International Journal on Emerging Technologies 11(5): 536-544(2020)
ISSN No. (Print): 0975-8364
ISSN No. (Online): 2249-3255

Non-Linear Mathematical Modelling for Quarter Car Suspension Model

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(Received 08 September 2020, Revised 20 October 2020, Accepted 13 November 2020)
(Published by Research Trend, Website: www.researchtrend.net)

ABSTRACT: Numerical displaying is the most significant strategies were considered in the examination to foresee the conduct of the real framework under determined information conditions. The greater part of the analysts accepted the vehicle suspension framework as a linear model. Notwithstanding, when the vehicle suspension exposed to various road excitation it displays a non-linear qualities. The goal of this work is to build up a non-linear numerical displaying of quarter vehicle aloof suspension framework to get the nonlinear attributes of a vehicle suspension segments which are not drew by previous researchers. The ride quality including comfort and road holding is the primary factor that is targeted in the design of an effective suspension system. The objective of this study is to analyse the non-linear behaviour of basic components of the suspension system. In order to know the level of nonlinearities on the suspension parts, the curve fitting dependent on the test graphical information utilizing MATLAB fitting instrument has been performed. At the result, the non-linear numerical model dependent on the consequence of curve fitting for a quarter vehicle suspension framework is created. The mathematical model of the quarter-car is derived, and the dynamics are evaluated in terms of the sprung and unsprung mass displacement, velocity and acceleration. The simulation is performed for two different types of ground vehicles, light-duty and heavyduty vehicles. Results show that the addition of the nonlinear suspension components in the dynamic model of the two vehicles decreases the vibration of the sprung and unsprung mass to meet better ride comfort and road holding performance of the vehicle respectively. This implies that nonlinear suspension system is better than that of the conventional linear suspension system.

Keywords: Quarter vehicle suspension, Mathematical modeling, Linear model, Non-linear model, Data fitting, Linear curve fitting, Non-linear curve fitting.

I. INTRODUCTION

The examination of a vibrating framework normally includes numerical modeling, inference of the administering conditions, performing reproduction to get arrangement of the conditions, and translation of the outcomes. By utilizing the identical estimations of the mass, stiffness, and damping of the framework, a numerical model for a specific application can be gotten. In this examination, the quarter vehicle model is spoken to by a two-degree of freedom with spring-mass-damper framework. It comprises of double masses, double springs and single damper. The lower mass is un-sprung mass (M_{us}) that speaks to the haggle get together mass, and the upper one is sprung mass (M_s) that speaks to roughly quarter of the staying all out vehicle mass. The vertical movements of the two masses are portrayed by the displacement factors X_2 and X_1 for sprung and unsprung mass, respectively. The lower spring is portrayed by the tire stiffness kt. while the upper spring is spoken to by the coil spring stiffness (K). The un-sprung mass is energized by the road surface input (r).

II. RELATED WORK

Shpetim Lajqi *et al.,* & Dinçer Ozcan [1, 2] they have done an exploration on plans and enhancements of dynamic and semi-dynamic non-direct suspension frameworks for a landscape vehicle. They presented a plan and enhancement strategy for dynamic and semi-dynamic non-straight suspension frameworks with respect to landscape vehicles. Their exploration depended on the numerical model of nonlinear semi-dynamic and dynamic quarter vehicle suspension with twofold cosine street knocks and conditions of movement are determined and settled by utilizing MATLAB/Simulink. The reproduction result gave that dynamic nonlinear suspension framework gives preferable ride comfort execution over semi-dynamic and aloof suspension framework. At long last the specialists inferred that dynamic suspension framework improved the driving trademark by up to 80% contrasted with the uninvolved one.

Zhiyong *et al.,* [3] have depicted vibration concealment of four level of-opportunity nonlinear vehicle suspension model energized by the continuous hindrances. They have considered the chance of disorderly vibration of the four level of-opportunity half vehicle model under continuous speed control bumps on the interstate. At that point, an immediate variable input control was proposed to take out the impact of confusion on vehicle nonlinear vibrations and how to choose appropriate control boundaries was examined.

Chen *et al.,* [4] were exhibited in both reenactment and genuine vehicle explores that there is a conspicuous contrast between the ride comfort expectation of direct suspension and genuine vehicle test. The suspension's non-linearization factors can't be dismissed in the plan and utilization of suspension. The reproduction result and genuine vehicle explore show that just if the suspension's non-linearization factors are assessed when displaying the ride comfort, will the ride comfort execution be anticipated all the more definitely, along these lines adding to directing the plan and improvement of vehicles experimentally.

Mahesh *et al*., [5] have been explored on the non-direct quarter vehicle framework of seat and driver model and executed for unreal plan, for non-straight thought purposed a square of tire firmness and third degree solidness in suspension spring, edge, and seat pad with 4 degrees of opportunity driver model was introduced for streamlining and examination. In view of the Simulation result they presumed that ideal plan factors improve ride solace and wellbeing rules over traditional plan factors.

Bahman *et al*., [6] have proposed another way to deal with ideal control of non-direct dynamic vehicle suspension framework with input imperative. Another successful strategy was proposed for compelled ideal control of a vehicle suspension framework including non-strait qualities for flexible damping components. They contrasted the proposed compelled regulator and the unconstrained ideal regulator in the instances of costly and modest control for two sorts of street input. The outcomes showed that the obliged regulator uses the greatest limit of outer powers and thus achieves a superior presentation.

