International Journal on Emerging Technologies **10**(2b): 48-53(2019)

ISSN No. (Print) : 0975-8364
ISSN No. (Online) : 2249-3255

Influence of Irregularity and Rigid Boundary on Surface Wave Propagation under Initially Stressed Porous Medium

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(Received 02 May 2019, Revised 10 July 2019, Accepted 08 August 2019)
(Published by Research Trend, Website: www.researchtrend.net)

ABSTRACT: In this paper we study surface wave propagation in a transverse anisotropic fluid saturated porous layer lying between homogeneous layer with rigid boundary and a non-homogeneous elastic half space. This seems a realistic model for surface wave propagation. We derive the dispersion equation and discuss some particular cases for torsional surface wave under initial stress present in the porous layered medium. The effects of inhomogeneity, irregularity and initial stress have been studied. The phase velocity of the surface wave propagation is plotted against the wave number with the help of MATLAB graphical routines. Finally a conclusion is presented.

Keywords: Porous Medium, Initial Stress, Surface Waves, Torsional Waves, Inhomogeneity, Phase velocity.

I. INTRODUCTION

Seismology plays a vital role in the more general fields of geophysics and Earth sciences. In the nineteenth and twenties centuries seismology became a self-governing science. Due to the fact that the internal structure of the Earth is heterogeneous along with a totally rough layer, the medium porosity and the rigid boundary have an important role in the propagation of the seismic waves. Some valuable information concerning seismic wave propagation is available in the famous books of Ewing *et al*., [1] and Achenbach [2]. A large number of papers have been published on torsional wave propagation in elastic medium with exceptional forms of heterogeneity. Love waves over a pre-stressed elastic half space have been investigated by [3]. The rigidity effect on Love wave propagation in a non -homogenous layer under an initial stressed half space have been studied [4]. The surface wave solutions obtained by solving analytically the frequency equation in a transversely isotropic dissipative medium and the effect of transverse isotropy and initial stress parameter observed on the Rayleigh wave velocity [5]. The propagation of Love waves in a homogeneous irregular layer over an elastic porous half-space, under the effect of initial stress for both layer and half-space have been studied [6]. The SH-type surface wave propagation in a layered medium with an invariant initial stress, wherein a piezo-electric powered thin layer is perfectly bonded on a piezomagnetic substrate [7]. The effect of initial stress on the propagation of torsional waves in an anisotropic porous layer lying between a homogeneous and heterogeneous half space had been discussed [8]. It was observed that torsional wave propagation was much influenced by initial stress in an anisotropic layer lying between two half-spaces [9]. The dispersion curves plotted for torsional surface wave propagation in an inhomogeneous crustal layer under an initially stressed viscoelastic half-space [10]. The effect of initial stress and inhomogeneity parameter on Love wave propagation for different type of mediums was also discussed by [11, 13]. To investigate the effect of surface heterogeneities, irregularity and rigidity in fluid saturated porous layer over homogeneous and non-homogeneous half spaces, studies have also been performed by [14-22] and developed the corresponding frequency equations in terms of phase velocity and wave number.

In the present paper, we study the propagation of surface waves in an initially stressed transversely anisotropic fluid saturated porous layer lying between isotropic layer and inhomogeneous half space with a rectangular irregularity present in the half space under rigid boundary. The study shows that the inhomogeneity, rigidity and initial stress have significant effects on the propagation of surface waves. The results obtained in this study presented graphically with the help of MATLAB graphical routines. Some particular cases have also been discussed and corresponding dispersion equation is derived.

II. FORMULATION OF PROBLEM

In this problem we have taken the cylindrical coordinate system (r, θ, z) for a model consisting a transversely anisotropic liquid-saturated porous layer of thickness H_2 under an initial stress $P = -\tau_{rr}$ in direction of radial coordinate r which is lying between the elastic isotropic homogeneous rigid layer with thickness H_1 and non-homogeneous elastic half-space with rectangular irregularity of length 2m and height h at the lower interface. We have taken two dimensional (r-z plane) where the circumferential coordinate θ is independent in r and z directions and the centre of the cylindrical coordinate system is situated on the middle of irregularity at the interface of lower half space, and z-coordinate is downward and r-coordinate is parallel to the disturbance. Let the topmost elastic isotropic homogeneous rigid layer be assumed as the medium M1:$-\,(H_1+\,H_2)\,\leq z\,\leq 0$; the intermediate transversely isotropic liquid saturated porous layer be the medium M2:$-H_1 \leq z \leq\; 0$ and half space be the medium M3: $0 \leq z\, \leq \infty$. The inhomogeneity is present in both rigidity and mass density. The variations in rigidity and mass density have been taken into consideration. The geometry for the considered model is presented in Fig. 1.

