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Application of Higher Order Shear Deformation Theory in the Analysis of thick Rectangular Plate

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ABSTRACT: The study presents the bending solutions of thick rectangular plates with clamped at three edges and simply supported at the remaining one edge (CCCS) carrying a uniformly distributed load applying energy method. Higher order shear deformation plate theory was used to formulate the governing differential equation by applying the principles of elasticity. Total potential energy equation of a thick plate was formulated from the constitutive relations thereafter the three general governing differential equations for the determination of the out of plane displacement and shear deformations rotation along the direction of x and y coordinates were obtained. The total potential energy was in the same way used by the method of direct variation to obtain three simultaneous direct governing equations for the determination of deflection and shear deformations coefficients. From the formulated relationship, formulars for calculation of the in-plane and out plane displacement shear deformation rotations along x and y axis, stresses, moments and stress-resultants expressions were deduced. Unlike First order shear deformation theory (FSDT), this higher order shear deformation theory as was proposed does not require shear correction factor in the analysis. It can be concluded that the rectangular plate can be classified as thick plate, when the plate span-to-depth ratios is less or equal to twenty ($a/t \leq 20$); it can be classified as moderately thick plate, when the plate span-to-depth ratios is between twenty and fifty: ($20 \leq a/t \leq 50$); when the plate span-to-depth ratios is greater or equal to twenty plate: $a/t \geq 50$. This assertion can be used to shows the boundary between thin and thick plate. Furthermore, it is seen that at the 96 % confidence level, the values from the present study are the same with those from of previous studies, confirming the accuracy and reliability of the derived relationships.

Keywords: CCCS plates, traditional higher-order shear deformation plate theory, shear correction factors, moment, and stresses.

I. INTRODUCTION

Thick plate bending problems are basic issues in mechanical and civil engineering as well as in structural engineering. Plates has wide applications in floor slabs for buildings, bridge decks and flat panels for aircrafts. In view of their relevance, the problems regarding its bending due to applied load required much attention. Attentions have actually been given to the bending problems of thin plate by using classical plate theorems (CPT) for its analysis. The CPT which assumes that normal to middle plane did not bend after deformation shows that the transverse shear deformation was ignored during formulation makes the theory inaccurate and unreliable when dealing with thick plate.

First order shear deformation theory (FSDT) has been employed by many researchers to analyze thick plates. The theory unlike classical plate theory (CPT) took account of the shear deformation by introducing shear correction factor to satisfy the constitutive relations for transverse shear stresses and shear strains. Mindlin's [4-6] and Reissner's theory [10, 12] adopted FSDT in their work by employing a stress and displacement based approach respectively, which incorporates the effect of shear deformation. FSDT is discovered to have assumed transverse shear stress to be constant through the thickness of the plate, which violates the shear stress free surface conditions on the top and bottom surfaces of the plate.

"Reddy's third-order [9], and other higher-order shear deformation plate theory [2, 3, 9, 10, 12 and 13].Ibearugbulem and Onyeka [13] applied nonlinear strain–displacement polynomial third order shear deformation theory for rectangular thick plate under uniformly distributed load case".

Onyeka and Ibearugbulem [2] applied nonlinear strain–displacement polynomial third order shear deformation theory for rectangular thick plate under uniformly distributed load case. Their work did not consider thick rectangular plates with clamped at three edges and simply supported at the remaining one edge.

This study uses fourth order polynomial shape function of the plate and applied in the shear deformation theory to analyze the bending behavior of thick rectangular plate with clamped at three edges and simply supported at the remaining one edge (CCCS) carrying uniformly distributed load. From the theory, the shear deformation profile for vertical shear stress through the thickness of the plate was formulated from the first principle as the deformation line as given in Equation 5 and incorporate to the energy equation established.

II. FORMULATION OF TOTAL POTENTIAL ENERGY

Our formulation of the direct governing equation for thick plate under pure bending is based on Fig. 1 below.

