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# A Study of the Black Scholes Pricing Model by Applying Estimated Volatility in Indian Securities

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**ABSTRACT: In this article, we have studied the Black-Scholes partial differential equation (BSPDE) which is a fundamental model in financial mathematics that describes the behaviour of option pricing. One important application of the Black-Scholes equation is in the pricing of European call and put options traded in the stock markets. Traders and financial analysts use this model to estimate the fair value of options, helping them to decide when to buy or sell these contracts. Under the theoretical framework of BS model supporting the efficient functioning of financial markets, we can have accurate risk assessment and execute perfect investment strategies. In this study, we have employed the Crank-Nicolson (CN) method to solve the Black-Scholes equation for real volatility. An accurate second-order solution in both asset space and time dimension are provided by the finite difference Crank-Nicolson method. We have applied a tri-diagonal matrix algorithm to solve the Black-Scholes PDE using the Crank-Nicolson method, in which a system of linear equations is constructed. In real-time situation, the accurate volatility estimation is crucial because it deeply affects the option pricing and the risk assessment. Volatility reflects the uncertainty or risk associated with the price movement of an underlying asset. Its precise estimate ensures that traders take well-informed decisions. Using accurate volatility estimates, investors can better manage their portfolios and hedge against potential market fluctuations, leading to more stable, predictable, and profitable outcomes. The Crank-Nicolson method provides an accurate and efficient solution to the Black-Scholes equation for real volatility. The method is stable and can easily be extended to more complex financial models. Our scheme is designed to describe the behaviour of financial options in the presence of real-world volatility, which is often characterized by a nonconstant volatility. Here, we have applied the CN method to solve BS PDE numerically and simulated the call option results through MATLAB. We have also presented the graphical and numerical solutions to the option pricing problem.**

**Keywords:** Black-Scholes partial differential equation, European Option Pricing, Call option, Numerical scheme, CN Method, Indian financial market.

## INTRODUCTION

In this paper, we examine Black Scholes partial differential equations (BSPDEs) which is a fundamental model in financial mathematics that describes the behaviour of Pricing options. The BSPDE can be used to price European-style options, and can also be used to model other financial derivatives such as mortgage-backed securities. The BSPDE is a powerful tool for understanding the behaviour of financial markets. The BSPDE is used to analyse the pricing of financial derivatives such as stock options and futures contracts. It can also be used to model the pricing of financial assets such as stocks and bonds. The BSPDE is also commonly used to model the dynamics of stock prices. It is a powerful tool for financial modelling and can be used to price a wide range of financial instruments, including options, derivatives, and bonds. Its flexibility and accuracy make it an essential component in the toolkit of financial analysts and risk managers. By leveraging this tool, professionals can make more informed decisions and optimize their investment strategies, and also use the estimated volatility in the BSPDE of the real stock price data. This method provides a comprehensive approach to understanding market dynamics and making informed investment decisions. By incorporating real stock price data, the model offers a more accurate reflection of market conditions, enhancing its predictive capabilities and reliability and solve the BSPDE using the Crank-Nicholson method. The BSPDE can also be used to analyse the effects of monetary policy on stock prices. In addition, the BSPDE can provide insight into the optimal investment strategy for an investor. The study of volatility and fluctuations of stock prices is important, as various models for determination of option pricing like the classic BS model help investors and traders in determining the fair value of financial derivatives based on the underlying assets.

FIIs and MF are the major institutional investors that play a vital role in Indian stock market. The role of technology in shaping Mutual funds flow in India is studied by Jain (2019). In her study the correlation between FIIs and MF flows in equity, and debt market is found to be significant and positive. It is concluded that the investment made by mutual funds has surpassed FIIs investment with technology playing a crucial role in developing mutual funds as a stronger institutional investor in the market.

Volatility plays a crucial role in risk management by providing insights into the potential price movements of financial assets. It allows investors and risk managers to assess the level of uncertainty and potential risk associated with an asset's future performance. By understanding volatility, they can implement strategies such as hedges to mitigate potential losses and optimize their investment portfolios. Volatility directly impacts option pricing through its influence on the option's premium. As volatility increases, option premiums typically rise as the uncertainty increases the possibility of significant price swings, therefore increasing profitability. Alternatively, low volatility leads to low option premiums since price movements are less drastic. Historical data plays a vital role in predicting volatility as it provides a record of past price movements and market behaviour. By analysing historical trends and patterns, investors and analysts can estimate future volatility levels and make informed decisions about potential risk exposure. Additionally, historical data helps in calibrating models to forecast future price fluctuations and enhance the accuracy of volatility predictions. Historical data also helps to identify trends and patterns that can be used to develop trading strategies. It can also be used to identify potential market entry and exit points. By accurately pricing options, the BS model enables market participants to make informed decisions about hedging, speculation, and risk management. Understanding the volatility and price fluctuations of stocks is essential for the effective application of the model. The classic Black and Scholes model employs a continuous-time framework with several assumptions, like constant volatility and constant interest rates, to derive analytical solution for option pricing Black and Scholes (1973). The model has been refined and modified over time to account for more complex market conditions. Modern versions of the model also consider the effects of transaction costs and time decay. Alternatively, the Cox, Ross, Rubinstein (CRR) binomial model uses a discrete-time approach. A binomial tree is used in this approach in order to simulate probable price movements over time (Cox *et al.*, 1979).

Both models provide foundational tools for evaluating options, though they have been expanded and adapted to account for more complex market conditions that are fundamentally based on the estimation of asset's volatility during the life time of the option concerned Black and Scholes (1973); Cox *et al.* (1979); Yamasaki, *et al.* (2005).