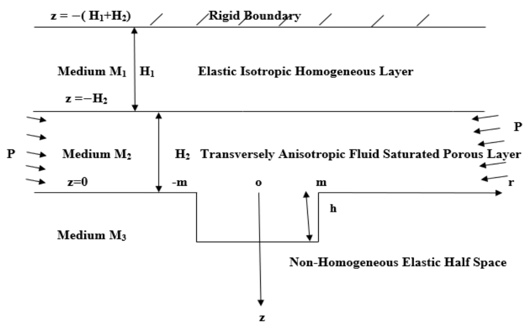

Fig. 1. Model for the Considered Problem.

The expressions for the rigidity μ and density ρ for considered mediums are given as

for medium M$_1$

$$\mu = \mu_1,\quad \rho = \rho_1, \tag{1}$$

for medium M$_2$

$$\mu = \mu_2,\quad \rho = \rho_2, \tag{2}$$

for medium M$_3$

$$\mu = \mu_3(1+az)^2,\quad \rho = \rho_3(1+bz)^2, \tag{3}$$

where $a > 0$, $b > 0$ are constants of inverse length.

Let the equation of irregularity for considered model be

$$z = \varepsilon F(r) \tag{4}$$

Where $F(r) = \begin{cases} h & for\ |r| \le m, \\ 0 & for\ |r| > m, \end{cases}$

$\varepsilon = \frac{h}{2m}$, $\varepsilon << 1.$

III. GOVERNING EQUATIONS

The dynamical equations of motions for wave propagating in the direction of radial and circumferential coordinates are [23]

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{r\theta}}{\partial \theta} + \frac{\partial \sigma_{rz}}{\partial z} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = \rho \frac{\partial^2 u}{\partial t^2},$$
$$\frac{\partial \sigma_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{\theta\theta}}{\partial \theta} + \frac{\partial \sigma_{\theta z}}{\partial z} + \frac{2\sigma_{r\theta}}{r} = \rho \frac{\partial^2 v}{\partial t^2}, \tag{5}$$
$$\frac{\partial \sigma_{rz}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{\theta z}}{\partial \theta} + \frac{\partial \sigma_{zz}}{\partial z} + \frac{\sigma_{rz}}{r} = \rho \frac{\partial^2 w}{\partial t^2},$$

where $\sigma_{rr}, \sigma_{r\theta}, \sigma_{rz}, \sigma_{\theta\theta}, \sigma_{\theta z}, \sigma_{zz}$ are the respective stress components and u, v, w are the respective components of displacement;

Now the stress strain relation is given by

$$\sigma_{ij} = \lambda \Omega \delta_{ij} + 2\mu e_{ij}, \tag{6}$$

where, λ and μ both are Lame's elastic constants and $\delta_{ij} = \begin{cases} 1; i \ne j \\ 0; i = j \end{cases}$ is the Kronecker delta,

$e_{ii} = \Omega$ is the dilatation,

and

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \tag{7}$$

Now, for wave propagating in the direction r and displacement in z direction, we have

$$u_1 = u = 0,\quad w_3 = w = 0,\quad v_2 = v(r,z,t) \tag{8}$$

By using Eqns. (7) and (8) in Eqn. (5) we obtain the following equation of motion without body forces:

$$\frac{\partial \sigma_{r\theta}}{\partial r} + \frac{\partial \sigma_{\theta z}}{\partial z} + \frac{2\sigma_{r\theta}}{r} = \rho \frac{\partial^2 v}{\partial t^2} \tag{9}$$