Jun Yao *et al*., & Jia Hong Yu *et al*., [7, 8] have been dissected strength of vehicle concerning the moderate changing sprung mass dependent on two degrees of opportunity quarter-vehicle model. The reproduction results demonstrated that presence of static just as unique bifurcation and the outcome prompts an adjustment in the last steady vibration of the suspension. Indeed, even the little vibration of the sprung mass will prompt plenty fullness transformation, prompting the sprung mass unsteadiness.

Puneet *et al.,* [9] considered damping coefficient values for quarter car analysis using Matlab simulink model. Vehicle speed, sprung mass, spring constant are the other parameters considered along with damping constant.

Al-Ashmori and Xu Wang [10] have proposing a model predictive controller for active vibration control of seating suspension systems.

Jamdar *et al.,* [11] have developed Bingham model and equivalent damping model. These models are then used to simulate the magneto-rheological damper in a quarter car model with four degree of freedom featuring semi-active seat suspension that is subjected to bump road input and random road input.

Mahmoodabadi and Javanbakht [12] introduces an optimal adaptive fuzzy controller consists of two fuzzy systems which each of them includes 1 output, 2 inputs, and 25 fuzzy rules. The body acceleration and the relative displacement between the tire and the sprung mass are utilized to form a proper objective function in the optimization procedure.

Sistla *et al.,* [13] experimentally validated on a scaled model of a quarter-car active suspension system with different road profiles, varying load conditions, and noise and delay in the sensor measurements and actuator respectively. The results are compared with that of an uncontrolled system with linear quadratic regulator and sliding mode control.

The major research gap identified from the different literatures is that many researchers considered vehicle suspension as a linear system for the matter of simplicity of mathematical modelling. But, in practical terms vehicle suspension is a nonlinear system because it consists of flexible suspension tires and other components which have nonlinear properties, such as nonlinear spring and damper. Also, it is found that most of the vehicles now a days have a passive suspension system, which is simulated and designed without the consideration of nonlinearities on the spring and damper. That design affects the ride comfort performance of the vehicle, especially when the spring and damper are subjected to large deflection and velocity respectively due to different off road conditions.

III. PROPOSED METHODOLOGY

To accomplish the goals of this investigation diverse examination works or writings identified with the current examination are first surveyed. The methodology is to build up a non-linear numerical model by utilizing MATLAB data fitting apparatuses for the adaptable. Segments of the suspension framework dependent on the test information accumulated from the individual written works.

A. Identification of Suspension spring Nonlinearity

Mahmoud [9] was played out the spring adjustment under his PhD postulation. Alignment work was done in the labs to decide the firmness of the spring utilizing a ductile/Compression test rig as appeared in Fig. 1. Springs were embedded into the apparatus and utilizing a heap measure and the distortion of the spring, a power versus uprooting was plotted consequently utilizing the information securing arrangement of the apparatus. The outcome got a non-straight burden versus redirection chart as appeared in Fig. 1.

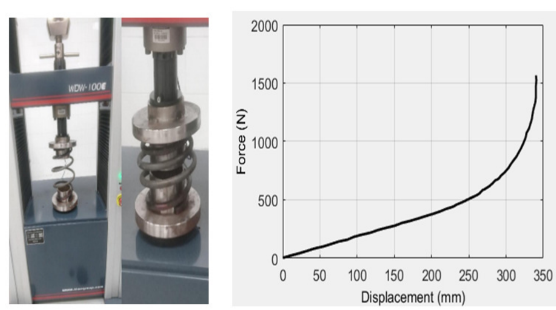


Fig. 1. Suspension spring experimental test set up and force vs displacement relation of spring [14].

The majority of the vehicle suspension frameworks have a helical spring that is additionally utilized in numerous mechanical frameworks. That acts a nonlinear conduct while in activity or moving. This implies that the spring solidness isn't steady however relies upon the pressure. This nonlinear conduct happens when the quantity of dynamic loops diminishes or increments with changing pressure. The non-straight impacts remembered for the spring power are because of fluctuating the loop width, changing the pitch, shifting the mean spring breadth pivotal way, knock stops. These non-straight impacts can be remembered for spring power with non-straight trademark versus suspension spring dislodging the vertical way [14].

In the current examination a cone shaped spring is chosen to demonstrate a non-direct quarter vehicle suspension frameworks because of certain favorable circumstances contrasted with round and hollow springs. Extending cone shaped springs can have a higher sideways security, so they will better oppose clasping. Since the mean measurement of a cone like spring fluctuating pivotal way appeared in Fig. 2 the non-straight conduct will be accomplished without any problem. So that, a tapered extending spring with a consistent pitch and a steady curl distance across is utilized.

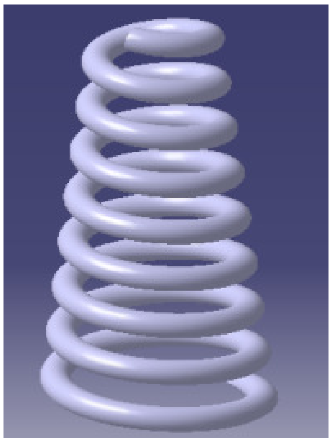


Fig. 2. CAD model of suspension spring.