The outcome of the unpredictable movement of stock prices can be explained with the help of the evolution of stochastic processes like Brownian, geometric Brownian, and Levi process having i. i. d. increments Geman and Yor (1993); Madan (2010). Wiener process has been defined on the basis of probabilities assigned to the sets concerning process path. It has normally distributed increments which are independent, with a zero mean and the variance that is proportional to the time interval Madan (2010). For European call and put options, the BS model, as well as some other alternative models, can also be solved analytically. It is difficult to have an analytical solution for path-dependent options, like American or barrier options. However, numerical methods, such as Monte Carlo simulations or binomial trees, can be used to approximate their values. These methods provide a practical way to handle the complexity of path-dependent options. Numerical methods for these contracts have their own limitations Boyle (1997). Now a days jump diffusion models have become important as these models are more realistic. Jump diffusion models incorporate sudden, large changes in asset prices, which reflect real-world market events like economic announcements or geopolitical events. Unlike traditional models that assume continuous price changes, jump diffusion models capture the erratic and unpredictable nature of financial markets. This makes them more effective in modelling and forecasting asset price movements Kou (2002), also studies a jump diffusion model for option pricing. Asati *et al.* (2024) have presented a comprehensive analysis of Black Scholes Model encompassing its historical context, theoretical foundations, practical applications, limitations, and presented a critical review of the existing literature on the exact as well as the numerical solutions to the Black-Scholes model and discussed recent advancements in the field. Gangele and Asati (2024) have studied the Black-Scholes pricing model under varying dividend condition. Anwar and Andallah (2018) have studied some numerical solution of Black-Scholes model.

## LITERATURE REVIEW

Partial differential equations are very useful tools in mathematical modelling. The BSPDE is the most popular PDE for determining the price of an option contract Wilmott *et al.* (1995). Various researchers have also applied finite element methods to solve the BSPDE numerically Emmanuel *et al.* (2012); Thomas (1995); Babasola *et al.* (2018); Wilmott *et al.* (1995); Wade *et al.* (2007); Umeorah and Mashele (2019). Other than the finite element method, researchers often use finite difference methods, which approximate derivatives using difference equations on a grid. Singar *et al.* (2020) have studied the key generation using featured based Finite Element Method. The Finite Element Method (FEM) has turned into an effective and well-known apparatus to tackle the problems and find the solution of Engineering mathematics and financial world.

Spectral methods are another alternative, where the solution is represented as a sum of basic functions, allowing for high accuracy with fewer grid points. Furthermore, the finite volume method conserves energy and mass over control volumes in computational fluid dynamics. Spectral methods have been widely used in fluid dynamics, where they provide accurate solutions for problems involving turbulence and complex boundary conditions. They are also applied in meteorology for weather prediction models, where they help capture the global features of atmospheric phenomena. Additionally, spectral methods are utilized in quantum mechanics to solve eigenvalue problems with high precision. They are also used in image processing and computer vision, where they can be used to recognize objects in images. Spectral methods are also used in many other areas of science and engineering. Spectral methods are also widely used in finance, where they are used to solve complex optimization problems. They are also used in signal processing and data analysis, where they are used to detect patterns in data. Spectral methods are also widely used in weather forecasting, where they help to predict and analyse weather patterns. They are also used in medical imaging, where they are used to analyse medical images. The convection diffusion equation that is converted into a fully discrete problem with the help of the compact approximations for spatial discretization and Crank-Nicolson scheme for temporal discretization is studied by Goswami and Patil (2022). They have used a matrix method approach to establish the stability of Crank-Nicolson compact schemes for convection-diffusion equation. Emmanuel *et al.* (2012) have studied some finite difference methods like implicit method and Crank-Nicolson methods for determination of option pricing. It is concluded that the Crank-Nicolson finite difference method is more stable, more accurate and converges faster than that of the implicit method. Wade *et al.* (2007) have applied an improved smoothing strategy for the Crank–Nicolson method which is unique in achieving optimal order convergence for barrier option problems. This strategy enhances the accuracy of numerical solutions by effectively addressing the discontinuities at barrier boundaries, making it a significant advancement in financial mathematics. Furthermore, the numerical experiments for one asset and two assets have been discussed. Umeorah and Mashele (2019) have employed the Crank-Nicolson finite difference scheme to estimate the prices of rebate barrier options. They have also discussed the effect of rebates on barrier option values. The numerical solution of the Black-Scholes PDE is an important area of research, with contributions from several notable authors, *viz*., Smith (1985); Smith *et al.* (2018); Jones and Brown (2016). These authors have explored various numerical methods and their applications in financial mathematics, further advancing the field Jeong *et al.* (2018); Babasola *et al.* (2018). These authors have explored the finite difference methods like Crank-Nicholson with their convergence and stability for the option pricing problem regarding European options. Kumar and Agrawal (2017) have studied the computational efficiency of various numerical schemes for solving the BS equation. They have extended the BS model, incorporating dividend yields and stochastic volatility in the model. The application of the Crank-Nicolson method in case of these situations makes it an efficient instrument for financial world problems. Ganga Ram *et al.* (2022) have studied the exotic options, such as barrier and lookback. They emphasized the ability of the Crank-Nicolson method to handle non-smooth payoff structures effectively. Amadi *et al.* (2020) have also presented the applications of numerical methods applicable in the case of real-world market problems for high frequency trading. Banu (2019a) have studied the analyse of the effect of monetary policy instruments on the bank's revenue and profitability of the India's largest public bank "State Bank of India". Banks played an un-denying relentless role in the banking system of modern India. It is not only commendable but also adorable. The efficiency of the banks is crucial for the existence of smooth flow of trade locally and internationally. The results indicate that there exists a strong correlation among public, private and foreign sector banks with regard to return on equity, return on investments, return on assets and return on advances on the profitability position Banu and Vepa (2019b).

Wokoma *et al.* (2020) have presented the estimation of Stock Prices using Black-Scholes Partial Differential Equation for Put Option.