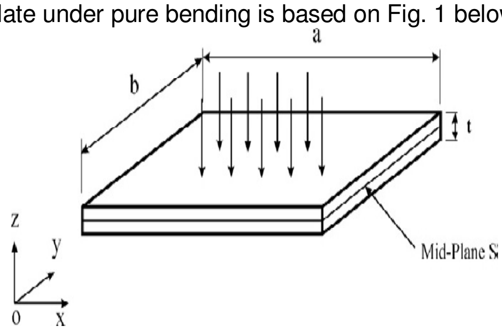

Fig. 1. An element of thick rectangular plate showing middle surface.

The total potential energy expression for a thick rectangular plate with length, a; width, b and at constant thickness with mid-plane layer separation under uniformly distributed load [8], is presented as:

$$\Pi = \int_0^1\int_0^1 \Big[\left(g_1 {A_1}^2 k_1 - 2g_2 A_1 A_2 k_1 + g_3 {A_2}^2 k_1\right) + \left(2g_1 \tfrac{{A_1}^2}{\propto^2} k_2 - 2g_2 \tfrac{A_1A_2}{\propto^2} k_2 - 2g_2 \tfrac{A_1A_3}{\propto^2} k_2\right) + \left([1+\mu] g_3 \tfrac{A_2A_3}{\propto^2} k_2\right) + \tfrac{[1-\mu]}{2}\left(g_3 \tfrac{{A_2}^2}{\propto^2} k_2 + g_3 \tfrac{{A_3}^2}{\propto^2} k_2\right) + \left(g_1 \tfrac{{A_1}^2}{\propto^4} k_3 - 2g_2 \tfrac{A_1A_3}{\propto^4} k_3 + g_3 \tfrac{{A_3}^2}{\propto^4} k_3\right) + \tfrac{[1-\mu]}{2}\left(\rho^2 g_4 {A_2}^2 k_4 + \tfrac{\rho^2 g_4 {A_3}^2}{\propto^2} k_5\right)\Big] abdR\partial Q - \int_0^1\int_0^1 \left[\tfrac{2qa^4}{D} A_1 k_6\right] ab\partial R\partial Q \qquad (1)$$

Where the length-breadth and span-thickness aspect ratio are given as:

$$\propto = \frac{b}{a} \; and \; \rho = \frac{a}{t}$$

III. GOVERNING DIFFERENTIAL EQUATION

The general polynomial deflection function deflection equation of a rectangular plate is defined as:

$$w = \frac{F_{a4}.F_{b4}}{1152}(1.5R^2 - 2.5R^3 + R^4) \times (Q^2 - 2Q^3 + Q^4) \qquad (2)$$

Let the amplitude,

$$A_1 = \frac{F_{a4}}{48} \times \frac{F_{b4}}{24} = \frac{F_{a4}.F_{b4}}{1152} \qquad (3)$$

and;

$$h = (1.5R^2 - 2.5R^3 + R^4) \times (Q^2 - 2Q^3 + Q^4) \qquad (4)$$

Shear deformation profile of the thick rectangular section of plate used in this study is given as:

$$F(z) = \frac{5}{3}\left(z^3 - \frac{2z}{t^3}\right) \qquad (5)$$

To get the solution of the governing equation, equations 1 must be differentiated with respect to coefficient of deflection A_1, coefficient of shear deformation at x – axis A_2 and coefficient of shear deformation at y – axis A_3 to have:

$$r_{11}A_1 - r_{12}A_2 - r_{13}A_3 = \frac{qa^4}{D} k_q \qquad (6)$$

$$-r_{21}A_1 + r_{22}A_2 + r_{23}A_3 = 0 \qquad (7)$$

$$-r_{31}A_1 + r_{32}A_2 + r_{33}A_3 = 0 \qquad (8)$$

By solving equations 6, 7 and 8 simultaneously, we get;

$$A_1 = \frac{qa^4}{D}\left(\frac{k_q}{r_{11}T_1 - r_{12}T_2 - r_{13}T_3}\right) \qquad (9)$$

Let:

$$\overline{A_1} = \left(\frac{k_q}{r_{11}T_1 - r_{12}T_2 - r_{13}T_3}\right) \qquad (10)$$