The stress components related to the displacement vectors, for the elastic medium are

$$\sigma_{rr} = \sigma_{\theta\theta} = \sigma_{zz} = \sigma_{rz} = 0$$
$$\sigma_{r\theta} = \mu\left(\frac{\partial v}{\partial r} - \frac{v}{r}\right)$$
$$\sigma_{z\theta} = \mu \frac{\partial v}{\partial z} \tag{10}$$

With the help of Eqn. (10), (9) takes the form of

$$\mu(z)\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - \frac{1}{r^2}\right)v + \frac{\partial}{\partial z}\left(\mu(z)\frac{\partial v}{\partial z}\right) = \rho(z)\frac{\partial^2 v}{\partial t^2} \tag{11}$$

For waves varying harmonically with time, and moving parallel to the radial vector r, we take

$$v = V(z) J_1(kr) e^{i\omega t} \tag{12}$$

where, k and ω are the wave number and angular frequency respectively, $J_1(kr)$ is the first order Bessel's function and therefore Eqn. (12) becomes

$$\frac{d^2V}{dz^2} + \frac{\mu'(z)}{\mu(z)}\frac{dV}{dz} - k^2\left(1 - \frac{c^2}{c_s^2}\right)V(z) = 0 \tag{13}$$

where, c_s and c are the shear velocity and wave velocity respectively such as:

$$c_s = \sqrt{\frac{\mu}{\rho}}, \text{ and } c = (\omega/k) \tag{14}$$

For Medium M$_1$

By using Eqn. (1), Eqn.(13) reduces to

$$\frac{d^2V}{dz^2} - m_1^2 V(z) = 0 \tag{15}$$

where $m_1^2 = k^2\left(1 - \frac{c^2}{c_1^2}\right)$, c_1 and c are the shear velocity and torsional wave velocity respectively for medium M$_1$ and is given

$$c_1 = \sqrt{\frac{\mu_1}{\rho_1}},\quad c = (\omega/k) \tag{16}$$

Solution of Eqn. (15) is given by

$$V(z) = A_1 e^{m_1 z} + A_2 e^{-m_1 z} \tag{17}$$

where A_1 and A_2 are arbitrary constants.

Thus, displacement vector for medium M$_1$ given by equation (12) takes the form of

$$v = v_1(say) = A_1 e^{m_1 z} J_1(kr) e^{i\omega t} + A_2 e^{-m_1 z} J_1(kr) e^{i\omega t} \tag{18}$$

For Medium M$_3$

The Eqn. (14) with the help of (3) takes the form of

$$\frac{d^2V}{dz^2} + \frac{2a}{(1+az)}\frac{dV}{dz} - k^2\left(1 - \frac{c^2(1+bz)^2}{c_3^2(1+az)^2}\right)V(z) = 0 \tag{19}$$

where c_3 is shear wave velocity for medium M$_3$ and is given by

$$c_3 = \sqrt{\frac{\mu_3}{\rho_3}} \tag{20}$$

To eliminate the term $\frac{dV}{dz}$ from equation (19), we take $V(z) = \phi(z)/(1+az)$ and thus

$$\frac{d^2\phi(z)}{dz^2} - k^2\left(1 - \frac{c^2(1+bz)^2}{c_3^2(1+az)^2}\right)\phi(z) = 0 \tag{21}$$

Now, we take dimensionless quantities $\gamma = \sqrt{1 - \frac{bc^2}{ac_3{}^2}}$ and $\eta = \frac{\gamma k(1+2az)}{a}$ in Eqn. (21), it reduces to

$$\frac{d^2\phi}{d\eta^2} + \left(-\frac{1}{4} + \frac{R}{\eta}\right)\phi(\eta) = 0 \tag{22}$$

where, $R = \frac{c^2(a-b)k}{4c_3{}^2a^2\gamma}$

The solution of differential Eqn. (22) will be of the form

$$\phi(\eta) = A_5 W_{R,1/2}(\eta) + A_6 W_{R,-1/2}(\eta) \tag{23}$$

where A_5, A_6 are constants and $W_{R,1/2}(\eta)$ is the Whittaker function [24].

