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A Corner Point Quadrature Method for 4 Node Quadrilateral Element for the Evaluation of Element Stiffness Matrix

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ABSTRACT: Most of the recent studies aim in the developments of new elements in finite element analysis This paper focuses on development of new quadrature for the 4- node quadrilateral element for the purpose evaluation of element stiffness matrix. The corner point Quadrature is a mimics of Gauss numerical integration scheme. This integration scheme consists of 5 sampling points and weights where four sampling points are at the corner and one at the Centre of the element. Accuracy of results, Convergence of the results and stability of values been tested using standard benchmarked problems defined by various research studies.

Keywords: Finite Element, Numerical integration, Quadrature, Sampling Point, Stiffness Matrix, Triangular Element

Abbreviations: FEA, Finite Element Analysis; CEM, Corner Edge Method.

I. INTRODUCTION

For the computation of complex problems, Finite Element Analysis (FEA) is an approximation approach used in engineering. Solving problems in FEA needs more computational efforts because of handling large varying data. Sampling points and weights plays a key role in the calculation of element stiffness matrix because it needs to deal with large varying data with more accuracy. For evaluation of integrals over the finite elements, numerical integration is considered to be the important method. In general, for finite elements, the load vectors and expressions for the integration of element stiffness matrices cannot be done analytically way. Instead of doing analytically the element stiffness and load vectors matrices are evaluated by using some numerical integration rule. For estimation of element matrices and vectors usually we use numerical integration methods for different types of finite elements. Many researchers had done study on finding new elements in finite element analysis. The most basic quadrilateral element in finite element method is Q4. The studies had shown that the performance of Q4 element was fond to be poor while computing in bending problems and distorted meshes. Wilson *et al*., (1973) had proposed a new non-conforming element Q6 with additional internal degrees of freedom [25]. Taylor (1976) has proposed new element QM6 for passing the patch test [24]. As a modification of Q6. developed another element Nq6 using additional linear terms [6].
Various schemes of integration techniques have made significant role in the development of finite element methods [11, 27]. Recently it is found from various studies deals with reduced integration schemes which are more frequently been used with a combination of time domain in application of explicit integration and stabilization methods. This lead to improvement in the computational efficiency while simulating demanding models computationally [1, 2, 15]
Introduction of drilling degrees of freedom were introduced in many literature studies for the purpose of obtaining better results and performance [3, 5, 10, 17, 22] those elements the final outcome is found to be rank deficiency while solving using reduced integration schemes. Studies shown that there is difficulty of "trapezoidal locking" in the 4-node quadrilateral element when each node remains with two degrees of freedom. This will be based on the element which passes the strong patch test, the studies shown that MacNeal’s slender beam will be shear locking under the trapezoidal grid. However, some researchers used special treatments to eliminate the trapezoidal locking. Using the method of selective scaling technique has developed an element SPS [23].
Recently studies were done on unsymmetrical element TQ4 developed by [26] and US-ATFQ4 proposed by [4]. The elements stiffness matrices for those elements are unsymmetrical. Various development in new quadrature for the three dimensional elements has been developed by various authors [13, 21]. An alternate adjustment in patch test has been done to improve the convergence of results by [12]. A simple and robust method for the development of quadrature’s for quadrilateral element was done by [20]. This research papers deals with a development of new quadrature on par with Gauss quadrature. In order compute the element stiffness matrix with large varying data the new quadrature element Corner method is designed with four sampling points at the corners of the element and one point at the center of the element.