The force uprooting trademark curve having illustrative conduct, acquired from the trial information gives the premise to considering the spring non-linear trademark in the framework. The test explanatory bend [9] was fitted by the assistance of MATLAB programming with two unique curves, linear and non-linear, to acquire a numerical condition for the force-displacement connection. At that point the non-linear attribute of the suspension spring is taken care of to a basic quarter vehicle framework to anticipate the impact of considering the linear just as non- linear spring.

B. Linear suspension spring model

It considers linear spring force (f_s) versus displacement(x) connection with steady stiffness coefficient (k). A linear fitting is performed on experimental force-displacement plot appeared in Fig. 3. The fundamental fitting recreation result shows the lingering focuses or the fitting mistake the genuine trial curve was a lot higher than that of the proposed nonlinear model.

From the data fitting equation $\mathrm{y} = 3.7\mathrm{x} - 220$, to verify the linear fitting equation it is possible to substitute the value of x in to the fitting equation and cross check the spring force from y-axis. As shown in the Table 1 the equation is correctly represents the linear graph.

So that, mathematically the linear spring force can be expressed as;

$f_s = k_1 x$ (1)

Table 1: Force- displacement relation of linear spring model.

Spring force (f_s) (N)	-35	150	335	520	705	890	1075
Displacement (x) (mm)	50	100	150	200	250	300	350

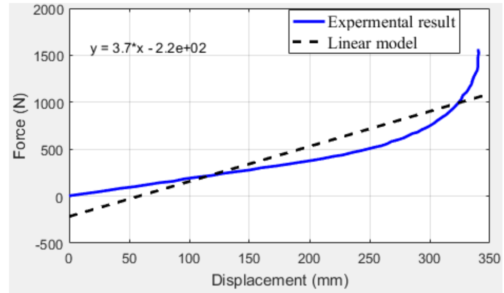


Fig. 3. Linear suspension spring model fitting with experimental force-displacement plot.

C. Non-linear suspension spring model

Non-linear model is relatively complex than linear model. That captures the non-linear properties of the suspension spring much better than the linear model as shown in the Fig. 4.

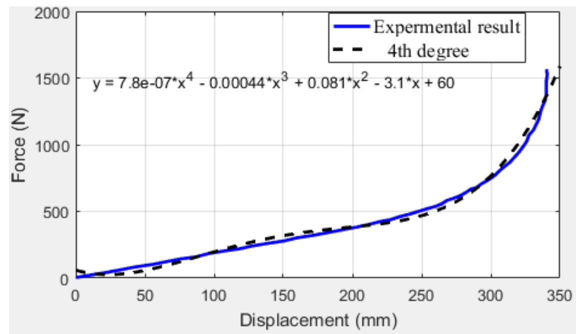


Fig. 4. Non-linear suspension spring model fitting with experimental force-displacement plot.

This information was curve fit to polynomials of second up to tenth order polynomials. Level of fitting mistake for linear curve is around 69.86%, for second order 62.6%, for third order 38.8%, for fourth order 27.01%, for fifth order 26.1%, for sixth order 25.9%.

In view of this fitting blunder and coefficient of determination (R2) given in Table 2 it was found that a fourth order polynomial adequately depicts the information. More than fourth order is pointless in light of the fact that the all-out mistake doesn't diminish by a critical sum past this. Notwithstanding, the fourth order polynomial has a lot more modest fitting mistake than the first, second or third order conditions. In the overall investigation of the suspension spring utilizing a linear model, forces fluctuate straightly with the spring misshaping. By and by, when the spring distortion is little, the spring qualities are near those of the linear model. Regardless, when the spring misshaping is enormous, critical non-linear attributes are displayed, as appeared in Fig. 4.

Table 2: Force-displacement relation of non-linear spring model.

Spring force (f_s)(N)	57.4	198	327.4	408	519.4	858	1737.4
Displacement (x)(mm)	50	100	150	200	250	300	350

From the data fitting condition y= 〖 $7.8*10^{(-7)}x^4-0.00044x^3+0.081x^2-3.1x+60$, y-speaks to the spring force and x-speaks to the displacement of the spring.

To confirm the non-linear fitting condition it is conceivable to substitute the estimation of x in to the fitting condition and cross check the spring force from y-pivot. As appeared in the Table 2 the condition is effectively speaks to the non-linear curve.

So that, mathematically the proposed polynomial model by fitting the force-displacement curve of the actual spring characteristics is expressed as follows.

$$f_s = k_1\mathrm{x} + \mathrm{k}_2\mathrm{x}^2 + \mathrm{k}_3\mathrm{x}^3 + \mathrm{k}_4\mathrm{x}^4 \quad (2)$$

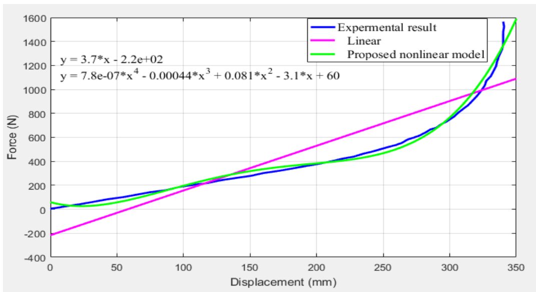


Fig. 5. Linear and non-linear model fitting comparison with experimental force-displacement curve.