Then, the solution vanishes at $z \to \infty$, that means, for $\eta \to \infty$, this can be written as:

$$\phi(\eta) = A_5 W_{R,1/2}(\eta) \tag{24}$$

Now, by expanding Whittaker function in linear terms, we obtain the displacement vector for medium M$_3$ as

$$v = v_3 = A_5 e^{-\beta z} J_1(kr) e^{i\omega t} \tag{25}$$

where

$$\beta = \frac{1}{1+az} e^{\frac{-\gamma k(1+2az)}{2a}} \frac{\gamma k(1+2az)}{a} . \left[1 + \frac{\gamma k A}{a}(1+2az)\right]$$

For Medium M_2

The governing equations for medium M_2 in the absence of body forces, under anisotropic porous medium under the initial stress P, are given [25] by

$$\frac{\partial \tau_{rr}}{\partial r}+\frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta}+\frac{\partial \tau_{rz}}{\partial z}+\frac{\tau_{rr}-\tau_{\theta\theta}}{r}-P\frac{\partial w_\theta{}'}{\partial z}=\frac{\partial^2}{\partial t^2}(\rho_{rr}u+\rho_{r\theta}U),$$
$$\frac{\partial \tau_{r\theta}}{\partial r}+\frac{1}{r}\frac{\partial \tau_{\theta\theta}}{\partial \theta}+\frac{\partial \tau_{\theta z}}{\partial z}+\frac{2\tau_{r\theta}}{r}-P\frac{\partial w_z{}'}{\partial r}=\frac{\partial^2}{\partial t^2}(\rho_{rr}v+\rho_{r\theta}V), \quad (26)$$
$$\frac{\partial \tau_{rz}}{\partial r}+\frac{1}{r}\frac{\partial \tau_{\theta z}}{\partial \theta}+\frac{\partial \tau_{zz}}{\partial z}+\frac{\tau_{rz}}{r}-P\frac{\partial w_\theta{}'}{\partial r}=\frac{\partial^2}{\partial t^2}(\rho_{rr}w+\rho_{r\theta}W),$$

and

$$\frac{\partial \tau}{\partial r}=\frac{\partial^2}{\partial t^2}(\rho_{r\theta}u+\rho_{\theta\theta}U),$$
$$\frac{\partial \tau}{\partial \theta}=\frac{\partial^2}{\partial t^2}(\rho_{r\theta}u+\rho_{\theta\theta}V), \quad (27)$$
$$\frac{\partial \tau}{\partial z}=\frac{\partial^2}{\partial t^2}(\rho_{r\theta}w+\rho_{\theta\theta}W),$$

where, τ_{rr},$\tau_{\theta\theta}$,τ_{zz},τ_{rz},$\tau_{r\theta}$,$\tau_{\theta z}$ are the stress components, (u, v, w) and (U, V, W) are the displacement vectors for solid and fluid respectively and the stress component for liquid is τ.

$$w_r{}'=\frac{1}{2r}\left(\frac{\partial w}{\partial \theta}-r\frac{\partial v}{\partial z}\right),$$
$$w_\theta{}'=\frac{1}{2}\left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial r}\right), \quad (28)$$
$$w_z{}'=\frac{1}{2r}\left(\frac{\partial (rv)}{\partial r}-\frac{\partial v}{\partial \theta}\right),$$

are components of the rotational vector w'.

The constitutive relations for medium M_2 are

$$\tau_{rr}=(A+P)e_{rr}+(A-2N+P)e_{\theta\theta}+(F+P)e_{zz}+Q_\varepsilon,$$
$$\tau_{\theta\theta}=(A-2N)e_{rr}+Ae_{\theta\theta}+Fe_{zz}+Q_\varepsilon,$$
$$\tau_{zz}=Fe_{rr}+Fe_{\theta\theta}+Ce_{zz}+Q_\varepsilon, \quad (29)$$
$$\tau_{r\theta}=2Ne_{r\theta},$$
$$\tau_{\theta z}=2Ge_{\theta z},$$
$$\tau_{rz}=2Ge_{zr},$$

where A, F, C N and G are the elastic constants for medium M_2. Q_ε is the coupling parameter between change of volume of solid and liquid. The vector τ and the fluid pressure P' are related as

$$-\tau=fP', \quad (30)$$

where, f is porosity of the layer. ρ_{rr}, $\rho_{r\theta}$ *and* $\rho_{\theta\theta}$ are the mass coefficients associated with the densities ρ, ρ_s *and* ρ_w of the layer, solid and water respectively.