## MATHEMATICAL DETAILS

Black and Scholes (1973) derived a complete option pricing model depending only on observable variables Duffy (2013). Their work revolutionized financial markets by providing a theoretical framework to price options, which was previously considered an art rather than a science. This model, known as the Black-Scholes model, assumes that the price of the underlying asset follows a geometric Brownian motion, leading to a log-normal distribution of prices. The stock value in the model follows a log-normal diffusion process

$$dS/S = \mu\, dt + \sigma\, dW\,. \qquad (1)$$

Let $S_t$ be the value of the stock at the time t. The stock price can fluctuate due to various factors such as market conditions, company performance, and investor sentiment. By analysing these factors, investors can make informed decisions about buying or selling the stock. Change in this value from the previous value in percentage terms from $t$ to $t + dt$ is represented as $(S_{t+dt} - S_t)/S_t$. A log-normal distribution for $dS/S$ consists of two components: a drift term $\mu\, dt$, and a normally distributed stochastic term $\sigma\, dW$. The stochastic term is independent for each state, distributed normally with mean zero and variance $\sigma^2 t$. Hence, the percentage change in stock value from $t$ to $t + dt$ is normally distributed with mean $\mu\, dt$ and variance $\sigma^2\, dt$. This implies that over a small-time interval, the stock price follows a geometric Brownian motion, which is a common model for stock price movements in financial mathematics. As $dt$ gets small (tends to zero), $S_{t+dt}$ will not differ much from $S_t$. This represents the characteristics of a diffusion process, which is a continuous, frictional kind of random walk around a trend term (Cox *et al.*, 1979).

## NOTATIONS AND SYMBOLS

In this Paper we have used the following Mathematical notations

(1) $V_1\ (S, t)$: Theoretical value of the option.

(2) $K$ : Arbitrary fixed choice of strike price.
(3) $S$ : Spot price, in addition, we use S by $S_t$ for simplicity.
(4) $e^{-r(T-t)}$ : Discounting factor used for continuous discounting.
(5) $r$ : risk-free interest rate.
(6) $t$: Exercise time.
(7) $\sigma$ : Volatility of the stock.
(8) $d_1$ and $d_2$ : Intermediate variables (constants).
(9) $N(d_1)$ and $N(d_2)$: Values of the cumulative distribution functions (CDF) of the standard normal distribution at $d_1$ and $d_2$ respectively.
(10) $P$: Value of the put option.
(11) $V_1$ : Value of the call option.
(12) $PV_1(K)$ : Present value of the strike price.

## DERIVATION OF THE BLACK SCHOLES PARTIAL DIFFERENTIAL EQUATION

As per the assumptions of the Black-Scholes model, the price of the underlying asset (typically a stock) follows a geometric Brownian motion Wilmott *et al.* (1995); Jeong *et al.* (2018); Madan (2010). That is,

$$dS = S\,(\mu\, dt + \sigma\, dW) \tag{2}$$

where $W$ is a stochastic (Brownian) process, and also called the Winner Process. The uncertainty originates due to the infinitesimal increment in $W$, represented as $dW$. It is the only source of uncertainty in the stock price. Hence, $W$ is a process that wiggles up and down in such a random way that its expected change over any time interval approaches zero. This property makes $W$ a martingale, a process where future values are conditionally independent of past values, given the present. Consequently, predicting the future stock price based solely on its past performance is impossible. In addition, its variance over time $T$ remains non-zero. A good discrete analog for $W$ is a simple random walk. This implies that while the stock price can fluctuate unpredictably in the short term, its long-term trend is unpredictable. According to the random walk model, future stock prices cannot be predictably predicted based on past price movements. Thus, the above Eq. (2) states that the infinitesimal rate of return on the stock has an expected value μ and variance $\sigma^2 t$. The payoff of an option $V_1(S,t)$ at maturity is known. To find its value at an earlier time, we need to know how $V_1$ evolves as a function of $S$ and $t$. Based on Ito's lemma and stock value $S$ and time $t$, we have

$$dV_1 = \left( \frac{\partial V_1}{\partial t} + \mu S \frac{\partial V_1}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V_1}{\partial S^2} \right) dt + \sigma S \frac{\partial V_1}{\partial S}\, dW \tag{3}$$

Now, we can have a portfolio, called the delta-hedging portfolio, consisting of a short position in one option and a long position of $\frac{\partial V_1}{\partial S}$ shares at time $t$. Alternatively, we can also apply the concept of the replicating portfolio. To introduce a replicating portfolio to equate to $V_1$ as a function of stock price and time, we arrive at the *BSPDE*. For constructing the replicating portfolio, a mixture of investments is constructed to produce a net return equivalent to the option segment. This mixed investment combination behaves like a replicating financial derivative. Let us consider the initial stock value $S_0$, which is the stock price at $t = 0$. The stock can move to possible values at time $t = \tau$, one being $S_u$ and the other $S_d$. Here, a financial derivative for the option value $V_1$ depends on time $t$ and is also dependent on the performance of the stock value $S$. If $S$ goes up, the value of $V_1(S,t)$ will be reflected as $U$. If $S$ goes down, the value $V_1(S,t)$ will be equal to $D$. For the analysis, we also need the riskless interest rate $r$. This rate serves as a benchmark for evaluating investment opportunities and calculating the present value of future cash flows. It represents the return on an investment with no risk of financial loss. Typically, the riskless interest rate is derived from government bonds, such as U.S. treasury securities, which are considered free from default risk. This rate is crucial in determining the discount rate used in discounted cash flow (*DCF*) analysis. The riskless interest rate is a fundamental component in financial modelling, providing a baseline for assessing the potential returns of various investments. Its stability and predictability make it an essential tool for investors seeking to make informed decisions in the market. Consider a short-term bond with an initial value of one dollar. With continuous compounding, the value of such a bond at time $t$ is given by $e^{rt}$. Let us now construct a portfolio consisting of "$a$" units of the stock and "$b$" units of the bond, each worth one dollar. Thus, the portfolio's value at time $t = 0$ equals

$$P_0 = a\,S_0 + \mathrm{b}.1$$

Based on a stock model, we can predict two future portfolio values at time $\tau$. At time , the portfolio value is as follows:

$$P_\tau = a\,S_u + \mathrm{b}.e^{r\,t} \text{ (up state)}, \quad P_\tau = U,$$
$$P_\tau = a\,S_d + \mathrm{b}.e^{r\,t} \text{ (down state)}, \quad P_\tau = D$$

