



An Innovative Approach for Ranking Hexagonal Fuzzy Numbers to Solve Linear Programming Problems

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(Received 26 December 2019, Revised 02 March 2020, Accepted 03 March 2020)

(Published by Research Trend, Website: www.researchtrend.net)

ABSTRACT: Applications of LP are spread in various fields which includes decision making in business, industry etc. To find the finest optimal solution than existing system by converting it to crisp form by ranking method become more challenging. In this paper, we have introduced a fuzzy LPP with normal HFNs having HFNs as the parameters. An alternative simplex method is used to conclude the optimal solution by new two ranking approaches with its comparison. By using all two ranking approaches, individually, any hexagonal fuzzy LPP can be converted into crisp value LPP to find an optimum solution.

Keywords: Fuzzy Linear Programming Problem, Hexagonal Fuzzy Number, Ranking Technique, Triangular Fuzzy Number.

Abbreviations: OR, Operations Research; LP, Linear Programming; LPP, Linear Programming Problem; FN, Fuzzy Number; TFN, Triangular fuzzy Number; HFN, Hexagonal Fuzzy Number.

I. INTRODUCTION

Optimization is the way of life. We always strive to get maximum out of finite resources and time. LPP is the simplest ways to achieve optimization. It aids you unravel some very multifaceted optimization problems by making a few simplifying assumptions. LPP is modest method in which we designate multifaceted associations through linear functions to find the best solutions. Applications of LP are everywhere. The course of selecting the paramount route is called OR. OR is a tactic to decision-making, which involves a set of approaches to drive a system. LP is used for determining the closest viable clarification for a problem with available limitations.

Tanaka and Asai (1984) introduced the concept of FLPP [17]. Nasser (2008) proposed that classical LP can be used for solving FLPP [10]. Khobragade *et al.*, (2009) presented another approach to revised simplex method of LPP in which proper selection of pivot vector by new rule is presented to minimize the iterations [9]. Kumar *et al.*, (2010) projected a novel system with TFN for solving FFLP problems with inequality constraints [7]. Kumar and Kaur (2011) introduced a novel system for FLP with trapezoidal FN [8]. Veeramani and Duraisamy (2012) suggested a first-hand methodology of solving FFLP problems using nearest symmetric TFN approximation with preserve expected interval [18]. Rajarajeswari *et al.*, (2013) presented a novel operation for elementary mathematical operations of HFN on α -cut basis [12]. Khobragade *et al.*, (2014) offered a unique alternative algorithm for simplex and two-phase simplex methods solving LP problems [6]. Rajarajeswari and Sangeeta (2015) used hexagonal FTP for nearest optimal solution and found BCM is the best option [13]. Saberi *et al.*, (2016) proposed an innovative effective technique with equality constraints using unrestricted variables and parameters [16]. Das *et al.*, (2017) shown that linear programming can be used to optimize the use of resources, i.e. Human Resources so that the profit margin of any business can be increased [2]. Sahaya Sudha *et al.*, (2017) compared three ranking approaches for solving FLPP using

pentagonal FNs [14]. Selvam *et al.*, (2017) proposed a novel ranking approach on the in centre of centroids and the new basic mathematical operations by using pentagonal fuzzy numbers [15].

Recently Ghadle *et al.*, (2017) suggested an innovative technique namely "Pathade and Ghadle one's BCM" for finding finest solution for transshipment problem which provides the remarkable solutions on balanced and unbalanced FTP [3]. Deshmukh *et al.*, (2020) suggested ranking technique to get an optimal solution using FFLPP with symmetric HFNs [1]. Ingle and Ghadle (2020) discovered the finest possible solution to a balanced fuzzy assignment model using a novel technique [5]. Very recently Pathade *et al.*, (2020) developed an innovative algorithm for BCM to solve mixed constraint fuzzy balanced and unbalanced TP using trivial trapezoidal fuzzy numbers by ranking method to obtain an optimal solution [11]. Here, an attempt is made to apply the proposed ranking technique. The FLPP are normal HFNs. We alter the FLPP into crisp value LPP problem to discover the solution of the given problem. The advantage of the offered technique is its usefulness to decision makers during uncertainty.