D. Identification of Suspension damper Nonlinearity

Suspension damper (Fig. 7) have non-linear qualities while in activity and its damping attributes influences the plan and generally ride execution of the vehicle. The force in the damper from liquid impacts other than thickness can be assembled in with grating between the seals and the cylinder and pole, violent stream, and cavitation add to its nonlinearities. In a latent damper these are of the damper or damper liquid that cause the connection between relative velocity and yield force to be nonlinear.

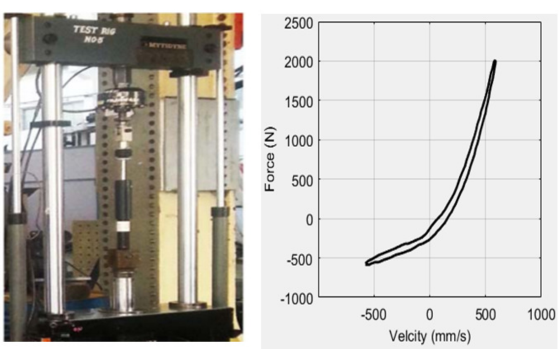


Fig. 6. Suspension spring experimental test set up and force vs displacement relation of spring [10].

The tried outcomes show that the damping power has the average highlights of nonlinearity, non-balance and hysteresis connection with the cylinder speed. The Data from experimentation is spoken to in graphical structure as appeared in the Fig. 6 [15].

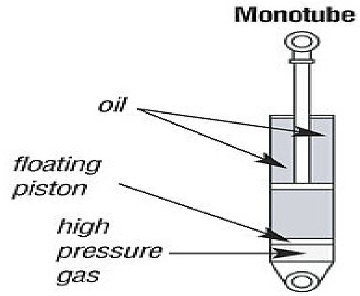


Fig. 7. Schematic model of suspension damper.

By and large the water driven damping Characteristic bend shows non-straight conduct in both pressure and expansion locale with some impact of hysteresis, yet it is unmistakably observed from the diagram that damping is kept more in augmentation than that of pressure segment. The force velocity trademark having hysteresis in conduct, gotten from the exploratory information [15] gives the premise to considering the damping non-linear trademark in the framework. The trial hysteresis curve was fitted by the assistance of MATLAB programming with two unique curves, linear and polynomial, to acquire a numerical condition for the force-velocity connection. At that point the non-linear quality of the damping framework is taken care of to a basic quarter vehicle framework to anticipate the impact of considering the linear just as non-linear damper.

E. Linear damper model

Linear model is the least demanding model utilized in the current postulation. It considers linear damping force versus velocity connection with steady damping coefficient. A linear fitting is performed on trial force velocity plot [15] appeared in Fig. 8 and the damping coefficient for linear conduct is distinguished. The essential fitting reproduction result shows the remaining focuses or the fitting mistake with the genuine test curve was a lot higher than that of the proposed nonlinear model.

Nonetheless, the third order polynomial has a lot more modest fitting blunder than the first or second order conditions.

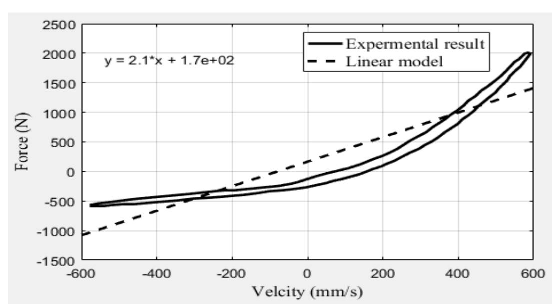


Fig. 8. Linear suspension damping model fitting with experimental force-velocity curve.

From the information fitting condition y= 〖 $2.7*10^{(-6)}x^3+0.0026x^2+1.3x-190$, y-speaks to the damping force and x-speaks to the velocity of the cylinder.

To confirm the non-linear fitting condition it is conceivable to substitute the estimation of x in to the fitting condition and cross check the damping force from y-hub. As appeared in the Table 4 the condition is accurately speaks to the non-linear curve.

As shown in the Table 3 the equation is correctly represents the linear graph.

So that, mathematically the linear damping force with respect to the piston velocity can be expressed as;

$$f_d = c_1 \quad (3)$$

Table 3: Force- velocity relation of linear damper model.

Damping force (f_d) (N)	-1090	-670	-250	590	1010	1430
Velocity (v) (mm/s)	-600	-400	-200	200	400	600

F. Non-linear damper model

This model is relatively complex than linear model. That captures the non-linear properties of the damper much better than the linear model as shown in the Fig. 9. And the coefficients of the polynomial are determined by fitting the experimental force-velocity curve.

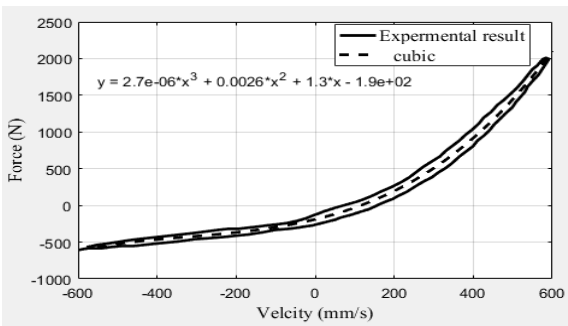


Fig. 9. Non-linear suspension damping model fitting with experimental force-velocity curve.

This data was curve fit to polynomials of second up to tenth order polynomials. Level of fitting blunder for linear curve is around 59.8%, for second order 35.4%, for third order 23.4%, for fourth order 23.39%. In view of this fitting blunder and coefficient of determination (R2) given in Table 2, it was found that a third order polynomial adequately portrays the information. More than this is pointless on the grounds that the complete blunder doesn't diminish by a critical sum past this.