Thus, the value of our portfolio $P$ is identical to the derivative security. In this case, the portfolio is said to replicate $V_1$. This replicating portfolio is a very powerful tool. Now, we search for a suitable combination of stock and bond investments. At any time, the net worth of this investment is the target value $V_1(S,t)$. Assume that $V_1(S,t)$ is some given smooth function of the variables $S$ and $t$. Define $\phi(t)$ as the number of shares of stock and $\psi(t)$ as the number of bonds. Therefore, the portfolio value is

$$V_1(S,t) = \emptyset.S + \psi.P_t \quad \text{for} \quad 0 \le t \le T,$$

where $P_t$ is the value of a bond. The following equation represents the changes in the net worth of the portfolio

$$dV_1 = \phi\, \mathrm{d}S + \psi\, \mathrm{d}P_t \tag{4}$$

For the stock and bond derivatives, we have

$$dS = \mu\, S\, dt + \sigma\, S\, dW.$$

and

$$dP = r\,P\,dt. \tag{5}$$

Thus, eq. (4) becomes

$$dV_1 = \phi\,(\mu\, S\, dt + \sigma\, S\, dW) + \psi\,(r\, P\, dt)$$

Simplifying, we get

$$\begin{aligned} dV_1 &= \phi\,(\mu\, S\, dt + \sigma\, S\, dW) + \psi\,(r\, P dt) \\ &= (\mu\phi\, S + r\psi\, P) dt + \sigma\,\phi S\, dW \end{aligned} \tag{6}$$

Substitute this into Eq. (3) we obtain

$$(\mu\phi\, S + r\psi P)\, dt + \sigma\phi S\, dW = \left(\frac{\partial V_1}{\partial t} + \mu\, S\frac{\partial V_1}{\partial S}\right) dt$$

$$+\left(\frac{1}{2}\sigma^2\, S^2 \frac{\partial^2 V_1}{\partial S^2}\right) dt + \sigma\, S\frac{\partial V_1}{\partial S}\, dW \tag{7}$$

Assuming $\phi$ as

$$\phi\,(t) = \frac{\partial V_1}{\partial S} \tag{8}$$

Substituting the value of $\phi$ into the Eq. (7)

$$r\psi P\; dt = \left(\frac{\partial V_1}{\partial t} + \frac{1}{2}\sigma^2\, S^2\frac{\partial^2 V_1}{\partial S^2}\right) dt \tag{9}$$

Since $V_1(S, t) = \phi S + \psi P_t$ ,we have $\psi P = V_1 - \phi\, S$.
From eq. (8)

$$\psi P = V_1 - S\frac{\partial V_1}{\partial S}$$

Substituting this into Eq. (9)

$$r\left(V_1 - S\,\frac{\partial V_1}{\partial S}\right) dt = \left(\frac{\partial V_1}{\partial t} + \frac{1}{2}\sigma^2\, S^2\frac{\partial^2 V_1}{\partial S^2}\right) dt \tag{10}$$

Simplifying, eq. (10) the option price PDE is obtained as follows

$$\frac{\partial V_1}{\partial t} + \frac{1}{2}\,\sigma^2\, S^{\,2}\,\frac{\partial^2 V_1}{\partial S^2} + r\, S\,\frac{\partial V_1}{\partial S} - r\, V_1 = 0 \tag{11}$$

This is known as the *BSPDE* for option pricing. Under the assumptions of the *BSPDE*, this second-order partial differential equation provides the option value (call or put) for any European option, as long as its price function $V_1\,(S, t)$ is twice differentiable with respect to $S$ and once differentiable with respect to $t$. The *BSPDE* is a powerful tool in financial mathematics, offering a rigorous framework for pricing options. Its ability to handle a wide range of European options makes it a cornerstone of modern option pricing theory, that is based on the payoff function at expiry and boundary conditions Smith (1985); Øksendal (2003).

## THE BLACK-SCHOLES FORMULA

The Black–Scholes formula evaluates the price of European put and call options. The obtained option price is consistent with the *BSPDE (11)* that we have already derived. This consistency confirms the validity of the formula and its practical application in financial markets. Furthermore, it allows traders to accurately price options, facilitating informed decision-making and risk management. In order to obtain this formula, one must solve the BSPDE for the terminal and boundary conditions given by Black and Scholes (1973); Hull and Basu (2017); Kumar and Agrawal (2017) as follow

$V_1(\,0, t\,) \;=\; 0$ for all t,

$V_1\,(\,S\,, t\,) \to S - K\;\; as\, S \;\to \infty$,

$V_1\,(\,S\,,T\,) \;=\; max\,\{\,S - K\,, 0\,\}$.

The value of a call option for a non-dividend-paying underlying stock in terms of the Black–Scholes parameters is

$$V_1(S\,, t\,) \;= N\,(d_1)S \;-\; N\,(d_2)K e^{-r(T-t)}\,,$$

Where

$$d_1 = \frac{1}{\sigma\sqrt{T-t}}\;\left[\ln\left(\frac{S}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)(T-t)\right],$$

$$d_2 = d_1 - \sigma\sqrt{T-t}\,.$$

The put–call parity condition allows us to evaluate the price of a corresponding put option based on put–call parity, with a discount factor $e^{-r(T-t)}$. Hence, the value of the put option is given by

$$P\,(S, T) = K\, e^{-r(T-t)} - S + \,V_1(S\,, t)\,.$$

This can also be written as

$$P \;= N(-d_2\,)\,K\, e^{-r(T-t)} - N(-d_1\,)S$$

Alternatively, the value of put option can be expressed as

$$P \,=\, V_1 + \;PV_1(K) - S\,.$$

## DERIVATION OF CRANK-NICOLSON SCHEME

The Black-Scholes PDE that is the eq. (11) for call option pricing is given as Dura *et al.* (2010); Rana and Ahmad (2012); Wade *et al.* (2007)

$$\frac{\partial V_1}{\partial t} + \frac{1}{2}\,\sigma^2\, S^{\,2}\,\frac{\partial^2 V_1}{\partial S^2} + r\, S\,\frac{\partial V_1}{\partial S} - r\, V_1 = 0 \tag{12}$$