II. PRELIMINARIES

A. FS [1]

FS is characterized by a membership function mapping element of the universe of discourse X to the unit interval [0,1]. (i.e.) $A = \{(x, \mu_A(x)) | x \in X\}$. Here $\mu_A: X \rightarrow [0,1]$ is a mapping called the degree of membership function of the FS A and $\mu_A(x)$ is called the membership value of $x \in X$ in the FS of A. These membership grades are often symbolized by real numbers ranking from [0,1].

B. FM[1]

A fuzzy set A defined on the set of real numbers R is said to be a FN if its membership function $\mu_A: R \rightarrow [0,1]$ has the subsequent attributes:

- Convex and Normal of fuzzy set.
- A is piecewise continuous.

C. TFN[1]

A FN $\tilde{A} = (m_1, m_2, m_3)$ is said to be a TFN if its membership function is given by:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x \leq m_1 \\ \frac{x - m_1}{m_2 - m_1}, & m_1 \leq x \leq m_2 \\ 1, & x = m_2 \\ \frac{m_3 - x}{m_3 - m_2}, & m_2 \leq x \leq m_3 \\ 0, & x > m_3 \end{cases}$$

D. HFN [1]

A fuzzy number \tilde{A}_H is a HFN expressed by $\tilde{A}_H = (m_1, m_2, m_3, m_4, m_5, m_6)$ where $m_1, m_2, m_3, m_4, m_5, m_6$ are real numbers and its membership function $\mu_{\tilde{A}}(x)$ is given by:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & \text{for } x < m_1 \\ \frac{1}{2} \frac{(x - m_1)}{(m_2 - m_1)}, & \text{for } m_1 \leq x \leq m_2 \\ \frac{1}{2} + \frac{1}{2} \frac{(x - m_2)}{(m_3 - m_2)}, & \text{for } m_2 \leq x \leq m_3 \\ 1, & \text{for } m_3 \leq x \leq m_4 \\ 1 - \frac{1}{2} \frac{(x - m_4)}{(m_5 - m_4)}, & \text{for } m_4 \leq x \leq m_5 \\ \frac{1}{2} \frac{(m_6 - x)}{(m_6 - m_5)}, & \text{for } m_5 \leq x \leq m_6 \\ 0, & \text{for } x > m_6 \end{cases}$$

E. Ranking Function [1]

Let $F(R)$ is a set of FNs defined on the set of real numbers and the ranking of a FN is actually a function \mathcal{R} from $F(R)$ to R , which maps each FN into the real line.

If $\tilde{A} = (m_1, m_2, m_3, m_4, m_5, m_6)$ and $\tilde{B} = (n_1, n_2, n_3, n_4, n_5, n_6)$ any two HFNs then the correlation between those two FNs are, specified below:

- if $\mathcal{R}(\tilde{A}) < \mathcal{R}(\tilde{B})$ then $\tilde{A} < \tilde{B}$
- if $\mathcal{R}(\tilde{A}) > \mathcal{R}(\tilde{B})$ then $\tilde{A} > \tilde{B}$
- if $\mathcal{R}(\tilde{A}) \approx \mathcal{R}(\tilde{B})$ then $\tilde{A} \approx \tilde{B}$

III. PROPOSED RANKING METHOD

A. Ranking Method (I)

Let $\tilde{A}_H = (m_1, m_2, m_3, m_4, m_5, m_6)$ be HFN, then the proposed ranking for (Fig. 1).

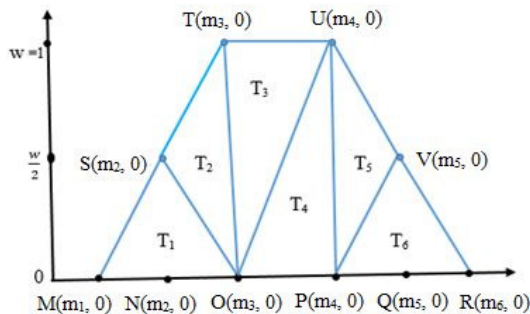


Fig. 1. Graphical representation of the HFN with six Triangles.

