partner, whereas type-B builds upon both the sufficiency and the stage of participation work. In expansion, the thought of rotational invariance for entropy measures of complex fuzzy sets is analyzed. It is demonstrated that two equations about the type-A entropy measures fulfils the asset of rotational invariance. In any case, in complex fuzzy sets, how to get it the acceptance degree of a protest to a set is analyzed by a complex number. An unused sort of set, the complex fuzzy set is displayed. The complex fuzzy set speaks to a unique way to the theory of participation through permitting it to be portrayed in terms of a complex number. A comprehensive ponder of the numerical assets of complex fuzzy sets is displayed. This paper has to be considered as a presentation about complex fuzzy sets. Hence thoroughly, the complex fuzzy sets appear to be an encouraging unused theory, clearing the approach to the various conceivable outcomes for future research.

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CONFLICT OF INTEREST

Authors have no any conflict of interest.

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