Using the transformation $\tau \;= T - t,$ eq. (12) becomes

$$\frac{\partial V_1}{\partial \tau} - \frac{1}{2}\,\sigma^2\, S^{\,2}\,\frac{\partial^2 V_1}{\partial S^2} - r\, S\,\frac{\partial V_1}{\partial S} + r\, V_1 = 0 \tag{13}$$

Using the initial and boundary conditions as

$$V_1(S, 0) = \; max\,(S(0) - K, 0) \tag{14}$$

Our aim is to solve the *BSPDE* on a discrete asset-time grid. The domain of the problem is defined as $[0, S_{max}] \times [0, T]$. We discretize this domain using uniform asset and time strata with steps $\Delta S$ and $\Delta t$, respectively. The payoff at time $T$ is known, and thus the solution involves applying the concept of backward iteration to the square or rectangular grid until the expiration time $T$. Let us denote the option value at a given time and asset price as $V_{i,k} = V(t_i, S_k\,)$, where $k = 0, \dots, m$ , and $i = 0, \dots, n,$ and $S_k = k\,\Delta S$ is the asset price at step $k,$ $t_i \;= i\,\Delta t$ is the time at step $i$. The discretization forms a grid where the backward iteration starts with the payoff at $t \;=\; T$ and proceeds step-by-step to $t \;=\; 0$.

Crank and Nicolson (1947) introduced a numerical solution to a PDE arising from heat conduction problems. Their method, known as the Crank-Nicolson scheme, is an implicit finite difference approach that offers greater stability and accuracy compared to explicit methods. This technique is widely used in computational finance, engineering, and physics due to its robustness in handling complex boundary conditions and time-dependent problems. It is an unconditionally stable method.

Using the explicit finite difference (forward) scheme to transform the BSPDE (13), we obtain its discretized form as follows Hull and Basu (2003); Crank and Nicolson (1947). In order to simplify the notation of symbols, $V_1\ (S\ ,T\ )$ to $V\ (S,t)$ is used.

$$\frac{V_{i,k}-V_{i-1,k}}{\Delta t}+rk\ \Delta S\ \frac{V_{i-1,k+1}-V_{i-1,k-1}}{2\Delta S}+\frac{(\sigma k\ \Delta S)^2}{2}\ \frac{V_{i-1,k+1}-2V_{i-1,k}+V_{i-1,k-1}}{(\Delta S)^2}=rV_{i-1,k} \quad (15)$$

Now, applying the implicit finite difference (backward) scheme to the transformed BSPDE (13) we obtain

$$\frac{V_{i,k}-V_{i-1,k}}{\Delta t}+rk\ \Delta S\ \frac{V_{i,k+1}-V_{i,k-1}}{2\Delta S}+\frac{(\sigma k\ \Delta S)^2}{2}\ \frac{V_{i,k+1}-2V_{i,k}+V_{i,k-1}}{(\Delta S)^2}=rV_{i,k} \quad (16)$$

Now, considering the average of (15), (16) and rearranging the terms, the numerical scheme becomes

$$-\alpha_k V_{i-1,k-1}+(-1-\beta_k)\,V_{i-1,k}-\gamma_k\,V_{i-1,k+1}=\alpha_k V_{i,k-1}+(1+\beta_k)\,V_{i,k}-\gamma_k\,V_{i,k+1} \quad (17)$$

The discretized finite difference scheme can be written in the matrix form as

$$\boldsymbol{A}\,\boldsymbol{V}_{i+1}=\boldsymbol{B}\boldsymbol{V}_i\ ,$$

where $\boldsymbol{A}$ and $\boldsymbol{B}$ are tridiagonal matrices, and $\boldsymbol{V}_i$ is the vector of option values at time step $i$. The tridiagonal matrix $\boldsymbol{A}$ is given as

$$\boldsymbol{A}=\begin{bmatrix}-1-\beta_1 & -\gamma_1 & 0 & \dots & 0\\ -\alpha_2 & -1-\beta_2 & -\gamma_2 & \dots & 0\\ 0 & -\alpha_3 & -1-\beta_3 & \dots & 0\\ \vdots & \vdots & \vdots & \ddots & -\gamma_{m-1}\\ 0 & 0 & 0 & -\alpha_m & -1-\beta_m\end{bmatrix}$$

and the tridiagonal matrix $\boldsymbol{B}$ is given as

$$\boldsymbol{B}=\begin{bmatrix}1+\beta_1 & \gamma_1 & 0 & \dots & 0\\ \alpha_2 & 1+\beta_2 & \gamma_2 & \dots & 0\\ 0 & \alpha_3 & 1+\beta_3 & \dots & 0\\ \vdots & \vdots & \vdots & \ddots & \gamma_{m-1}\\ 0 & 0 & 0 & \alpha_m & 1+\beta_m\end{bmatrix}$$

where,

(a) $m$ : Number of asset price steps.
(b) $n$ : Number of time steps.
(c) $S_k = k\Delta S$ : Asset price at step $k$.
(d) $t_i = i\Delta t$ : Time at step $i$.
(e) $\alpha_k=\frac{\Delta t}{4}\ (rk-\sigma^2\ k^2)$ : Coefficient determined by the discretization.
(f) $\beta_k=\frac{\Delta t}{2}\ (r+\sigma^2\ k^2)$ : Coefficient determined by the discretization.
(g) $\gamma_k=\frac{-\Delta t}{4}\ (rk+\sigma^2\ k^2)$ : Coefficient determined by the discretization.
(h) $k$ : Index of the asset price step.
(i) $V_{i,k}$: Option value at time step $i$ and asset price step k.
(j) $\Delta t$ :Time step size.
(k) $\Delta S$ :Asset price step size.