The hexagon has been divided into six triangles (MSO, OST, TUO, OPU, PUV and PVR). Then the ranking function have been taken for the triangles and the averages are taken.

$$T_1 = \left(\frac{m_1 + m_2 + m_3}{3}, \frac{w}{6} \right), T_2 = \left(\frac{m_2 + 2m_3}{3}, \frac{w}{2} \right),$$

$$T_3 = \left(\frac{2m_3 + m_4}{3}, \frac{2w}{3} \right), T_4 = \left(\frac{m_3 + 2m_4}{3}, \frac{w}{3} \right)$$

$$T_5 = \left(\frac{2m_4 + m_5}{3}, \frac{w}{2} \right) \text{ and } T_6 = \left(\frac{m_4 + m_5 + m_6}{3}, \frac{w}{6} \right)$$

By adding T_1, T_2, T_3, T_4, T_5 and T_6 , we get

$$\mathcal{R}(\tilde{A}_H) = \left(\frac{m_1 + 2m_2 + 6m_3 + 6m_4 + 2m_5 + m_6}{3}, \frac{14w}{6} \right) \quad (1)$$

B. Ranking Method (II)

Let $\tilde{A}_H = (m_1, m_2, m_3, m_4, m_5, m_6)$ be HFN, then the proposed ranking for (Fig. 2).

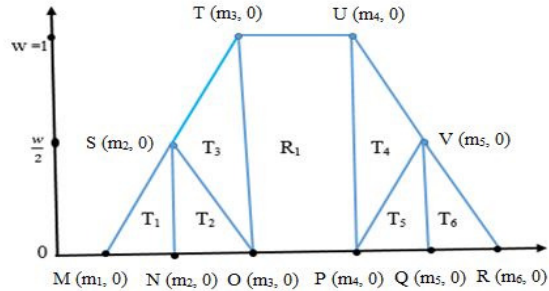


Fig. 2. Graphical representation of the HFN with six triangle and one trapezoid.

The hexagon has been divided into six triangles (MSN, NSO, OST, PUV, PVQ, QVR and OTUP). Then the ranking functions have been taken for the triangles and trapezoid and the averages are taken.

$$T_1 = \left(\frac{m_1 + 2m_2}{3}, \frac{w}{6} \right), T_2 = \left(\frac{2m_2 + m_3}{3}, \frac{w}{6} \right),$$

$$T_3 = \left(\frac{m_2 + 2m_3}{3}, \frac{w}{2} \right), T_4 = \left(\frac{2m_4 + m_5}{3}, \frac{w}{2} \right)$$

$$T_5 = \left(\frac{m_4 + 2m_5}{3}, \frac{w}{6} \right), T_6 = \left(\frac{2m_5 + m_6}{3}, \frac{w}{6} \right) \text{ and}$$

$$R_1 = \left(\frac{m_3 + m_4}{2}, \frac{w}{2} \right)$$

By adding $T_1, T_2, T_3, T_4, T_5, T_6$ and R_1 we get

$$\mathcal{R}(\tilde{A}_H) = \left(\frac{2m_1 + 10m_2 + 9m_3 + 9m_4 + 10m_5 + 2m_6}{6}, \frac{13w}{6} \right) \quad (2)$$

Remark: When $w=1$ the HFN is a normal FN.

IV. NUMERICAL EXAMPLES

Example 4.1

Explain FLP problem:

$$\text{Maximize } \tilde{Z} = (-1, 0, 1, 1, 2, 3; 1)\tilde{x}_1 + (0, 1, 2, 2, 3, 4; 1)\tilde{x}_2$$

$$\text{Subject to: } 5\tilde{x}_1 + 2\tilde{x}_2 \leq (3, 4, 5, 5, 6, 7; 1)$$

$$5\tilde{x}_1 + 15\tilde{x}_2 \leq (8, 9, 10, 10, 11, 12; 1)$$

$$\tilde{x}_1, \tilde{x}_2 \geq 0$$

Solution:

Standard form of FLP Problem:

$$\text{Maximize } \tilde{Z} = (-1, 0, 1, 1, 2, 3; 1)\tilde{x}_1 + (0, 1, 2, 2, 3, 4; 1)\tilde{x}_2 + 0\tilde{x}_3 + 0\tilde{x}_4$$

$$\text{Subject to: } 5\tilde{x}_1 + 2\tilde{x}_2 + \tilde{x}_3 = (3, 4, 5, 5, 6, 7; 1)$$

$$5\tilde{x}_1 + 15\tilde{x}_2 + \tilde{x}_4 = (8, 9, 10, 10, 11, 12; 1)$$

$$\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4 \geq 0$$

Using Ranking Function (1)

$$\text{Maximize } \tilde{Z} = 14\tilde{x}_1 + 28\tilde{x}_2 + 0\tilde{x}_3 + 0\tilde{x}_4$$

$$\text{Subject to: } 5\tilde{x}_1 + 2\tilde{x}_2 + \tilde{x}_3 = 70$$

$$5\tilde{x}_1 + 15\tilde{x}_2 + \tilde{x}_4 = 140$$

$$\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4 \geq 0$$

Using (Ghadle et al., [4]), we get

Table 1: Iteration.

\tilde{c}_B	\tilde{y}_B	\tilde{x}_B	\tilde{x}_1	\tilde{x}_2	\tilde{x}_3	\tilde{x}_4
0	\tilde{x}_3	70	5	2	1	0
0	\tilde{x}_4	140	5	15*	0	1
0	\tilde{x}_3	51.34	13/3*	0	1	-2/15
28	\tilde{x}_2	9.33	1/3	1	0	1/15
12.33	\tilde{x}_1	11.84	1	0	3/13	-6/195
24.66	\tilde{x}_2	5.39	0	1	-1/13	1/13

As $\tilde{z} \geq 0$, the recent fuzzy optimum solution of FLP problem is given as
 $\tilde{x}_1 = 11.84, \tilde{x}_2 = 5.39$ and Maximize $\tilde{z} = 316.68$

Using Ranking Function (2)

Maximize $\tilde{Z} = 15.16\tilde{x}_1 + 30.33\tilde{x}_2 + 0\tilde{x}_3 + 0\tilde{x}_4$
 Subject to: $5\tilde{x}_1 + 2\tilde{x}_2 + \tilde{x}_3 = 75.83$
 $5\tilde{x}_1 + 15\tilde{x}_2 + \tilde{x}_4 = 151.66$
 $\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4 \geq 0$
 Using Ghadle *et al.*, [4], we get

Table 2: Iteration.

\tilde{c}_B	\tilde{y}_B	\tilde{x}_B	\tilde{x}_1	\tilde{x}_2	\tilde{x}_3	\tilde{x}_4
0	\tilde{x}_3	75.83	5	2	1	0
0	\tilde{x}_4	151.66	5	15*	0	1
0	\tilde{x}_3	55.61	13/3*	0	1	-2/15
30.33	\tilde{x}_2	10.11	1/3	1	0	1/15
12.33	\tilde{x}_1	12.83	1	0	3/13	-6/195
24.66	\tilde{x}_2	5.84	0	1	-1/13	1/13

As $\tilde{z} \geq 0$, the recent fuzzy optimum solution of FLP problem is given as
 $\tilde{x}_1 = 12.83, \tilde{x}_2 = 5.84$ and Maximize $\tilde{z} = 371.62$.

Example 4.2

Explain FLP problem:

Minimize $\tilde{Z} =$

$-(0,1,2,2,3,4; 1)\tilde{x}_1 - (1, 2, 3, 3, 4, 5; 1)\tilde{x}_2$

Subject to: $6\tilde{x}_1 + 3\tilde{x}_2 \leq (4, 5, 6, 6, 7, 8; 1)$

$6\tilde{x}_1 + 16\tilde{x}_2 \leq (9, 10, 11, 11, 12, 13; 1)$

$\tilde{x}_1, \tilde{x}_2 \geq 0$

Solution:

Standard form of FLP Problem:

Maximize $\tilde{Z} = (0, 1, 2, 2, 3, 4; 1)\tilde{x}_1 +$

$(1, 2, 3, 3, 4, 5; 1)\tilde{x}_2 + 0\tilde{x}_3 + 0\tilde{x}_4$

Subject to: $6\tilde{x}_1 + 3\tilde{x}_2 + \tilde{x}_3 = (4, 5, 6, 6, 7, 8; 1)$

$5\tilde{x}_1 + 15\tilde{x}_2 + \tilde{x}_4 = (9, 10, 11, 11, 12, 13; 1)$

$\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4 \geq 0$

Using Ranking Function (1)

Maximize $\tilde{Z} = 28\tilde{x}_1 + 42\tilde{x}_2 + 0\tilde{x}_3 + 0\tilde{x}_4$

Subject to: $6\tilde{x}_1 + 3\tilde{x}_2 + \tilde{x}_3 = 84$

$6\tilde{x}_1 + 16\tilde{x}_2 + \tilde{x}_4 = 154$

$\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4 \geq 0$

Using (Ghadle *et al.*, [4]), we get

Table 3: Iteration.