That is:

$$A_1 = \overline{A_1}\left(\frac{qa^4}{D}\right) \qquad (11)$$

Similarly;

$$A_2 = \bar{A}\left(\frac{qa^4}{D}\right) \qquad (12)$$

Similarly;

$$A_3 = \overline{A_3}\left(\frac{qa^4}{D}\right) \qquad (13)$$

IV. DISPLACEMENT, STRESSES AND STRESS RESULTANT ANALYSIS OF THE PLATE

The expressions for the moment, displacement, stresses and stress resultants of isotropic rectangular thick plate are developed using the elastic principles as given:

$$w = \overline{A_1} h\left(\frac{qa^4}{D}\right) \qquad (14)$$

$$M_x = \left(-g_1\overline{A_1}\left[\frac{d^2h}{dR^2} + \mu\frac{d^2h}{dQ^2}\right] + g_2\left[\overline{A_2}\frac{d^2h}{dR^2} + \mu\overline{A_3}\frac{d^2h}{dQ^2}\right]\right)qa^2 \qquad (15)$$

That is:

$$M_x = \overline{M_x}qa^2 \qquad (16)$$

Where;

$$D = \frac{Et^3}{12(1-\mu^2)};\; D_1 = g_1 D; and\; D_2 = g_2 D \qquad (17)$$

Similarly;

$$M_y = \left(-g_1\bar{A}\left[\frac{d^2h}{dQ^2} + \mu\frac{d^2h}{dR^2}\right] + g_2\left[\overline{A_3}\frac{d^2h}{dQ^2} + \mu\overline{A_2}\frac{d^2h}{dR^2}\right]\right)qa^2 \qquad (18)$$

That is:

$$M_y = \overline{M_y}qa^2 \qquad (19)$$

Similarly;

$$Q_x = qa\left(-\overline{A_1}\left[\frac{\partial^3h}{\partial R^3} + \mu\frac{\partial^3h}{\partial Q^3}\right] + \left[\overline{A_2}\frac{\partial^3h}{\partial R^3} + \mu\overline{A_3}\frac{\partial^3h}{\partial Q^3}\right]\right) \qquad (20)$$

That is:

$$Q_x = \overline{Q_x}qa \qquad (21)$$

Similarly;

$$Q_y = qa\left(-\overline{A_1}\left[\frac{\partial^3h}{\partial R^3} + \mu\frac{\partial^3h}{\partial Q^3}\right] + \left[\overline{A_2}\frac{\partial^3h}{\partial R^3} + \mu\overline{A_3}\frac{\partial^3h}{\partial Q^3}\right]\right) \qquad (22)$$

That is:

$$Q_y = \overline{Q_y}qa \qquad (23)$$

$$u = [-\overline{A_1}s + \overline{A_2}F(s)]\frac{dh}{dR}\left(\frac{qa^4}{\rho D}\right) \qquad (24)$$

Similarly;

$$v = \frac{1}{\propto}[-\overline{A_1}s + \overline{A_3}F(s)]\frac{dh}{dQ}\left(\frac{tqa^3}{D}\right) \qquad (25)$$

$$\sigma_x = 12\left[[-\overline{A_1}s + \overline{A_2}F(s)]\frac{d^2h}{dR^2} + \frac{\mu}{\propto^2}[-\overline{A_1}s + \overline{A_3}F(s)]\frac{d^2h}{dQ^2}\right](q\rho^2) \quad (26)$$

Similarly;

$$\sigma_y = q\rho^2\left[12\left[\mu[-\overline{A_1}s + \overline{A_2}F(s)]\frac{d^2h}{dR^2}\right] + \frac{\mu}{\propto^2}[-\overline{A_1}s + \overline{A_3}F(s)]\frac{d^2h}{dQ^2}\right] \quad (27)$$

Similarly;

$$\tau_{xy} = 6\frac{(1-\mu)}{\propto}\left[-2\overline{A_1}s + \overline{A_2}F(s) + \overline{A_3}F(s).\frac{1}{\propto}\right]\frac{d^2h}{\partial R\partial Q}(q\rho^2) \quad (28)$$

Similarly;

$$\tau_{xz} = 6(1-\mu)\overline{A_2}\frac{dF(z)}{dz}\frac{dh}{dR}(q\rho^2) \quad (29)$$