Now, we simulate the Crank-Nicolson scheme in matrix form to generate graphs of option price with respect to stock price and time to be presented in the subsequent analysis in depth.

## EMPERICAL STUDY OF CALL OPTION SBI DATA

Accurate volatility estimation is crucial because it directly impacts the pricing of options and the assessment of risk in financial markets. Volatility reflects the uncertainty or risk associated with the price movement of an asset. Precise estimate ensures that traders can make well-informed decisions. By using accurate volatility estimates, investors can better manage their portfolios and hedge against potential market fluctuations, leading to more stable and predictable financial outcomes. We have evaluated the volatility of the State Bank of India (SBI) security prices over the duration of 20 sessions during April–May 2024. Here, we have applied various parameters and variable values as $S_{min}=0$, $S_{max}=1520$, $K=760$ and $T=1/12$. In this study, we have considered grid size $m=600, n=800$. The estimated volatility of SBI is 0.016 ($\sigma$), and the risk-free interest rate in the Indian context is chosen as $r=0.065$. Here, we have estimated the volatility of three stocks. We used the standard deviation of daily returns over the past year to estimate the volatility. This method provides insight into how much the stock prices have fluctuated during this period. By calculating the standard deviation (volatility), we can assess the risk associated with each stock. In the case of these three stocks, we calculate real volatility and plot the results.

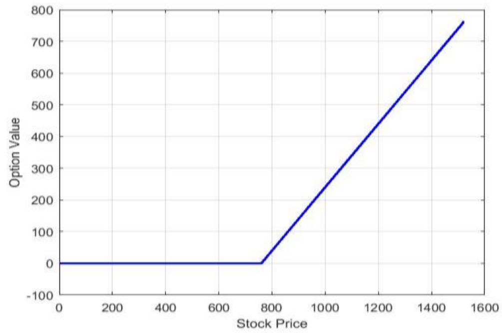


**Fig. 1.** The graph showing option value corresponding to the stock price.

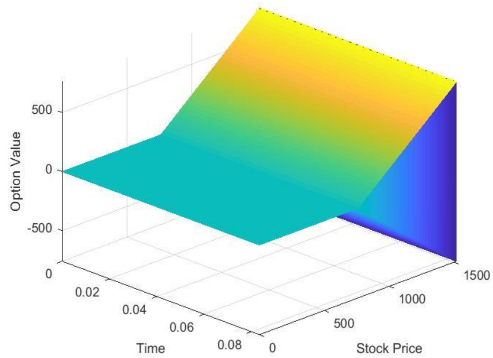


**Fig. 2.** The graph showing option value corresponding to the stock price and time.

Table 1: Data: One-month Historical data of Tata Consultancy Services.

| Date | Open | High | Low | Close | Volume | Change (%) |
|---|---|---|---|---|---|---|
| 27/12/24 | 4,164.85 | 4,163.00 | 4,180.95 | 4,147.25 | 858.10K | **-0.25** |
| 30/12/24 | 4,158.80 | 4,151.00 | 4,199.30 | 4,112.00 | 1.53M | **-0.10** |
| 31/12/24 | 4,094.80 | 4,135.00 | 4,139.95 | 4,032.05 | 1.56M | **-1.54** |
| 01/01/25 | 4,112.45 | 4,094.40 | 4,134.00 | 4,085.45 | 763.16K | **+0.43** |
| 02/01/24 | 4,175.75 | 4,120.00 | 4,183.00 | 4,096.95 | 1.72M | **+1.54** |
| 03/01/24 | 4,099.90 | 4,179.95 | 4,179.95 | 4,092.30 | 1.79M | **-1.82** |
| 06/01/24 | 4,095.00 | 4,105.50 | 4,149.65 | 4,066.40 | 2.09M | **-0.12** |
| 07/01/24 | 4,028.30 | 4,114.95 | 4,140.35 | 4,011.55 | 2.67M | **-1.63** |
| 08/01/24 | 4,108.40 | 4,034.90 | 4,126.00 | 4,017.75 | 2.18M | **+1.99** |
| 09/01/24 | 4,038.85 | 4,101.00 | 4,137.75 | 4,025.30 | 2.39M | **-1.69** |
| 10/01/24 | 4,198.72 | 4,134.10 | 4,229.58 | 4,104.57 | 7.89M | **+3.96** |
| 13/01/24 | 4,223.77 | 4,178.39 | 4,255.12 | 4,161.07 | 4.00M | **+0.60** |
| 14/01/24 | 4,166.63 | 4,234.50 | 4,246.24 | 4,152.85 | 3.44M | **-1.35** |
| 15/01/24 | 4,182.92 | 4,151.82 | 4,189.66 | 4,141.43 | 1.65M | **+0.39** |
| 16/01/24 | 4,140.30 | 4,193.16 | 4,226.62 | 4,126.08 | 2.64M | **-1.02** |
| 17/01/24 | 4,124.30 | 4,159.00 | 4,160.60 | 4,100.05 | 1.76M | **-0.39** |
| 20/01/24 | 4,077.80 | 4,145.80 | 4,145.80 | 4,068.10 | 1.29M | **-1.13** |
| 21/01/24 | 4,035.85 | 4,095.30 | 4,111.00 | 4,030.00 | 1.97M | **-1.03** |
| 22/01/24 | 4,156.60 | 4,050.00 | 4,163.15 | 4,044.20 | 2.41M | **+2.99** |
| 23/01/24 | 4,169.70 | 4,168.00 | 4,187.95 | 4,155.95 | 932.74K | **+0.32** |