\tilde{c}_B	\tilde{y}_B	\tilde{x}_B	\tilde{x}_1	\tilde{x}_2	\tilde{x}_3	\tilde{x}_4
0	\tilde{x}_3	84	6	3	1	0
0	\tilde{x}_4	154	6	16*	0	1
0	\tilde{x}_3	441/8	39/8*	0	1	-3/16
42	\tilde{x}_2	77/8	3/8	1	0	1/16
28	\tilde{x}_1	11.30	1	0	8/39	-1/26
42	\tilde{x}_2	5.38	0	1	-3/39	1/13

As $\tilde{z} \geq 0$, the recent fuzzy optimum solution of FLP problem is given as
 $\tilde{x}_1 = 11.30, \tilde{x}_2 = 5.38$ and Minimize $\tilde{z} = -542.30$

Using Ranking Function (2)

Maximize $\tilde{Z} = 30.33\tilde{x}_1 + 45.5\tilde{x}_2 + 0\tilde{x}_3 + 0\tilde{x}_4$

Subject to: $6\tilde{x}_1 + 3\tilde{x}_2 + \tilde{x}_3 = 91$

$6\tilde{x}_1 + 16\tilde{x}_2 + \tilde{x}_4 = 166.83$

$\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4 \geq 0$

Using (Ghadle *et al.*, [4]), we get

Table 4: Iteration.

\tilde{c}_B	\tilde{y}_B	\tilde{x}_B	\tilde{x}_1	\tilde{x}_2	\tilde{x}_3	\tilde{x}_4
0	\tilde{x}_3	91	6	3	1	0
0	\tilde{x}_4	166.83	6	16*	0	1
0	\tilde{x}_3	59.74	39/8*	0	1	-3/16
45.5	\tilde{x}_2	10.42	3/8	1	0	1/16
30.33	\tilde{x}_1	12.25	1	0	8/39	-1/26
45.5	\tilde{x}_2	5.83	0	1	-3/39	1/13

As $\tilde{Z} \geq 0$, the recent fuzzy optimum solution of FLP problem is given as $\tilde{x}_1 = 12.25$, $\tilde{x}_2 = 5.83$ and Minimize $\tilde{Z} = -636.8$

V. COMPARISON STUDY

Table 5: Comparison Table.

Examples	Ranking Methods	X1	X2	Z
4.1	Method (I)	11.84	5.39	316.68
	Method (II)	12.83	5.84	371.62
4.2	Method (I)	11.30	5.38	-542.3
	Method (II)	12.25	5.83	-636.8

Table 5 explains the comparison as well as validation results. It is evident from the outcomes that, for both the cases, i.e. for Maximization and Minimization type of cases, the proposed Method-II is efficient as it gives finest optimal results with the same number of iterations.

VI. CONCLUSION AND FUTURE SCOPE

We have proposed a new ranking technique for normal HFNs by using alternative simplex method and obtained a fuzzy basic feasible solution and optimal solution. After comparison, it is found that the second ranking technique compared to the other one, gives the maximum value as well as minimum value, hence it gives finest solution for maximization problem and minimization problem. In future, it may be tried to improve a system which can be used directly to discover fuzzy optimal solution of the FLP problems without transforming it into crisp LPP.

ACKNOWLEDGEMENT

I would like to thank Professor Kirtiwant P. Ghadle, Dept. of Mathematics, Dr. BAMU, Aurangabad for his guidance and Asst. Prof. K. R. Jagtap for his valuable suggestions in the preparation of this paper.

Conflict of Interest. The authors announce that there is no conflict of interest about the publication of this paper.

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How to cite this article: Deshmukh, M. C., Ghadle, K. P. and Jadhav, O. S. (2020). An Innovative Approach for Ranking Hexagonal Fuzzy Numbers to Solve Linear Programming Problems. *International Journal on Emerging Technologies, 11*(2): 385-388.