Similarly;

$$\tau_{yz} = \frac{6(1-\mu)}{\propto}\overline{A_3}\frac{dF(z)}{dz}\frac{dh}{dQ}(q\rho^2) \quad (30)$$

V. NUMERICAL ANALYSIS

Determine the deflection at the center $(\frac{1}{2}, \frac{1}{2}, \text{and } 0)$ of CCCS thick plate. A polynomial displacement function as developed in Equations 2, 3 and 4 shall be used to determine the value of stiffness coefficient which is presented in Table 1. The values of displacements, stresses and stress resultants at the edges of the plate are presented in the Table 2 to 6. More so, a comparison is made with those from [3] and [1] considering center deflection of CCCS square rectangular thick plate to check the accuracy of this method. The result of this comparison is presented in the Table 7.

VI. RESULTS AND DISCUSSION

The result of non-dimensional center deflection, in plane displacement, moments, normal and shear stresses along x and y axes of CCCS rectangular plate as obtained are presented in the Tables 2 to 5.The result reveals that the non-dimensional values of in-plane displacement quantities, u and v, and that of out-of-plane displacement quantities, w decrease as the span-thickness ratio increases. The value of these quantities increases as the length-width ratio increases.

Close look at Table 3, it can be seen that at span to thickness ratio more than 55, the value of transverse shear stress τ_{yz} is about 0.00001 when corrected to 5 decimal place. But for the span to thickness between 20 and 55, the value of transverse shear stress τ_{yz} varies between 0.0007 and 0.0001 respectively. Also, at span to thickness ratio less than 20, the value of transverse shear stress τ_{yz} is above 0.001. In the same way, from Table 4; it can be seen that at span to thickness ratio more than 55, the value of transverse shear stress τ_{yz} is about 0.00001 when corrected to 5 decimal place. But for the span to thickness between 20 and 55, the value of transverse shear stress τ_{yz} varies between 0.0006 and 0.0001 respectively. Also, at span to thickness ratio less than 20, the value of transverse shear stress τ_{yz} is above 0.001. Similarly, from table 5; it can be seen that at span to thickness ratio more than 55, the value of transverse shear stress τ_{yz} is about 0.00001 when corrected to 5 decimal place. But for the span to thickness between 20 and 45, the value of transverse shear stress τ_{yz} varies between 0.0003 and 0.0001 respectively. Also, at span to thickness ratio less than 20, the value of transverse shear stress τ_{yz} is above 0.001. From the result, it can be discovered that there are three classes of rectangular plate. The plates whose vertical shear stressdo not differ much from zero shall be classified as thin plates because its value are almost equal to the value of CPT. The ones that differ very well from zero shall be classified as thick plates. In between the thick plate and thin plate is the class for moderately thick plate. Thus, the span-to-depth ratios for these classes of rectangular plate are: Thick plate: $a/t \leq 20$; moderately thick plate: $20 \leq a/t \leq 50$; thin plate: $a/t \geq 50$. This assertion can be used to shows the boundary between thin and thick plate.

As per Ibearugbulem and Onyeka rectangular plate under uniformly distributed load case showed the advantages of polynomial function over trigonometric and exponential when analyzing a more complex or thick plate problem" [13].

From Table 6, comparison made from the present study for CCCS plate and those from past scholars (when multiplied by 100) shows that, present theory predicts a slightly higher value of in-plane displacement, transverse (central) deflection, in-plane normal stresses, in-plane shear stress and out plane shear stress for all aspect ratios, this proves some level of accuracy and safety of the analysis. The maximum percentage difference between the values from the present study and those of [3] is about 3.74%. Also, the maximum percentage difference between the values from the present study and those of [1] is about 3.86%. Consequently, the average total percentage difference between the values from the present study and those of [3, 1] is about 3.81 %. This means that at the 96 % confidence level, the values from the present study are the same with those from of previous studies. This value is satisfactory in the statistical analysis. Then, it can be said that the values obtained are in agreement with those obtained in the literature. Thus, confirming the accuracy and reliability of the derived relationships.

Table 1: Stiffness coefficient values for CCCS rectangular plate using orthogonal polynomial displacement function.