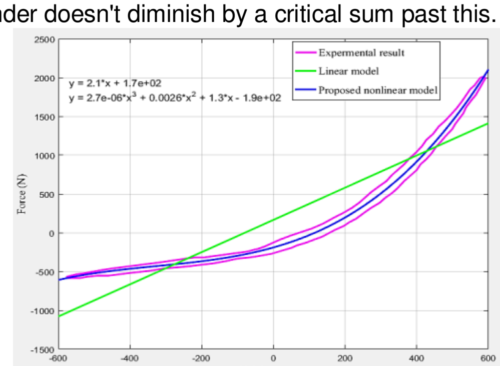


Fig. 10. Linear and non-linear model fitting comparison with experimental force-displacement curve of suspension damper.

The proposed polynomial model by fitting the force-velocity curve of the actual damper characteristics is expressed as follows.

$$f_d = c_1 v + c_2 v^2 + c_3 v^3 \quad (4)$$

G. Identification of Suspension tire spring Nonlinearity

Vertical solidness, or spring rate, is the proportion of vertical power to vertical avoidance of the tire, and it adds to the general suspension execution of the vehicle. All in all, spring rate increments with expansion pressure. The test result in [16] shows that traditional tire display a solid non-straight conduct. The vertical solidness is controlled by getting the best polynomial fit through the deliberate information utilizing MATLAB. Damien [16] under his Ph.D. proposition he was tried the tire both statically and progressively in segregation utilizing the ESH testing machine. The test arrangement can be found in Fig. 11.

Table 4: Force- velocity relation of non-linear damper model.

Damping force (f_d)(N)	-617.2	-466.8	-367.6	19.65	918.8	2109.2
Velocity (v) (mm/s)	-600	-400	-200	200	400	600

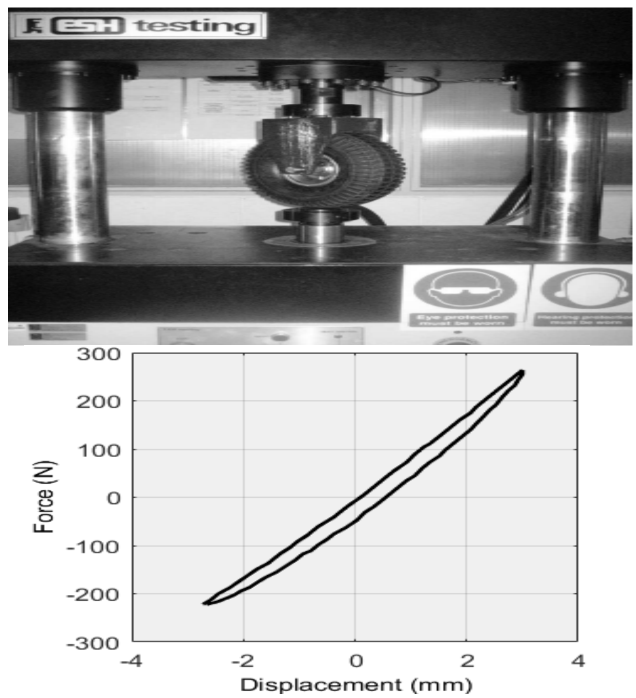


Fig. 11. Tire dynamic test setup and force vs displacement relation of tire [16].

H. Linear tire spring model

Linear model considers straight tire spring force versus displacement connection with steady tire stiffness coefficient. A linear fitting is performed on exploratory force relocation plot appeared in Fig. 12 and. The fundamental fitting reenactment result shows the lingering focuses or the fitting blunder with the genuine test curve was a lot higher than that of the proposed non-linear model.

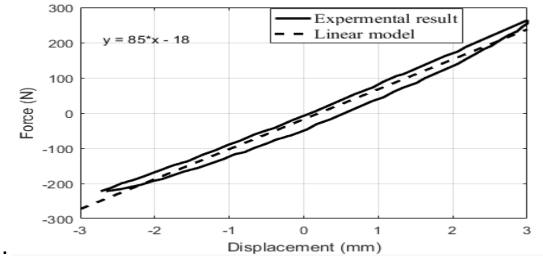


Fig. 12. Linear tire models fitting with experimental force-displacement curve.

From the data fitting equation y=85x-18 To verify the linear fitting equation it is possible to substitute the value of x in to the fitting equation and cross check the tire spring force from y-axis. As shown in the Table 5 the equation is correctly represents the linear graph.

Table 5: Force displacement relation of linear tire spring model.

Tire spring force (f_s)(N)	-273	-188	-103	67	152	237
Displacement (x) (mm)	-3	-2	-1	1	2	3

Linear force-displacement curve of the tire model is mathematically expressed as follows:

$$f_t = k_{t1}x \quad (5)$$

I. Non-linear tire spring model

Non-linear model is relatively complex than linear model. That captures the non-linear properties of the tire spring much better than the linear model as shown in the Fig. 13. The order of the polynomial is chosen as 2. And the coefficients of the polynomial are determined by fitting the experimental force-velocity curve.