II. FORMULATION OF ELEMENT STIFFNESS MATRIX FOR THE TRIANGULAR ELEMENTS

The computation of element stiffness matrix need to deal with handling large number of varying data. The standard element stiffness matrix is derived as follows Strain energy (U) can be defined as

$$U = \frac{1}{2}\int_v \sigma^T \varepsilon dv = \frac{1}{2}[u \quad v]^T [K][u \quad v] \tag{1}$$

$$\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\end{Bmatrix} = \frac{E}{(1+\mu)(1-(1+\gamma)\mu)}\begin{bmatrix}1-\gamma\mu & \mu & 0\\ \mu & 1-\gamma\mu & 0\\ 0 & 0 & \frac{1-(1+\gamma)\mu}{2}\end{bmatrix}\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{xy}\end{Bmatrix} \tag{2}$$

$$\{\sigma\} = [D]\{\varepsilon\} \tag{3}$$

A function of Young's modulus (E) and Poisson's ratio (μ) will for the material matrix (D). Eqn. 4 shows the various function for which are used for geometry defining field functions (x and y), the material functions (E and μ) and variable defining displacement function (u and v) are described.

$$u = \sum_{i=1}^{n} N(l \quad m)_i u_i \qquad v = \sum_{i=1}^{n} N(l \quad m)_i v_i$$

$$x = \sum_{i=1}^{n} N(l \quad m)_i x_i \qquad y = \sum_{i=1}^{n} N(l \quad m)_i y_i$$

$$E = \sum_{i=1}^{n} N(l \quad m)_i E_i \qquad \mu = \sum_{i=1}^{n} N(l \quad m)_i \mu_i \tag{4}$$

Mapping of elements should be done for the evaluation of matrix provided Cartesian coordinates should be only one set for each set of corresponding non-dimensional coordinates. For the process of mapping of elements, the following matrix is used (eqn. 5) which in turn known as Jacobian matrix.

$$J = \begin{pmatrix}\frac{\partial x}{\partial l} & \frac{\partial y}{\partial l}\\ \frac{\partial x}{\partial m} & \frac{\partial y}{\partial m}\end{pmatrix} \tag{5}$$

The general strain equation is defined in eqn. 6

$$\{\varepsilon\} = \begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \lambda_{xy}\end{Bmatrix} = \begin{Bmatrix}\frac{\partial u}{\partial x}\\ \frac{\partial v}{\partial y}\\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\end{Bmatrix} \tag{6}$$

$$\{\varepsilon\} = [B] * \lfloor u \quad v \rfloor^T \tag{7}$$

The plain stress and Plane strain were defined to a matrix called as strain displacement matrix [B] which is shown in eq. 8

$$[B] = [B_1]_{3\times 4} \times [B_2]_{4\times 4} \times [B_3]_{4\times 2n} \tag{8}$$

$$[u \quad v]^T = [u_1 \quad v_1 \quad u_2 \quad v_2 \quad u_3 \quad w_3 \quad . \quad . \quad . \quad . \quad u_n \quad v_n]^T \tag{9}$$

Where nodes per element is defined as "n"

$$[B_1] = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 1 & 1 & 0\end{bmatrix}$$

$$[B_2] = \begin{bmatrix}J_{11}' & J_{12}' & 0 & 0\\ J_{21}' & J_{22}' & 0 & 0\\ 0 & 0 & J_{11}' & J_{12}'\\ 0 & 0 & J_{21}' & J_{22}'\end{bmatrix}$$

$$B_3 = \begin{bmatrix}\frac{\partial N_1}{\partial l} & 0 & \frac{\partial N_2}{\partial l} & 0 & . & . & \frac{\partial N_n}{\partial l} & 0\\ \frac{\partial N_1}{\partial m} & 0 & \frac{\partial N_2}{\partial m} & 0 & . & . & \frac{\partial N_n}{\partial m} & 0\\ 0 & \frac{\partial N_1}{\partial l} & 0 & \frac{\partial N_2}{\partial l} & . & . & 0 & \frac{\partial N_n}{\partial l}\\ 0 & \frac{\partial N_1}{\partial m} & 0 & \frac{\partial N_2}{\partial m} & . & . & 0 & \frac{\partial N_n}{\partial m}\end{bmatrix} \tag{10}$$

The element stiffness matrix for quadrilateral element can be written as

$$[K^e] = \int_{-1}^{1}\int_{-1}^{1} [B^e(l,m)]^T [D^e(l,m)][B^e(l,m)] \det[J^e] dl dm \tag{11}$$