Type	Plate	k_1	k_2	k_3	k_4	k_5	k_6
1	CCCS	0.0028571	0.0016327	0.0060317	0.0001361	0.00014361	0.0025

Table 2: Bending Moments, Shear Force and Stress resultants of CCCS plate for b/a = 1.0.

$\rho = \frac{a}{t}$	$w = \bar{w}\left(\frac{qb^4}{D}\right)$	$M_x = \overline{M_x}qa^2$	$M_y = \overline{M_y}qa^2$	$Q_x = \overline{Q_x}qa$	$Q_y = \overline{Q_y}qa$
	$\bar{w}$	$\overline{M_x}$	$\overline{M_y}$	$\overline{Q_x}$	$\overline{Q_y}$
4	0.002987316	0.295421	0.298972	1.045017	2.292187
5	0.002785796	0.310939	0.323968	0.92112	2.017948
6	0.002391581	0.298228	0.320335	0.741935	1.619265
7	0.00216945	0.290704	0.318515	0.641014	1.393626
8	0.002030992	0.285851	0.317483	0.578129	1.252529
9	0.001938516	0.282528	0.316845	0.536139	1.15807
10	0.001873546	0.280151	0.316424	0.506643	1.091588
15	0.001723424	0.274508	0.315544	0.438509	0.816195
20	0.001672083	0.272526	0.315276	0.415214	0.816771
25	0.001648524	0.271606	0.315159	0.404526	0.817062
30	0.00163578	0.271106	0.315098	0.398745	0.817227
35	0.001628114	0.270805	0.315062	0.395267	0.817329
40	0.001623146	0.270609	0.315038	0.393013	0.817395
50	0.00161731	0.270378	0.315011	0.390366	0.817475
55	0.001615511	0.270307	0.315003	0.38955	0.8175
60	0.001614144	0.270253	0.314996	0.38893	0.817519
65	0.00161308	0.270211	0.314991	0.388447	0.817533
70	0.001612235	0.270178	0.314987	0.388064	0.817545
75	0.001610997	0.270129	0.314982	0.387503	0.817562
80	0.001610997	0.270129	0.314982	0.387503	0.817562
85	0.001610536	0.27011	0.31498	0.387293	0.817569
90	0.001610149	0.270095	0.314978	0.387118	0.817574
95	0.001609821	0.270082	0.314976	0.386969	0.817579
100	0.001609542	0.270071	0.314975	0.386842	0.817583
1000	0.001606981	0.26997	0.314963	0.077137	0.817619

Table 3: Non-displacement and Stresses of CCCS plate for b/a = 1. 0.