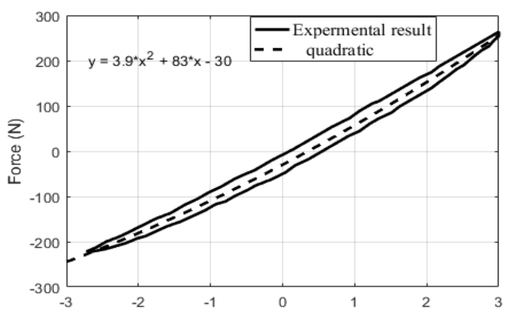


Fig. 13. Non-linear tire model fitting with experimental force-velocity curve.

This information was curve fit to polynomials of second up to tenth order polynomials. Level of fitting blunder for straight curve is around 41.2%, for second order 21.4%, for third order 21.1%, for fourth order 21%. In light of this fitting blunder and coefficient of determination (R2) given in Table 7 it was found that a second order polynomial adequately portrays the information. More than this is superfluous on the grounds that the all-out blunder doesn't diminish by a noteworthy sum past this. In any case, the second order polynomial has a lot more modest fitting blunder than the first order direct conditions.

From the information fitting condition y=3.9x^2 +83x-30, y-speaks to the tire spring force and x-speaks to the vertical displacement of the tire. To confirm the non-linear fitting condition it is conceivable to substitute the estimation of x in to the fitting condition and cross check the tire spring force from y-pivot. As appeared in the Table 6 the condition is effectively speaks to the non-linear curve.

Table 6: Force displacement relation of non-linear tire spring model.

Tire spring force (f_s) (N)	-243.9	-180.4	-109	56.9	151.6	254.1
Displace-ment (x) (mm)	-3	-2	-1	1	2	3

The proposed polynomial model by fitting the force-displacement curve of the actual tire characteristics is expressed as follows.

$$f_t = k_{t1}x + k_{t2}x^2 \quad (6)$$

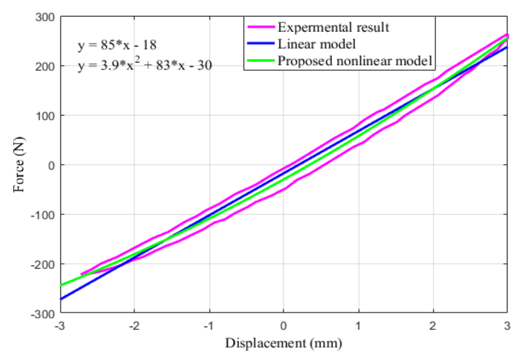


Fig. 14. Linear and nonlinear model fitting comparison with experimental force-displacement curve of tire.

Coefficient of determination(R^2) gives a sign of how well a straight or polynomial curve relapse predicts the watched information. So that, one proportion of decency of fit is the coefficient of determination.

R^2 esteem almost 0 shows that the fit isn't far superior to the model.

R^2 esteem almost 1 demonstrates that the free factor clarifies the majority of the fluctuation in the reliant variable.

Table 7: Coefficient of assurance (R^2) of the bend fitting for suspension part utilizing MATLAB.

Car suspension component	Degree of curve fitting	Value of R^2
Spring	Linear	0.301
	4th degree polynomial	0.729
Damper	Linear	0.402
	3rd degree polynomial	0.766
Tire	Linear	0.588
	quadratic	0.786

As per Cohen (1992) r-square worth 0.12 or underneath show low, between 0.13 to 0.25 qualities demonstrate medium, 0.26 or more qualities show high impact size. In this regard, R^2 esteem in Table 7 for direct and non-straight models of the vehicle suspension parts are high impact sizes. Notwithstanding, the non-straight models have higher R-square qualities this infers better to speak to the genuine model than direct model.

IV. NUMERICAL MODELING OF QUARTER CAR PASSIVE SUSPENSION SYSTEM

A. Mathematical Modeling of Linear Quarter Car Passive Suspension System

For the mater of straightforwardness of the numerical model a vehicle suspension framework can be expected as linear framework. The forces that demonstration in the suspension framework, for example, dynamic tire force F_t, spring force F_s, and safeguard damping force F_d are considered as a linear framework. The three dimensional CAD model of a quarter vehicle suspension framework can be appeared in Fig. 15.

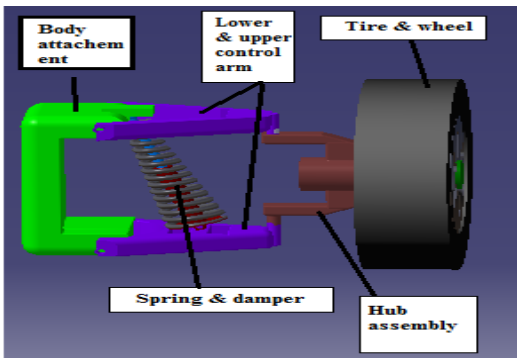
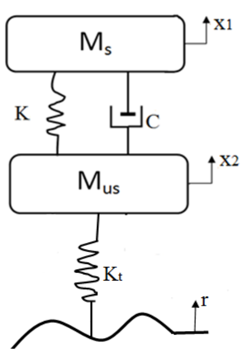


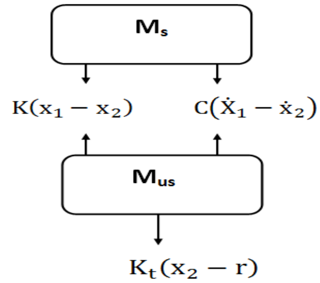
Fig. 15. Three dimensional CAD model of a quarter car suspension system.