$$\rho_{rr}+\rho_{r\theta}=(1-f)\rho_s,$$
$$\rho_{r\theta}+\rho_{\theta\theta}=f\rho_w, \quad (31)$$

Thus, the cumulative mass-density is

$$\rho'=\rho_s+f(\rho_w-\rho_s) \quad (32)$$

The inequalities for mass coefficient hold [21] as:

$$\rho_{rr}>0,\ \rho_{\theta\theta}>0,\ \rho_{r\theta}<0,\ \rho_{rr}\rho_{\theta\theta}-\rho_{r\theta}{}^2>0 \quad (33)$$

For torsional surface wave, we consider

$$u=0,\quad w=0,\quad v=v(r,z,t),$$
$$U=0,\quad W=0,\quad V=V(r.z,t) \quad (34)$$

which give the value of strain components as

$$e_{rr}=0,\ e_{\theta\theta}=e_{zz}=e_{zr}=0,$$
$$2e_{\theta z}=\frac{\partial v}{\partial z}, \quad (35)$$
$$2e_{r\theta}=\frac{\partial v}{\partial r}-\frac{v}{r},$$

By making use of Eqns. (34), (35) in (29), we get only two non-zero stress components as

$$\tau_{rr}=\tau_{\theta\theta}=\tau_{zz}=\tau_{rz}=0,$$
$$\tau_{\theta z}=G\frac{\partial v}{\partial z}, \quad (36)$$
$$\tau_{r\theta}=N\left(\frac{\partial v}{\partial r}-\frac{v}{r}\right),$$

Now by making use of Eqn. (36), Eqns. (26) and (27) take the form of

$$\left(N-\frac{P}{2}\right)\left(\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}-\frac{v}{r^2}\right)+G\left(\frac{\partial^2 v}{\partial z^2}\right)=\frac{\partial^2}{\partial t^2}(\rho_{rr}v+\rho_{r\theta}V) \quad (37)$$

and

$$\frac{\partial^2}{\partial t^2}(\rho_{r\theta}v+\rho_{\theta\theta}V)=0 \quad (38)$$

Eqn. (38) implies that

$$\rho_{r\theta}v+\rho_{\theta\theta}V=k_2\text{(say)} \quad (39)$$

By eliminating V from Eqns. (37), (39), we have

$$\left(N-\frac{P}{2}\right)\left(\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}-\frac{v}{r^2}\right)+G\left(\frac{\partial^2 v}{\partial z^2}\right)=d'\frac{\partial^2 v}{\partial t^2} \quad (40)$$

where

$$d'=\left(\rho_{rr}-\frac{\rho_{r\theta}{}^2}{\rho_{\theta\theta}}\right)$$

The equation (40) indicates that the shear wave velocity lies along radial direction and is given by

$$\sqrt{\frac{N-\frac{P}{2}}{d'}}=\sqrt{\frac{N-\zeta}{d}c_2{}^2},$$

where c_2,ζ and d are given by

$c_2=\sqrt{\frac{N}{\rho'}}$ and $\zeta=\frac{P}{2N}$ = dimensionless constraints due to the initial stress P, d = $\frac{d'}{\rho'}$

are dimensionless parameters for medium M_2.

To solve Eqn. (40), we consider

$$v=V(z)\mathrm{J}_1(kr)e^{i\omega t} \quad (41)$$

the Eqn. (40) with the help of Eqn. (41) becomes

$$\frac{d^2V}{dz^2}+q^2\mathrm{V}=0 \quad (42)$$

where $q=k[pd(\frac{c^2}{c_2{}^2}-\frac{1-\zeta}{d})]^{1/2}$ and $p=\frac{N}{G}$

The solution of differential Eqn. (42) is

$$\mathrm{V(z)}=A_3\cos qz+A_4\sin qz \quad (43)$$

where A_3,A_4 are arbitrary constants

Thus, the displacement vector for initially stressed medium M_2 is

$$v=v_2=[A_3\cos qz+A_4\sin qz]\,\mathrm{J}_1(kr)e^{i\omega t} \quad (44)$$

IV. BOUNDARY CONDITIONS

The suitable boundary conditions for the proposed model are

(a) The displacement component vanishes at the rigid surface $z=-(H_1+H_2)$,

i.e. $v(z=-(H_1+H_2))=0$, (45)