	$\propto = \frac{b}{a} = 1.0$							
$\rho = \frac{a}{t}$	$w = \bar{w}\left(\frac{qb^4}{D}\right)$	$u = \bar{u}\left(\frac{qa^4}{\rho D}\right)$	$v = \bar{v}\left(\frac{qa^4}{\rho D}\right)$	$\sigma_x = \overline{\sigma_x}(q\rho^2)$	$\sigma_y = \overline{\sigma_y}(q\rho^2)$	$\tau_{xy} = \overline{\tau_{xy}}(q\rho^2)$	$\tau_{xz} = \overline{\tau_{xz}}(q\rho^3)$	$\tau_{yz} = \overline{\tau_{yz}}(q\rho^3)$
	$\bar{w}$	$\bar{u}$	$\bar{v}$	$\overline{\sigma_x}$	$\overline{\sigma_y}$	$\overline{\tau_{xy}}$	$\overline{\tau_{xz}}$	$\overline{\tau_{yz}}$
4	0.002987316	-0.001187	-0.003272	0.232305	0.255794	-0.02906	0.0038617	0.0165864
5	0.002785796	-0.001163	-0.003254	0.22845	0.253594	-0.028669	0.0028994	0.0129176
6	0.002391581	-0.001046	-0.002986	0.206552	0.231778	-0.026031	0.0018921	0.0086656
7	0.002169450	-0.000979	-0.002837	0.194093	0.219562	-0.024539	0.0013383	0.0062451
8	0.002030992	-0.000936	-0.002745	0.186272	0.211981	-0.023606	0.0009993	0.0047250
9	0.001938516	-0.000908	-0.002684	0.181022	0.206935	-0.022982	0.0007760	0.0037042
10	0.001873546	-0.000888	-0.002641	0.177319	0.203399	-0.022543	0.0006208	0.0029841
15	0.001723424	-0.000840	-0.002543	0.168712	0.195259	-0.021526	0.0002677	0.00131
20	0.001672083	-0.000824	-0.00251	0.165751	0.192486	-0.021177	0.0001490	0.0007338
25	0.001648524	-0.000817	-0.002495	0.164389	0.191216	-0.021017	9.488E-05	0.0004687
30	0.00163578	-0.000813	-0.002487	0.163652	0.190529	-0.02093	6.571E-05	0.0003252
35	0.001628114	-0.00081	-0.002482	0.163208	0.190117	-0.020878	4.82E-05	0.0002388
40	0.001623146	-0.000809	-0.002479	0.16292	0.189849	-0.020844	3.686E-05	0.0001827
45	0.001619743	-0.000808	-0.002476	0.162723	0.189666	-0.020821	2.91E-05	0.0001443
50	0.00161731	-0.000807	-0.002475	0.162582	0.189535	-0.020804	2.356E-05	0.0001169
55	0.001615511	-0.000806	-0.002474	0.162477	0.189438	-0.020792	1.947E-05	9.659E-05
60	0.001614144	-0.000806	-0.002473	0.162398	0.189365	-0.020783	1.635E-05	8.116E-05
65	0.001612235	-0.000805	-0.002472	0.162287	0.189262	-0.020770	1.201E-05	5.962E-05
70	0.001610997	-0.000805	-0.002471	0.162216	0.189195	-0.020761	9.192E-06	4.564E-05
75	0.001610997	-0.000805	-0.002471	0.162216	0.189195	-0.020761	9.192E-06	4.564E-05
80	0.001610536	-0.000805	-0.002471	0.162189	0.189171	-0.020758	8.141E-06	4.043E-05
85	0.001610149	-0.000804	-0.00247	0.162166	0.18915	-0.020755	7.261E-06	3.606E-05
90	0.001609821	-0.000804	-0.00247	0.162147	0.189132	-0.020753	6.516E-06	3.236E-05
95	0.001609542	-0.000804	-0.00247	0.162131	0.189117	-0.020751	5.881E-06	2.921E-05
100	0.001606981	-0.000803	-0.002468	0.161983	0.188979	-0.020734	5.877E-08	2.92E-05
1000	0.001615511	-0.000806	-0.002474	0.162477	0.189438	-0.020792	1.947E-05	9.659E-05

Table 4: Non-displacement and Stresses of CCCS plate for b/a = 1. 5.