Fig. 15 shows the main components of quarter car passive suspension system. In the Fig. 16, the body attachment and upper control arm indicates the sprung mass. Whereas, the tire and wheel assembly, hub assembly and lower control arm indicates unsprung mass.

The free-body diagrams of M_s and M_{us} are shown in the Fig. 16. According to the assumption, the mass M_s moves faster than the mass M_{us}, and the elongation of the spring K is $x_1 - x_2$. The force exerted by the spring K on the mass Ms is downward, as it tends to restore to the un-deformed position. Because of Newton's third law, the force exerted by the spring K on the mass M_{us} has the same magnitude, but opposite in direction.



(a) Physical model of linear quarter car passive suspension model.



(b) Free body diagram of quarter car model.

Fig. 16.

The images in the Fig. 16 characterized as follows: where,

M_s-sprung mass, M_{us} unsprung mass, K-suspension stiffness, Kt-Tire stiffness, C-Damping coefficient of suspension, X_1-vertical sprung mass displacement, X_2-vertical unsprung mass displacement and r-street excitation of the framework and expecting it as sinusoidal and step input.

The tire spring forces and suspension damping forces can be resolved utilizing a similar method. The gravitational forces are excluded from the free body graphs.

Fig. 16, outlines the model qualities of the straight front quarter vehicle latent suspension. This is demonstrated as spring-mass-damper frameworks. They are spoken to in the model by two level of opportunity: vertical sprung mass displacement x_1 and vertical unsprung mass displacement x_2.

In a direct model, the outflow of a spring force is generally communicated by Hook's law: F_s=KX, where F_s speaks to the spring force K speaks to the spring stiffness, X speaking to the spring displacement. The outflow of damping force for moderate speed is communicated by Coulomb's law of rubbing: F_d=C, F_d speaks to the damping force, C speaks to the direct damping, speaking to the relative velocity.

Utilizing the direct plan of spring force, damping force and tire spring force referenced in condition 1, 3 and 5 separately and by applying Newton's second law of movement for both sprung mass, Ms and un-sprung mass, Mus; the conditions of movement of the two masses are given by:

$$M_S\ddot{x}_1 + K(x_1 - x_2) + C(\dot{X}_1 - \dot{x}_2) = 0 \qquad (7)$$

$$M_{us}\ddot{x}_2 + K_t(x_2 - r) - C(\dot{x}_1 - \dot{x}_2) - K(x_1 - x_2) = 0 \qquad (8)$$

In order to reduce the degrees of the higher orders of equation to the first order of differential equation. Let the states of the system be defined as the following state variables:

$y_1 = x_1$, $y_2 = \dot{x}_1$, $y_3 = x_2$ and $y_4 = \dot{x}_2$ this implies that:

$$\dot{y}_1 = \dot{x}_1$$
$$\dot{y}_2 = \ddot{x}_1$$
$$\dot{y}_3 = \dot{x}_2$$
$$\dot{y}_4 = \ddot{x}_2$$

Therefore, the system of first order differential equation becomes;

$$\dot{y}_1 = y_2 \qquad (9)$$

$$\dot{y}_2 = -\frac{K}{M_s}(y_1 - y_3) - \frac{C}{M_s}(y_2 - y_4) \qquad (10)$$

$$\dot{y}_3 = y_4 \qquad (11)$$

$$\dot{y}_4 = \frac{K}{M_{us}}(y_1 - y_3) - \frac{C}{M_{us}}(y_2 - y_4) - \frac{K_t}{M_{us}}(K_t - r) \quad (12)$$

B. Mathematical Modeling of Non-linear Quarter Car Suspension System

Vibration is mechanical marvel where by motions happen about a balance point. On account of the genuine vehicle suspension a straight model will be deficient to portray the dynamic conduct of the vehicle accurately. So it is imperative to present non-direct models of vehicle suspension which can foresee the dynamic conduct of the genuine framework. The investigation of non-direct conduct of spring, damper and tire considered in nonlinear suspension framework examination.

The suspension framework can be viewed as a likeness the spring-mass-damper model with a non-direct spring, damper and tire. In this proposal, in light of the information fitting outcomes which are referenced in (Fig. 5, 10 and 14) non-straight quarter vehicle suspension framework for the most part comprises of loop springs having fourth degree polynomial solidness, suspension damping having cubic nonlinearities and quadratic tire firmness nonlinearities. So as to demonstrate the nonlinear suspension for base excitation, consider the spring-mass-damper model appeared in Fig. 17.

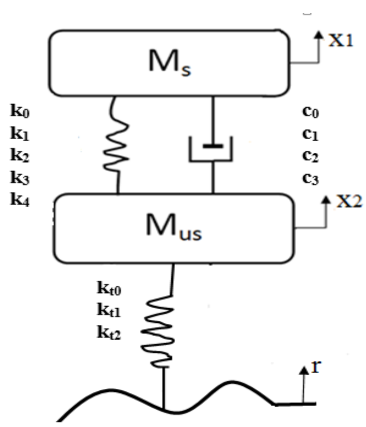


Fig. 17. Non-linear quarter car passive suspension model.

Fig. 17, illustrates the characteristics of the nonlinear quarter car passive suspension model. This is modeled as spring-mass-damper systems. Which is represented in the model by two degree of freedom: vertical sprung mass displacement and vertical unsprung mass displacement.