Table 2: Data: One-month Historical data of Infosys.

| Date | Open | High | Low | Close | Volume | Change (%) |
|---|---|---|---|---|---|---|
| 27/12/24 | 1,916.75 | 1,924.15 | 1,903.90 | 1,909.40 | 3.94M | **+0.49** |
| 30/12/24 | 1,906.00 | 1,916.00 | 1,886.50 | 1,915.70 | 7.79M | **-0.56** |
| 31/12/24 | 1,880.00 | 1,897.00 | 1,845.05 | 1,892.30 | 3.61M | **-1.36** |
| 01/01/25 | 1,882.85 | 1,892.95 | 1,874.00 | 1,874.00 | 1.84M | **+0.13** |
| 02/01/24 | 1,957.85 | 1,962.65 | 1,885.30 | 1,887.00 | 7.08M | **+4.00** |
| 03/01/24 | 1,938.75 | 1,952.95 | 1,922.00 | 1,952.95 | 6.22M | **-0.98** |
| 06/01/24 | 1,937.85 | 1,973.00 | 1,928.00 | 1,952.00 | 7.16M | **-0.05** |
| 07/01/24 | 1,930.85 | 1,958.55 | 1,923.65 | 1,945.00 | 4.29M | **-0.36** |
| 08/01/24 | 1,933.15 | 1,938.60 | 1,888.75 | 1,930.00 | 5.48M | **+0.12** |
| 09/01/24 | 1,917.30 | 1,951.85 | 1,910.55 | 1,934.05 | 6.83M | **-0.82** |
| 10/01/24 | 1,966.95 | 1,977.80 | 1,932.25 | 1,937.00 | 8.04M | **+2.59** |
| 13/01/24 | 1,962.20 | 1,982.80 | 1,949.00 | 1,956.00 | 5.80M | **-0.24** |
| 14/01/24 | 1,940.05 | 1,971.80 | 1,931.10 | 1,968.85 | 5.79M | **-1.13** |
| 15/01/24 | 1,949.65 | 1,958.05 | 1,937.10 | 1,947.00 | 2.99M | **+0.49** |
| 16/01/24 | 1,928.45 | 1,966.95 | 1,916.85 | 1,965.95 | 7.53M | **-1.09** |
| 17/01/24 | 1,815.45 | 1,858.00 | 1,812.00 | 1,851.00 | 16.41M | **-5.86** |
| 20/01/24 | 1,813.30 | 1,827.95 | 1,793.15 | 1,822.95 | 4.33M | **-0.12** |
| 21/01/24 | 1,800.70 | 1,831.65 | 1,793.05 | 1,819.05 | 7.17M | **-0.69** |
| 22/01/24 | 1,856.45 | 1,865.80 | 1,805.15 | 1,807.40 | 8.37M | **+3.10** |
| 23/01/24 | 1,874.35 | 1,879.55 | 1,853.45 | 1,858.00 | 3.04M | **+0.96** |

Table 3: Data: One-month historical data of Tech Mahindra.

| Date | Close | Open | High | Low | Volume | Change (%) |
|---|---|---|---|---|---|---|
| 27/12/24 | 1,711.65 | 1,704.95 | 1,716.50 | 1,698.70 | 647.95K | **+0.76** |
| 30/12/24 | 1,740.85 | 1,715.00 | 1,773.60 | 1,693.80 | 6.78M | **+1.71** |
| 31/12/24 | 1,706.20 | 1,729.00 | 1,729.00 | 1,682.45 | 1.46M | **-1.99** |
| 01/01/25 | 1,703.85 | 1,700.60 | 1,717.60 | 1,691.25 | 721.48K | **-0.14** |
| 02/01/24 | 1,726.95 | 1,709.00 | 1,735.70 | 1,696.00 | 1.24M | **+1.36** |
| 03/01/24 | 1,689.45 | 1,712.20 | 1,729.85 | 1,681.35 | 2.45M | **-2.17** |
| 06/01/24 | 1,686.30 | 1,704.75 | 1,711.30 | 1,675.00 | 945.69K | **-0.19** |
| 07/01/24 | 1,671.15 | 1,693.00 | 1,705.00 | 1,661.50 | 1.31M | **-0.90** |
| 08/01/24 | 1,663.75 | 1,667.95 | 1,669.95 | 1,634.00 | 673.34K | **-0.44** |
| 09/01/24 | 1,642.80 | 1,663.70 | 1,669.50 | 1,638.20 | 774.07K | **-1.26** |
| 10/01/24 | 1,705.60 | 1,653.10 | 1,714.00 | 1,636.30 | 2.21M | **+3.82** |
| 13/01/24 | 1,659.65 | 1,698.05 | 1,701.00 | 1,649.90 | 1.40M | **-2.69** |
| 14/01/24 | 1,647.50 | 1,669.90 | 1,670.45 | 1,621.30 | 2.48M | **-0.73** |
| 15/01/24 | 1,675.95 | 1,658.00 | 1,679.00 | 1,650.15 | 1.08M | **+1.73** |
| 16/01/24 | 1,687.65 | 1,695.00 | 1,722.85 | 1,670.95 | 2.33M | **+0.70** |
| 17/01/24 | 1,660.30 | 1,684.75 | 1,699.90 | 1,650.40 | 2.501M | **-1.62** |
| 20/01/24 | 1,674.60 | 1,663.00 | 1,987.15 | 1,625.00 | 4.86M | **+0.86** |
| 21/01/24 | 1,640.50 | 1,674.60 | 1,703.55 | 1,636.10 | 2.50M | **-2.04** |
| 22/01/24 | 1,683.95 | 1,642.70 | 1,688.75 | 1,635.25 | 4.86M | **+2.65** |
| 23/01/24 | 1,712.00 | 1,671.00 | 1,729.45 | 1,671.00 | 2.50M | **+1.67** |