	$\propto = \frac{b}{a} = 1.5$							
$\rho = \frac{a}{t}$	$w = \bar{w}\left(\frac{qb^4}{D}\right)$	$u = \bar{u}\left(\frac{qa^4}{\rho D}\right)$	$v = \bar{v}\left(\frac{qa^4}{\rho D}\right)$	$\sigma_x = \overline{\sigma_x}(q\rho^2)$	$\sigma_y = \overline{\sigma_y}(q\rho^2)$	$\tau_{xy} = \overline{\tau_{xy}}(q\rho^2)$	$\tau_{xz} = \overline{\tau_{xz}}(q\rho^3)$	$\tau_{yz} = \overline{\tau_{yz}}(q\rho^3)$
	$\bar{w}$	$\bar{u}$	$\bar{v}$	$\overline{\sigma_x}$	$\overline{\sigma_y}$	$\overline{\tau_{xy}}$	$\overline{\tau_{xz}}$	$\overline{\tau_{yz}}$
4	0.006011736	-0.002411	-0.004809	0.407324	0.304534	-0.040937	0.007493	0.016974
5	0.005048547	-0.002164	-0.004345	0.365879	0.274518	-0.036860	0.004544	0.010389
6	0.004564017	-0.002039	-0.004114	0.344981	0.259477	-0.034813	0.003067	0.007053
7	0.004283849	-0.001966	-0.003981	0.332879	0.250803	-0.033631	0.002215	0.005114
8	0.004106588	-0.001920	-0.003897	0.325213	0.245325	-0.032884	0.001678	0.003883
9	0.003987075	-0.001889	-0.003841	0.320040	0.241637	-0.032380	0.001316	0.003051
10	0.003902571	-0.001867	-0.003801	0.316381	0.239032	-0.032024	0.001060	0.002461
15	0.003705631	-0.001816	-0.003709	0.307845	0.232969	-0.031195	0.000465	0.001084
20	0.003637743	-0.001798	-0.003677	0.304900	0.230883	-0.030910	0.000261	0.000608
25	0.003606500	-0.001790	-0.003662	0.303544	0.229923	-0.030779	0.000166	0.000388
30	0.003589576	-0.001786	-0.003654	0.302809	0.229403	-0.030707	0.000115	0.000269
35	0.003579388	-0.001783	-0.00365	0.302367	0.229090	-0.030665	8.48E-05	0.000198
40	0.003572781	-0.001781	-0.003647	0.302080	0.228887	-0.030637	6.49E-05	0.000151
45	0.003568255	-0.001780	-0.003644	0.301884	0.228748	-0.030618	5.12E-05	0.000120
55	0.003562625	-0.001779	-0.003642	0.301639	0.228576	-0.030594	3.43E-05	8.01E-05
60	0.003560805	-0.001778	-0.003641	0.30156	0.228520	-0.030587	2.88E-05	6.73E-05
65	0.003559389	-0.001778	-0.003640	0.301499	0.228476	-0.030581	2.45E-05	5.73E-05
70	0.003558265	-0.001777	-0.003640	0.301450	0.228442	-0.030576	2.12E-05	4.94E-05
75	0.003556617	-0.001777	-0.003639	0.301378	0.228391	-0.030569	1.62E-05	3.78E-05
80	0.003556617	-0.001777	-0.003639	0.301378	0.228391	-0.030569	1.62E-05	3.78E-05
85	0.003556003	-0.001777	-0.003639	0.301352	0.228372	-0.030566	1.44E-05	3.35E-05
90	0.003555487	-0.001777	-0.003638	0.301329	0.228357	-0.030564	1.28E-05	2.99E-05
95	0.003555052	-0.001777	-0.003638	0.301310	0.228343	-0.030562	1.15E-05	2.68E-05
100	0.003554680	-0.001777	-0.003638	0.301294	0.228332	-0.030561	1.04E-05	2.42E-05
1000	0.003551270	-0.001776	-0.003636	0.301146	0.228227	-0.030546	1.04E-07	2.42E-07