Suitable quadrature can be defined following equation.

$$[K^e] = \sum_{i=1}^{P}\sum_{j=1}^{Q} W_i W_j K^e(l_i, m_j) dl dm \tag{12}$$

Here P and Q represents the sampling points number along the l, m and n directions and Wi, Wj represents the sampling weights for the respective Quadrature

III. FORMULATION OF ELEMENT CORNER METHOD

The aim of this research study is to find an alternate sampling point to 4-node quadrilateral element. The current study has used standard Gauss quadrature for the evaluation of stiffness matrix. Undetermined coefficients method is used for deriving new sampling points for the 4- node quadrilateral element. The main assumption which made here is the typical quadrilateral element is considered with 4 sampling points at the corners and one at the centre point. Fig.1 (a) and 1(b) shows the Gaussian quadrature methods and Fig. 1(c) – 1(e) shows the new set of sampling points locations. There are 5 sampling pointss, of which four are located at the corners of a quadrilateral element at a distance of (±a, ±a), each having a weight (W_a). The remaining point is located on the center of the axes each having a weight (W_b).

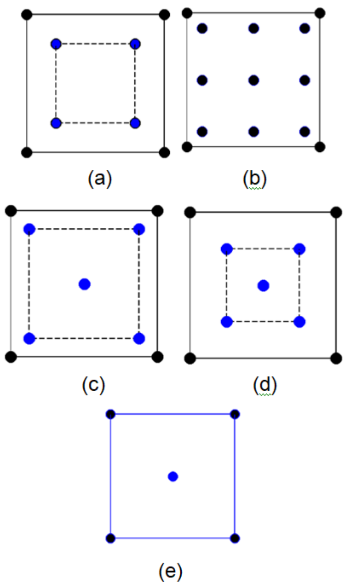

Fig. 1 (a) Gauss Two-point quadrature, (b) Gauss Three-point quadrature, (c) Element Corner method with location 0.5, (d) Element Corner method with location 0.5, and (e) Element Corner method with location 1.

The following polynomial is chosen for the deriving the new sampling locations at the corner point of quadrilateral

$$\Phi(f,g) = d_1 + d_2 f + d_3 g + d_4 f^2 + d_5 fg + d_6 g^2 + d_7 f^3 + .d_8 f^2 g + d_9 .fg^2 + d_{10} g^3 .... \tag{13}$$

Integrating equation (13) will get the following

$$\int_{-1}^{1}\int_{-1}^{1} \Phi(f,g) = d_1 + d_2 f + d_3 g + d_4 f^2 + d_5 fg + d_6 g^2 + ... \tag{14}$$

performing the numerical integration on equation (14) will provide

$$= d_1(4) + d_4\left(\frac{4}{3}\right) + d_6\left(\frac{4}{3}\right) + ... \tag{15}$$

The function Φ can be evaluated by using the following numerical form

$$\Phi = \sum_{i=1}^{n} W_1 \Phi(f,g,h) \tag{16}$$

Where n is considered as number of sampling points

Again substituting equation (13) in equation (16) we get

$$(W_1 + W_2 + W_3 + ...+ W_n)d_1 + (W_1 + W_2 + W_3 + ...+ W_n)d_2 + ...+ (W_1 + W_2 + W_3 + ...+ W_n)d_n$$

$$= W_1(d_1 + d_2 f + d_3 h + ...) + W_2(d_1 + d_2 f + d_3 h + ...) + ...+ W_n(d_1 + d_2 f + d_3 h + ...) \tag{17}$$

There can be only two sampling weights W_a and W_b on the basis of the location of sampling points. The coordinates and the weights are substituted simultaneously for each sampling integration points and on simplifying we end up with a set of simultaneous equations which are shown in equation 18.

$$4W_a + W_b = 4$$
$$4d^2 W_a = \frac{4}{3} \tag{18}$$

On substituting the points and solving the (19) the set of points are found and shown in Table 1. For two dimensional quadrilateral elements the reduced integration will be
For two dimensional quadrilateral elements, the reduced integration will be

$$I = \int_{-1}^{1}\int_{-1}^{1} f(l,m)\,dl\,dm = \sum_{i=1}^{5} f(l_i, m_i) W_i \tag{19}$$

Table 1: Quadrilateral element with new set of integration points.