(b) The continuity of displacement and stress components at the interface, z = $-H_2$, i.e.;

$$v_1=v_2 \quad (46)$$
$$\mu_1\tau_{32}=\mu_2\sigma_{32} \quad (47)$$

(c) The continuity of displacement and stress components at the interface, z = $\varepsilon F(r)$ i.e.;

$$v_2=v_3, \quad (48)$$
$$\mu_2\sigma_{32}=\mu_3 t_{32} \quad (49)$$

V. SOLUTION OF THE PROBLEM

By making use of above boundary conditions in Eqns. (10), (18), (25), (36), (44), we get five homogeneous equations in A_1,A_2,.....A_5.

$$A_1e^{-m_1(H_1+H_2)}+A_2e^{m_1(H_1+H_2)}=0 \quad (50)$$
$$A_1e^{-m_1(H_2)}+A_2e^{m_1(H_2)}-A_3\cos qH_2+A_4\sin qH_2=0 \quad (51)$$
$$A_1(\mu_1m_1e^{-m_1(H_2)})-A_2(\mu_1m_1e^{m_1(H_2)})-A_3(\mu_2 q\sin qH_2)+A_4(\mu_2 q\cos qH_2)=0 \quad (52)$$
$$A_3\cos(q\varepsilon F(r))+A_4\sin(q\varepsilon F(r))-A_5e^{-\beta(\varepsilon F(r))}=0 \quad (53)$$
$$-A_3[\mu_2 q\sin(q\varepsilon F(r))]+A_4[\mu_2 q\cos(q\varepsilon F(r))]+A_5\beta\mu_3e^{-\beta(\varepsilon F(r))}=0 \quad (54)$$

For non-zero solution of above homogeneous system of equations, we have

$$\begin{vmatrix} e^{-m_1(H_1+H_2)} & e^{m_1(H_1+H_2)} & 0 & 0 & 0 \\ e^{-m_1(H_2)} & e^{m_1(H_2)} & -\cos q(H_2) & \sin(qH_2) & 0 \\ m_1\mu_1e^{-m_1H_2} & -m_1\mu_1e^{m_1H_2} & -\mu_2q\sin(qH_2) & \mu_2q\cos qH_2 & 0 \\ 0 & 0 & \cos q(\varepsilon F(r)) & \sin q(\varepsilon F(r)) & -e^{-\beta(\varepsilon F(r))} \\ 0 & 0 & -q\mu_2\sin q(\varepsilon F(r)) & q\mu_2\cos q(\varepsilon F(r)) & \beta\mu_3e^{-\beta(\varepsilon F(r))} \end{vmatrix}=0 \quad (55)$$

On simplification, we get

$$\tan qH_2 = -\mu_2 q\left(1-e^{2m_1(H_1)}\right)\{\beta\mu_3 \cos q\,(\varepsilon F(r)) + \mu_2 q \sin q\, \varepsilon F(r)\}$$
$$-m_1\mu_1\left(1+e^{2m_1(H_1)}\right)\{\beta\mu_3 \sin q\,\varepsilon F(r) - \mu_2 q \cos q\,(\varepsilon F(r))\}$$
$$/\ \mu_2 q\left(1-e^{2m_1(H_1)}\right)\{\beta\mu_3 \sin q\,(\varepsilon F(r)) - \mu_2 q \cos q\,\varepsilon F(r)\}$$
$$+$$
$$m_1\mu_1\left(1+e^{2m_1(H_1)}\right)\{\beta\mu_3 \cos q\,\varepsilon F(r) + \mu_2 q \sin q\,(\varepsilon F(r))\} \quad (56)$$

Equation (56) represents the dispersion relation for torsional surface waves under the initially stressed porous layer.