Table 5: Non-displacement and Stresses of CCCS plate for b/a = 2.0

	$\propto = \frac{b}{a} = 2.0$							
$\rho = \frac{a}{t}$	$w = \bar{w}\left(\frac{qb^4}{D}\right)$	$u = \bar{u}\left(\frac{qa^4}{\rho D}\right)$	$v = \bar{v}\left(\frac{qa^4}{\rho D}\right)$	$\sigma_x = \overline{\sigma_x}(q\rho^2)$	$\sigma_y = \overline{\sigma_y}(q\rho^2)$	$\tau_{xy} = \overline{\tau_{xy}}(q\rho^2)$	$\tau_{xz} = \overline{\tau_{xz}}(q\rho^3)$	$\tau_{yz} = \overline{\tau_{yz}}(q\rho^3)$
	$\bar{w}$	$\bar{u}$	$\bar{v}$	$\overline{\sigma_x}$	$\overline{\sigma_y}$	$\overline{\tau_{xy}}$	$\overline{\tau_{xz}}$	$\overline{\tau_{yz}}$
4	0.007617847	-0.003098	-0.004838	0.491497	0.285037	-0.040308	0.0089553	0.012754
5	0.006536840	-0.002832	-0.004402	0.449086	0.259913	-0.036759	0.0054983	0.007788
6	0.005986527	-0.002697	-0.004179	0.427511	0.247095	-0.034949	0.0037361	0.005274
7	0.005666313	-0.002618	-0.004049	0.414963	0.239625	-0.033895	0.0027098	0.003817
9	0.005325508	-0.002534	-0.003910	0.401613	0.231665	-0.032771	0.0016167	0.002272
10	0.005228164	-0.002511	-0.003870	0.397801	0.229390	-0.032450	0.0013043	0.001832
15	0.005000772	-0.002455	-0.003777	0.388899	0.224070	-0.031699	0.0005743	0.000805
20	0.004922217	-0.002436	-0.003744	0.385824	0.222230	-0.031439	0.000322	0.000451
25	0.004886036	-0.002427	-0.003730	0.384408	0.221383	-0.031319	0.0002058	0.000288
30	0.004866429	-0.002422	-0.003722	0.383641	0.220924	-0.031255	0.0001428	0.000200
35	0.004854623	-0.002419	-0.003717	0.383179	0.220647	-0.031216	0.0001048	0.000147
40	0.004846967	-0.002417	-0.003714	0.382879	0.220468	-0.031190	8.025E-05	0.000112
45	0.00484172	-0.002416	-0.003711	0.382674	0.220345	-0.031173	6.339E-05	8.87E-05
50	0.004837969	-0.002415	-0.003710	0.382527	0.220257	-0.031160	5.134E-05	7.19E-05
55	0.004835195	-0.002414	-0.003709	0.382418	0.220192	-0.031151	4.242E-05	5.94E-05
60	0.004833085	-0.002414	-0.003708	0.382336	0.220143	-0.031144	3.565E-05	4.99E-05
65	0.004831443	-0.002413	-0.003707	0.382272	0.220104	-0.031139	3.037E-05	4.25E-05
70	0.004830140	-0.002413	-0.003707	0.382221	0.220074	-0.031135	2.619E-05	3.67E-05
75	0.004828230	-0.002413	-0.003706	0.382146	0.220029	-0.031128	2.005E-05	2.81E-05
85	0.004828230	-0.002413	-0.003706	0.382146	0.220029	-0.031128	2.005E-05	2.81E-05
90	0.004827517	-0.002412	-0.003706	0.382118	0.220012	-0.031126	1.776E-05	2.49E-05
95	0.004826920	-0.002412	-0.003705	0.382095	0.219998	-0.031124	1.584E-05	2.22E-05
100	0.004826415	-0.002412	-0.003705	0.382075	0.219986	-0.031122	1.421E-05	1.99E-05
1000	0.004825983	-0.002412	-0.003705	0.382058	0.219976	-0.031121	1.283E-05	1.80E-05

Table 6: Comparison of values of non-dimensional center deflection multiplied by 100 of CCCS square rectangular thick plate obtained herein with those from [1, 3].

Span-depth ratio (a/t)	Present	Ezeh *et al.* (2018) [3]	%Diff	Present	Li *et al.* (2015) [1]	%Diff
5	0. 2786	0.2434	14.46	0. 2786	0.2565	8.62
10	0. 1874	0.1816	3.19	0. 1874	0.1833	2.24
20	0. 1672	0.1660	0.72	0. 1672	0.1660	0.72
50	0. 1617	0.1615	0.12	0. 1617	-	-
100	0. 1607	0.1609	0.12	0. 1607	-	-
Average % Difference	**3.74**			**3.86**		
Total Average % Difference	3.81					

IV. CONCLUSION

It can be concluded that the rectangular plate can be classified as thick plate, when the plate span-to-depth ratios is less or equal to twenty $(a/t \leq 20)$; it can be classified as moderately thick plate, when the plate span-to-depth ratios is between twenty and fifty: $(20 \leq a/t \leq 50)$; when the plate span-to-depth ratios is greater or equal to twenty plate: $a/t \geq 50$ This assertion can be used to shows the boundary between thin and thick plate.

Furthermore, it is seen that at the 96 % confidence level, the values from the present study are the same with those from of previous studies, confirming the accuracy and reliability of the derived relationships. It is then, concluded that polynomial displacement function and higher order polynomial shear deformation theory can be used in confidence in the analysis of isotropic rectangular thick plate with clamped at three edges and simply supported at the remaining one edge (CCCS).

V. FUTURE SCOPE

Vibration analysis of rectangular thick plate applying trigonometric displacement function and higher order trigonometric shear deformation theory

Conflict of Interest. On behalf of all authors, I Onyeka Festus the corresponding author hereby states that there is no conflict of interest.

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