Table 4: Data: Estimated volatility of TCS, Infosys, Tech Mahindra Stocks.

| Sr. No. | Stocks | Estimated Volatility | Volatility in percentage | Market Capital |
|---|---|---|---|---|
| 1. | TCS | 0.015825 | 1.58 | 15.03LCr |
| 2. | Infosys | 0.019935 | 1.99 | 7.80LCr |
| 3. | Tech Mahindra | 0.017606 | 1.76 | 1.68LCr. |

## RESULTS AND DISCUSSION

A stock price cannot be negative because it represents the market value of a company's shares, which cannot be dropped below zero. If a stock were to reach zero, it would imply that the company has no remaining financial value, but it cannot go below zero as that would suggest an investor owes money simply for holding the stock. Therefore, $S_{min}$ is set at zero to reflect this logical constraint. In stock analysis, $S_{min}$ represents the theoretical minimum price a stock can reach, which is 0, indicating the company has lost all its value. This value is significant as it serves as a baseline for risk assessment, helping investors understand the worst-case scenario for their investments. It also helps in determining the stop-loss limits and evaluating the potential for a stock's recovery. The $S_{max}$ is chosen as 1520 by assuming that the underlying asset cannot reach twice its present value before the expiration period. For instance, if an investor is analysing a stock currently priced at 760 and they set $S_{max}$ as 1520, they are effectively considering scenarios where the stock could double in value. This upper limit is used to determine potential profit margins and to evaluate the feasibility of long-term investment strategies. By setting $S_{max}$, analysts can also assess the stock's volatility and potential for growth, providing a framework for decision-making in portfolio management. It is customary to choose a large value for $S_{max}$. In literature, it is usually chosen as triple the strike price. Using $S_{max}$ offers a straightforward approach to gauge the potential upside of a stock by setting a tangible ceiling that factors in significant growth scenarios. This method allows investors to easily visualize profit margins and align their strategies with market expectations, making it particularly useful for long-term planning. In contrast, other risk assessment methods, such as $value - at - risk$ $(VaR)$ or $beta\ analysis$, may provide some deeper insights into market volatility and correlation but can be more complex and less intuitive for quick decision-making. We have also investigated the graphical behaviour of option pricing at various grid sizes. Finer grid sizes provide more accurate approximations of option pricing, while coarser grids lead to less precise results. Additionally, it is also observed that the smaller grid size allows for faster convergence and better stability for option pricing problem. For visualization, in *figure 1* and *figure 2*, we have plotted the graphs for grid points $m = 600, n = 800$. When the strike price of the call option is less than or equal to the spot price of the underlying asset (out of the money), the call option has no value. The grid size selection procedure reflects the sensitivity to determine the optimal balance between computational efficiency and accuracy. Hence, testing a range of grid sizes is required to identify the point at which further refinement yields diminishing returns in terms of increased precision. Additionally, adaptive grid techniques, which dynamically adjust grid sizes based on the variability of the underlying asset's price, can be employed to enhance the accuracy of option pricing models while maintaining computational efficiency. If the value of the underlying asset reaches ahead of the strike price of the option (in the money case) then exercising the option would result in a profit for the holder. During the time period in which the underlying asset value exceeds the strike price of the option (in the money case), exercising the option would result in a profit for the holder. This scenario is advantageous for the investors as it allows them to capitalize on favourable market conditions. Being in the money can also increase the option's market value, making it more attractive for potential buyers. The variation in call option value with respect to both time and stock price is plotted in Fig. 2. The value of a call option decays dramatically, as time approaches expiry. Time decay, also known as theta, refers to the reduction in the value of an options contract as it approaches its expiration date. This occurs because options are time-sensitive financial instruments and the likelihood of the option being profitable decreases with the passage of time. As expiry nears, the remaining time for the underlying asset's price to move favourably as per expectation decreases. In this scenario, the option's extrinsic value decreases. The Fig. 2 provides crucial insights into the temporal dynamics of the call option value, illustrating how it declines significantly as the expiration date nears. This visualization highlights the sensitivity of option pricing to time decay, emphasizing the importance of timely decision-making for investors. By analysing these variations, traders can better understand the impact of time on option pricing, aiding in more informed investment strategies. The behaviour of the graph at other grid points remains the same. In our study, we have estimated the volatility of SBI on the basis of twenty real values (taken from NSE sources). The volatility $(\sigma)$ of SBI is estimated as $0.016$ using real data. Based on the Indian banking system, we have incorporated the value of the risk-free interest rate $(r)$ as $0.065$.

## CONCLUSIONS

The main objective of this paper is to investigate the European call option pricing problem by using differential equations. We presented the derivation of the numerical scheme and simulated it in MATLAB. The results demonstrated the effectiveness of the numerical scheme in approximating the option price. The findings suggest that differential equations provide a robust framework for understanding and solving complex financial problems. Using the data set of the State Bank of India security. We have also performed volatility estimation for the same data set. The estimated volatility turns out to be 0.016. It suggests that the underlying asset experiences relatively small price fluctuations over the time variable. This implies that the option traded is likely to have lower risk, which can affect its pricing and enhances its attractiveness for trading among investors. As a result, traders might adjust their strategies when considering to get into the option contracts offered by the SBI in view of its stable market condition. The parameter values on which option pricing depends are indicated in the section "*results & discussion*". Results obtained here through Crank-Nicolson method are in line with the results obtained by the analytical BS formula. The volatility of the underlying is a critical factor in option pricing as it measures the extent to which the price of the underlying asset is expected to fluctuate over time. Generally, higher volatility increases the option premium because it increases the likelihood of significant price movements, which could lead to more profitable outcomes. Conversely, lower volatility, as seen in this case, typically results in a lower premium, making the option less costly but also suggesting that major price changes are less likely.

## FUTURE SCOPE

The option pricing estimated in this study can also be extended for American options other more complex financial contracts. As a future project, we have planned to analyse stability as well as convergence for both European and American options

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