Integration point location	Weighting function (at a =1)	Weighting function (at a =0.75)	Weighting function (at a =0.5)
Corner points	0.3333	0.592592	1.3333
Center points	2.6667	1.629632	-1.33332

Numerical Examples

In this section Well defined standard problem defined by various authors are taken as reference problems to study the results by the new proposed element corner method for the 4 node quadrilateral element. Convergence of results plays a major role in finite element analysis.

Example 1. Macneal Slender Beam

This problem is for testing the shear locking and trapezoidal locking. There will be three meshes are considered (Regular Quadrilateral Mesh, Parallelogram Mesh, Trapezoidal Mesh). Accuracy of results is an important factor for a quadrature thus irregularity in meshes should not affect the results had defined this problem [19]. There will three meshes as shown in Fig. 2 (a), (b) and (c). The Poisson's ratio is 0.3, young's modulus for the problem is 100000 and the thickness will be 0.2 units. The reference for the evaluation of results is taken at the end point A. Table shows the normalized displacement results at point A using the Gauss numerical integration method and element Corner method.

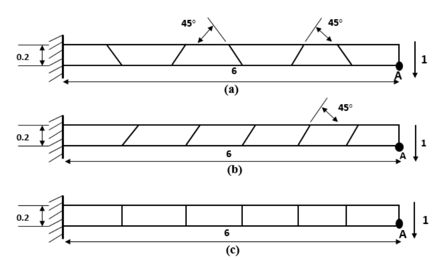

Fig 2. (a) Regular Mesh, (b) Parallelogram Mesh and (c)

The results show that on comparing other elements the proposed quadrature scheme results are on par with normalized displacement results of others methods and elements.

Example 2: Cooke's Skew Beam Problem

The example problem is defined by [8, 26]. This problem tested by improving the number of meshes thus it will lead to analyzing the convergence of displacement results. This example is being used for various researchers to study the anti-distortion performance. The Cooke's skew problem is shown in Fig. 3. The convergence behaviour of new proposed method is found to comparable with other elements. The results are tabulated in Table 3 and the convergence graph is shown in Fig. 4. This problem is considered to test the sensitivity of distortion of meshes. Distortion test was recommended by [7].

Example 3: Distortion Test

The problem outlines a beam discretized in two elements and the distortion of mesh is done by varying a parameter shew distance "e". Change of skew distance will have led to the distortion in meshes. Fig. 5 shows the distortion test problem.

Trapezoidal Mesh.

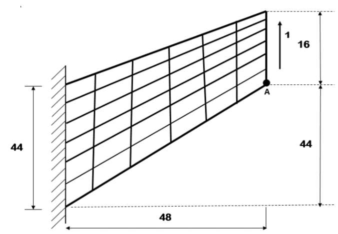

Fig. 3. Cooke's Skew Problem.

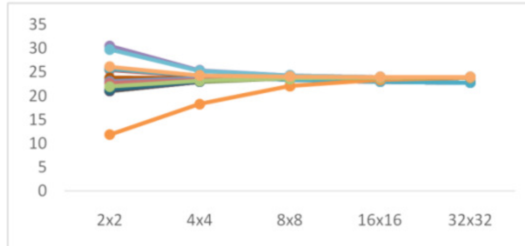

Fig. 4. Convergence plot of Cooke's skew problem.

Table 2: Normalized displacement results of Macneal slender beam problem.