VI. SPECIAL CASES

Case I: If $\varepsilon \to 0$ i.e., then Eqn. (56) takes the form

$$\tan qH_2 = -\mu_2 q\beta\mu_3\left(1-e^{2m_1(H_1)}\right) - m_1\mu_1\mu_2 q\left(1 + e^{2m_1(H_1)}\right)$$
$$/ -q^2{\mu_2}^2\left(1-e^{2m_1(H_1)}\right) + m_1\mu_1\beta\mu_3\left(1+e^{2m_1(H_1)}\right) \quad (57)$$

Equation (57) shows the frequency relation of torsional waves under an initially stressed anisotropic porous medium.

Case II: If $H_1 = 0$, Eqn. (56) reduces

$$\tan qH_2 = -\{\beta\mu_3 \sin q\,\varepsilon F(r) - \mu_2 q \cos q\,(\varepsilon F(r))\}$$
$$/\{\ \beta\mu_3 \cos q\,\varepsilon F(r)$$
$$\mu_2 q \sin q\,(\varepsilon F(r))\} \quad (58)$$

Equation (58) is the frequency relation of torsional surface waves in the transversely anisotropic liquid-saturated porous medium at interface.

VII. NUMERICAL PLOTTATIONS

Numerical computations are performed to demonstrate the effect of different values of the ratios of the heights of the layers with the fixed values of anisotropy parameter N/G, which is based on the dispersion Eqn. (56) for the considered problem. The numerical data has been taken from [26]. In the figures from 2 to 6, the dimensionless phase velocity c/c_1 is plotted against the dimensionless wave number for fixed values of N/G against different values of ratios of heights of the layers with the help of MATLAB graphical routines:

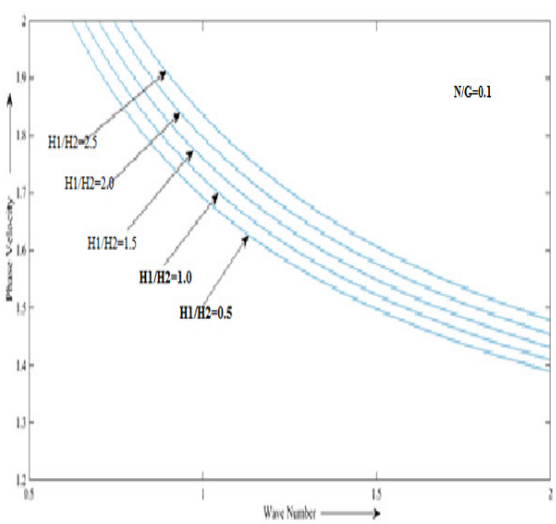

Fig. 2. Propagation of surface waves for different values of ratios of heights of the layers (H1/H2 = 0.5, 1.0, 1.5, 2.0, 2.5) against the anisotropy parameter N/G = 0.1.

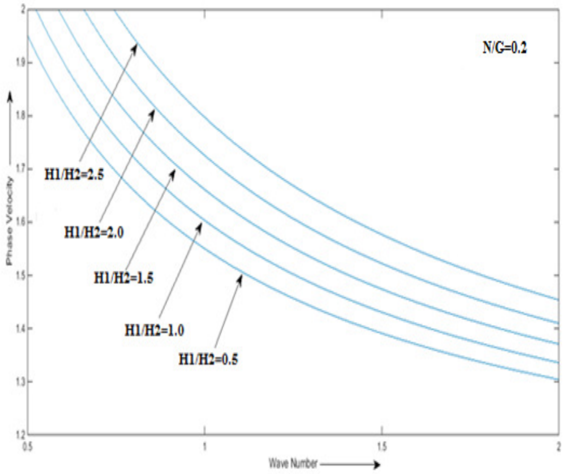

Fig. 3. Propagation of surface waves for different values of ratios of heights of the layers (H1/H2 = 0.5, 1.0, 1.5, 2.0, 2.5) against the anisotropy parameter N/G=0.2.