Using the non-linear formulation result from curve fitting spring force, damping force and tire spring force mentioned in equation 2, 4 and 6 respectively and by applying Newton's second law of motion for both sprung mass, M_s and unsprung mass, M_{us}; the equations of motion of the two masses are given by:

$$M_S\ddot{x}_1 + k_4(x_1 - x_2)^4 + k_3(x_1 - x_2)^3 + k_2(x_1 - x_2)^2 + k_1(x_1 - x_2) + c_3(\dot{X}_1 - \dot{x}_2)^3 + c_2(\dot{X}_1 - \dot{x}_2)^2 + c_1(\dot{X}_1 - \dot{x}_2) = 0 \quad (13)$$

$$M_{us}\ddot{x}_2 - k_4(x_1 - x_2)^4 - k_3(x_1 - x_2)^3 - k_2(x_1 - x_2)^2 - k_1(x_1 - x_2) - c_3(\dot{X}_1 - \dot{x}_2)^3 - c_2(\dot{X}_1 - \dot{x}_2)^2 - c_1(\dot{X}_1 - \dot{x}_2) + k_{t2}(x_2 - r)^2 + k_{t1}(x_2 - r) = 0 \quad (14)$$

In order to reduce the second order differential equation to first order differential equations using state space approach, let the states of the system be defined as the following state variables:

y1=x1, $y_2=\dot{x}_1$, $y_3=x_2$ and $y_4=\dot{x}_2$ this implies that:

$$\dot{y}_1 = \dot{x}_1$$
$$\dot{y}_2 = \ddot{x}_1$$
$$\dot{y}_3 = \dot{x}_2$$
$$\dot{y}_4 = \ddot{x}_2$$

Therefore, the system of first order differential equation becomes

$$\dot{y}_1 = y_2 \quad (15)$$

$$\dot{y}_2 = -\frac{1}{M_s}[k_4(y_1 - y_3)^4 + k_3(y_1 - y_3)^3 + k_2(y_1 - y_3)^2 + k_1(y_1 - y_3) + c_3(y_2 - y_4)^3 + c_2(y_2 - y_4)^2 + c_1(y_2 - y_4) \quad (16)$$

$$\dot{y}_3 = y_4 \quad (17)$$

$$\dot{y}_4 = \frac{1}{M_{us}}[k_4(y_1 - y_3)^4 + k_3(y_1 - y_3)^3 + k_2(y_1 - y_3)^2 + k_1(y_1 - y_3) + c_3(y_2 - y_4)^3 + c_2(y_2 - y_4)^2 + c_1(y_2 - y_4) - k_{t2}(y_3 - r)^2 - k_{t1}(y_3 - r) \quad (18)$$

V. DISCUSSION

This work compares the performance of linear and nonlinear passive vehicle suspension systems. A two-DOF light-duty and heavy-duty vehicle model has been used to model the vehicle suspension system. MATLAB solver is used to simulate the proposed mathematical model of the system for a linear and nonlinear suspension system. According to the simulation results, the performance of the nonlinear suspension system is capable of achieving better performance than the conventional linear suspension system in terms of sprung mass displacement, sprung mass acceleration and unsprung mass displacement for both types of vehicles under the two different road modeling. Comparing the figures and tabulated results, a considerable difference has been observed in the sprung and unsprung mass displacements and accelerations. The graphs of the nonlinear suspension system deviate from the graph of the linear suspension system. Maximum value of sprung and unsprung mass displacements and accelerations has been observed due to linear suspension model which yields lower performance of ride comfort and road holding of the vehicle. Therefore, it is very necessary to include non-linearity in suspension components during modeling and designing of the vehicle suspension system.

VI. CONCLUSION

This paper has done to develop the nonlinear quarter car passive suspension system model. The model is developed using experimental data generated from the previous research work. Based on the actual or experimental characteristics of suspension components the curve fitting on MATLAB has been performed. By using the result of curve fitting the non-linear car suspension model has been developed. The degree of effectiveness of the nonlinear suspension components has been investigated. From the result the degree of effectiveness observed more due to nonlinearity in damper and tire. Lower value of sprung mass displacement was observed due to nonlinearity in damper and tire stiffness as compared to the fully linear suspension model. Thus, it can be concluded that the nonlinearity in damper and tire are more significant than the nonlinearities in spring.

VII. FUTURE SCOPE

For the future work, a more advanced control structure and control design will be investigated under various maneuvers in order to improve the vehicle stability, which will not only benefit the handling aspect of the vehicle but also contribute to greater vehicle ride comfort. In order to carry out the more accurate simulation, new nonlinear suspension model on a half-car model or full car model to study extensively its effect on a larger model. Apply this work to other types of spring-mass system for example suspension building, trucks and machine vibration isolators. Finally combine a semi-active or active damper to further improve the study.

ACKNOWLEDGEMENT

This work is a part of Final year M.Sc. Thesis project. The authors are thankful to the Faculty of Mechanical Engineering, Jimma Institute of Technology, Jimma University, Ethiopia for their contribution in evaluating this project.

Conflict of Interest. There is no conflict of interest.

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How to cite this article: Abebe, B. A., Santhosh, J., Ahmed, A. A., Murugan, P. and Ashok, N. (2020). Non-Linear Mathematical Modelling for Quarter Car Suspension Model. *International Journal on Emerging Technologies, 11*(5): 536–544.