Quadrature	Mesh (a)	Mesh(b)	Mesh(c)
Q4	0.093	0.034	0.027
Q6	0.994	0.678	0.106
Qm6	0.993	0.623	0.044
NQ6	0.994	0.633	0.052
P-S	0.993	0.798	0.221
PEAS7	0.982	0.795	0.217
QC6N	0.993	1.000	0.987
AQGβ6-I(β=0)	0.993	0.994	0.994
US-ATFQ4	0.993	0.992	0.992
ESQ5α-M	0.993	0.973	0.967
rQm6(-2)	0.994	0.995	0.994
Nq6(-2)	0.994	0.995	0.995
iQ8	0.994	0.995	0.995
Gauss 2X2	1.000	1.0000	0.9991
ECM1	1.0001	0.9979	0.9949
ECM0.75	1.000	0.9993	0.9976
ECM0.5	1.0001	1.0003	0.9996
Gauss 3X3	0.9999	0.9998	0.9987

Table 3: Normalized displacement results.

Element/ Method	2 × 2	4 × 4	8 × 8	16 × 16	32 × 32
Gauss 2 × 2	25.79299	24.283	23.4579	23.02239	22.8016
ECM1	25.4765	24.237	23.4546	23.02286	22.8024
ECM0.75	25.6866	24.267	23.45649	23.02233	22.8016
ECM0.5	25.8328	24.290	23.45908	23.02304	22.8022
Gauss 3×3	25.709	24.2687	23.45449	23.01998	22.7993
Q4	11.85	18.30	22.08	23.43	23.82
Q6	22.94	23.48	23.81	23.91	23.95
Qm6	21.05	23.02			
NQ6	21.05	23.02	23.69	23.88	23.94
P-S	21.13	23.02		23.88	
QE2	21.35	23.04		23.88	
QC6N	23.78	23.72	23.87	23.93	
AQGβ6-I(β=0)	23.07	23.68	23.87	23.93	
US-ATFQ4	22.76	23.43	23.79	23.91	
ESQ5α-M	22.00	23.21	23.66	23.85	23.92
rQm6(-2)	30.55	25.37	24.28	24.03	23.98
rNQ6(-2)	29.78	25.32	24.28	24.03	23.98
iQ8	26.13	24.33	24.03	23.98	23.97

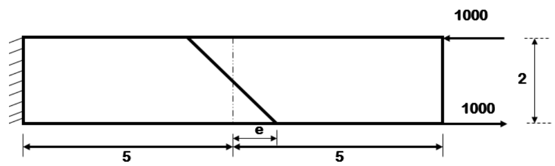

Fig. 5. Distortion test on beams.

Table 4: Normalized Displacement Results of distortion test.

Element / Method	e=0.5	e=1	e=2	e=3	e=4
2 × 2	100.0002	99.9996	100	99.7673	98.9325
ECM1	99.4394	98.2254	93.9983	84.734	69.3042
ECM0.75	99.8044	99.3894	97.9811	94.8177	89.2645
ECM0.5	100.0738	100.2229	100.7253	101.5175	102.3288
3 × 3	99.9171	99.7539	98.9307	96.4736	91.1123
Q6	93.21	86.90	92.67	102.4	110.5
Qm6		67.3	62.4	65.7	66.9
NQ6	83.91	67.29	62.42	65.66	66.95
iQ8	100	100	100	100	100
rQm6(-2)	100.1	99.91	97.32	91.53	83.34
rNQ6(-2)	100.1	100.2	99.89	99.85	101.0
AQGβ6-I(β=0)	100	100	100	100	100
US-ATFQ4	100	100	100	100	100
ESQ5α-M	99.99	99.97	99.88	99.82	99.79
P-S		67.5	63.1	67.2	70.0
ECQ4		87	93.1	103.1	111.4
HSF-Q4α-7β	99.93	99.47	95.95	87.14	71.87

The normalized displacement results of distortion test problem are tabulated in Table 4.