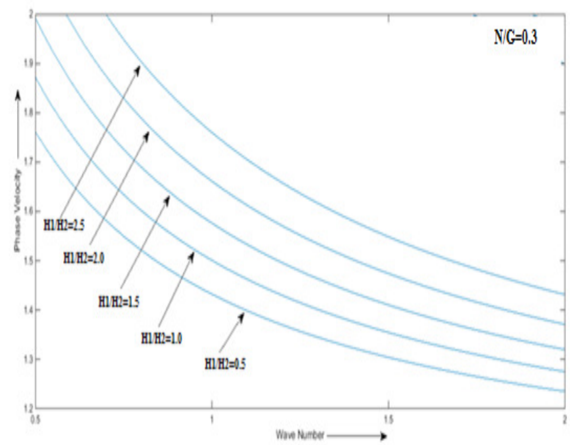

Fig. 4. Propagation of surface waves for different values of ratios of heights of the layers (H1/H2 = 0.5, 1.0, 1.5, 2.0, 2.5) against the anisotropy parameter N/G = 0.3.

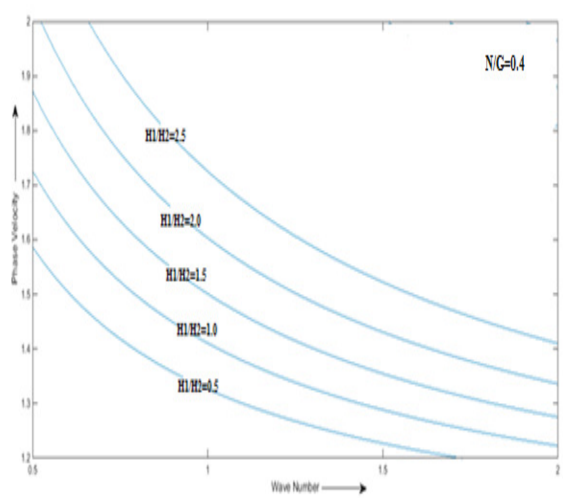

Fig. 5. Propagation of surface waves for different values of ratios of heights of the layers (H1/H2 = 0.5, 1.0, 1.5, 2.0, 2.5) against the anisotropy parameter N/G = 0.4.

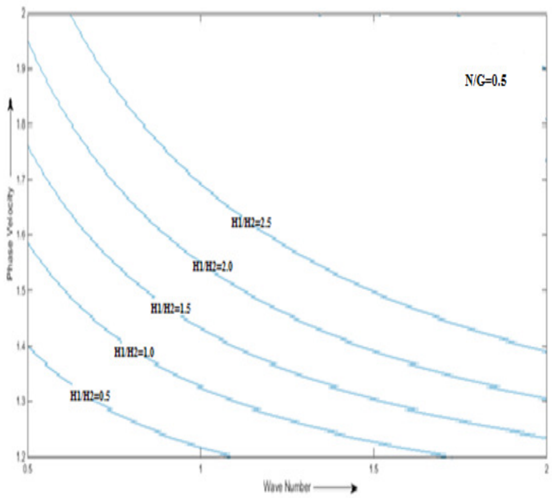

Fig. 6. Propagation of surface waves for different values of ratios of heights of the layers (H1/H2 = 0.5, 1.0, 1.5, 2.0, 2.5) against the anisotropy parameter N/G =0.5.

VIII. CONCLUSION

The propagation of surface waves in an initially stressed porous layer sandwiched between elastic isotropic homogeneous rigid surface and a non-homogeneous half- space had been studied in this paper. The frequency relation has been obtained analytically by using simple mathematical calculations. Some particulars cases have also been discussed. We have observed that

- ❖ The dimensionless phase velocity c/c_1 of surface waves increases with the decreases of the dimensionless wave number in figures 2-6.
- ❖ The phase velocity of the surface waves decreases with an increase in the anisotropic factor N/G.
- ❖ The ratios of the heights of the layers decrease with the increase in the anisotropic factor N/G in all figures.
- ❖ The size of irregularity and inhomogeneity parameter effects the dispersion relation for surface wave propagation for the considered model.

Hence the results obtained in this study may be useful for seismic wave propagation which are generated by artificial explosion and earthquakes. Due to varied application of seismology, this paper may be very helpful for researchers as well as post graduate students.

CONFLICT OF INTEREST

Authors have no any conflict of interest.

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