Example 4: Eigen Value Analysis

Eigen value is used to determine the rank deficiency of element stiffness matrix thus it is necessary to analyze the Eigen value [16]. Eigen value analysis is done to study the stability of the quadrature. A single standard quadrilateral element is chosen with unit length. The element stiffness matrix is calculated and the Eigen values are determined. Figure 6 shows standard 4-node quadrilateral element with unit length and Table 5 shows the Eigen value analysis results. From Table 5 we can infer that the proposed quadrature's are found to stable and which can be used for the evaluation of large fluctuating element stiffness matrix.

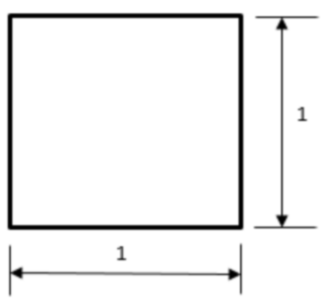

Fig. 6. Unit quadrilateral element.

Example 5 CPU Time Comparison

Computation of stiffness matrix will lead to deal with large fluctuating data thus computational time needs to be on par with existing quadrature methods. The code for the computational analysis is written on MATLAB [9, 14]. The CPU time comparison was done on a 4-node quadrilateral element with the element corner approach and gauss sampling approach using computer of configuration Intel(R), i3 2.4 GHz, 4 GB RAM.

Table 5: Eigen value analysis for the computation of element stiffness matrix using various integration schemes.

λ	2×2	ECM1	ECM0.75	ECM0.5	3×3
1	0	0	1.6	1.6	1.6002
2	0	0	0	0	0
3	0	0	0.8	0.8	0.8001
4	0.5333	0.5333	0.5333	0.5333	0.5334
5	0.5333	0.5333	0.8	0.8	0.8001
6	0.8	0.8	0.5333	0.5333	0.5334
7	0.8	0.8	0	0	0
8	1.6	1.5999	0	0	0

Table 6: CPU computational time analysis in seconds.

	1000	10000	25000	50000	100000
2×2	0.66966 5	3.50290 2	6.46176 8	11.6581 4	21.3700 8
ECM1	0.62962 3	4.23048 6	7.53503 5	14.2046 6	24.6222 8
ECM0.7 5	0.67593 1	4.16779 4	7.39426 8	13.6063 2	25.3281 7
ECM0.5	0.68726 9	4.36531 1	7.59341 2	13.2737 1	25.4918

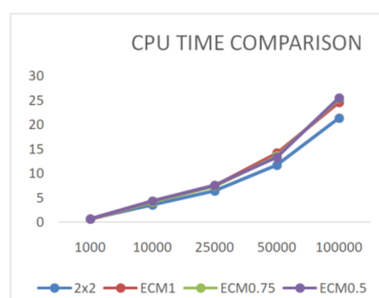

Fig. 7. CPU computational time plot of various quadrature schemes.

The coded program was run for 100,000 time in MATLAB to study the CPU time using different quadrature methods. The results are shown in following Table 6 and Fig. 7.

Example 6 patch test

Patch test is considered to be one of the important test in finite element analysis which deals with errors in stiffness matrix calculation. The patch test is conducted in distorted elements [6]. The problem chosen for patch test is shown in Fig. 8. A linear elastic material with Poisson's ratio (ϑ) of 0.25 and Young's modulus (E) of 1000000 is selected for the test. The patch test is carried out to determine the error in stiffness matrix on comparison with standard matrix calculated using standard quadrature methods here it is considered as Gauss quadrature. Equation 20 shows the stiffness matrix error for the quadrilateral elements. Stiffness matrix error (ε_s) is given by

$$\varepsilon_s = \frac{\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n}\left(K_{ij}^{Gauss} - K_{ij}^{Proposed}\right)^2}}{\sum_{i=1}^{n}\sum_{j=1}^{n}\left|K_{ij}^{Gauss}\right|} \quad (20)$$

Table 7 shows the stiffness matrix error of quadrilateral elements on comparing with gauss quadrature methods. The results show that on comparing with Gauss integration scheme, the integration points of element corner method are found to be negligible so it can be inferred that it had passed patch test.

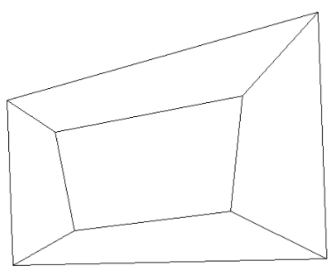

Fig. 8. Patch test with distorted elements.

Table 7: Stiffness Matrix Error.

ECM1	0.0043
ECM0.75	0.0013
ECM0.5	4.31E-04

Example 7: Convergence test

Convergence test for quadrilateral elements is conducted by increasing number of elements. Here on increasing the number of elements the distortion of elements getting reduced and converges to a value. Fig. 9 shows the test problem for convergence test. The displacement error norm is calculate using the equation 22.

$$\xi_d = \frac{\sqrt{\sum_{i=1}^{n}(U_i^{Gauss2x2} - U_i^{Proposed})^2}}{\sqrt{\sum_{i=1}^{n}(U_i^{Gauss2x2})^2}} \quad (22)$$

The displacement results are shown in table 8 and the results are plotted n Fig. 10. It can be seen that after increasing number of elements the results are getting converges to a value.

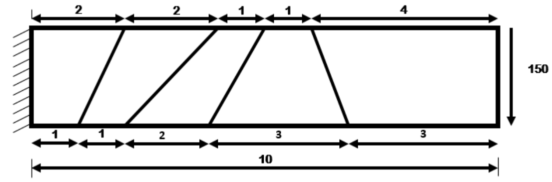

Fig. 9. Convergence test problem.

Table 8: Convergence of displacement results.

Quadrature	Number of Elements				
	5	20	80	320	1280
2 × 2	0.023472	0.003272	0.000375	0.00012	0.000101
ECM1	0.048097	0.006554	0.000438	9.66E-05	0.000131
ECM0.75	0.001024	8.98E-05	8.51E-05	9.96E-05	0.0001
ECM0.5	0.032345	0.004498	0.000505	0.000152	0.000127

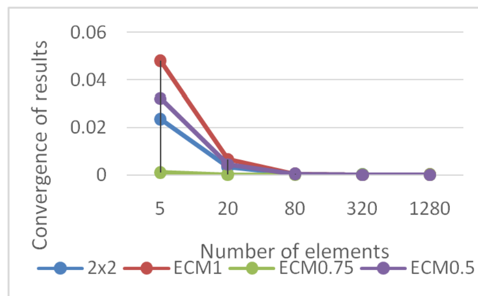

Fig. 10. Convergence of displacement results in graphical mode.

IV. DISCUSSION AND CONCLUSION

The proposed Element Corner Method for the 4 node quadrilateral element is been tested and compared with Gauss Numerical quadrature scheme. In view of the results, it is inferred that the proposed scheme is accurate and efficient. Because of large varying data while computing element stiffness matrix and post processing calculations will lead doors for complex calculations. Formulation of element corner was found simple. The results of the proposed schemes were compared with results of benchmarked problems defined by various authors. The following conclusions were inferred from the research study.

- The element corner method doesn't show any shear and trapezoidal locking problems and also the normalized displacement results were found to be on par with other standard results
- In the case of Cooke's skew problem, the convergence of results was happened on increasing the number of elements. The results shown that on increase of number of elements the results get converged to a value.
- The normalized displacement results of Cooke's problem show that the normalized displacement results are comparable with the existing gauss numerical scheme.

The sensitivity of meshes for the new quadrature was tested using distortion test. The Variation of skew parameter 'e' will lead for inaccurate results but in the proposed quadrature scheme the results are on par with the gauss quadrature.

The results of patch test were significantly inferred that the new scheme can handle complex problems and error in stiffness matrix were found to be too less thus the Element Corner method can handle distorted elements.

Based on the results we can conclude that the proposed Element corner method is found to be simple and accurate which can be used for evaluating the element stiffness matrix infinite element analysis.

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