Application of the New Integral “J-transform” in Cryptography

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ABSTRACT: Cryptography is the interchange of information between the users without leakage of information to others. In our present paper, a new kind of transform is introduced named as Jaya transform and abbreviated as J-transform. In this paper, we discuss some properties and the application of J-transform in cryptography. J-transform is used for encryption, and inverse J-transform is used for decryption and an example is presented to discuss the process of encrypting and decrypting the given data.

Keywords: ASCII code, Decryption, Encryption, J-transform.

I. INTRODUCTION

Integral transformations have been effectively used for the past two centuries in solving many problems in applied mathematics and engineering science. Because of their prominence, Integral transforms are known for their role to find the solution of linear differential equations, difference equations, integral equations, electric circuits and networks, vibration and wave propagation, heat conduction in solids, quantum mechanics, fractional calculus and fractional differential equations, dynamical systems, signal processing, physical chemistry, mathematical biology, probability and statistics, solid and fluid mechanics [2, 4, 5, 8, 9, 11].

In our presentation of the paper, a new transformation is established, Jaya transform abbreviated as J-transform. This is used as a security tool in coding theory for encrypting and decrypting a message. J-transform is originated from the standard Laplace integral. Based on the mathematical simplicity of this transform and its fundamental properties, the process of encryption and decryption algorithms to get the message turns out to be simple. This transform is established with the inspiration from Laplace, Elzaki, Sumudu, Aboodh, Kamal, and Mahgoub transform. During the discussion of our techniques, a new cryptography method is proposed using J-transform. This method is used for encrypting the plain text and corresponding inverse J-transform is used for decryption. In this procedure, encryption is performed by replacing each letter by the American Standard Code for Information Interchange (ASCII) values [1, 3, 6, 7, 10, 12, 13, 14].

II. DEFINITION AND PROPERTIES

We define J-transform for the function of exponential order. We consider functions in the set B defined by

$$B = \left\{ f(t) : \exists M, k_1, k_2 > 0, |f(t)| \leq Me^{-k_1t}, \text{if } t \in (-\infty, 0], x(0, \infty) \right\}$$

where M represents a finite number, $k_1$ and $k_2$ are constants.

This transform is denoted by the operator $J(\cdot)$ defined by the integral equation

$$J[f(t)] = F(v) = v^2 \int_0^\infty f(t)e^{-vt}dt, \text{ where } t \geq 0, \text{ and}$$

$$k_1 \leq v \leq k_2$$

(1)

A. Standard Function values of J-transforms

Suppose we assume that the integral Eqn. (1) exists for the given function $f(t)$, we find some function values of J-transform.

Let $f(t) = 1$

By the definition of J-transform

$$J[f(t)] = v^2 \int_0^{\infty} f(t)e^{-vt}dt = v^2 \left[ -\frac{t}{v} \right]_0^{\infty} = v^3.$$

Therefore, $J[1] = v^3$ and Inversion Formula is


Let $f(t) = t$

By the definition of J-transform

$$J[f(t)] = v^2 \int_0^{\infty} f(t)e^{-vt}dt = v^2 \left[ -\frac{t}{v} \right]_0^{\infty} = v^4.$$
Therefore, \( J[t] = v^4 \) and hence
\[ J^{-1}[v^4] = t. \]

Let \( f(t) = t^2 \).

By the definition of J-transform
\[ J[f(t)] = v^2 \int_0^\infty f(t) e^{-vt} dt \]
\[ J[t^2] = v^2 \int_0^\infty t^2 e^{-vt} dt \]
\[ = v^2 \left[ \frac{-t}{v^2} - \frac{t}{v} + \frac{2}{v^2} \right]_0^\infty = 2! v^5. \]

Therefore, \( J[t^2] = 2! v^5 \) and hence
\[ J^{-1}[v^5] = \frac{t^2}{2!}. \]

Let \( f(t) = t^3 \).

By the definition of J-transform
\[ J[f(t)] = v^2 \int_0^\infty f(t) e^{-vt} dt \]
\[ J[t^3] = v^2 \int_0^\infty (t^3) e^{-vt} dt \]
\[ = v^2 \left[ \frac{-t}{v^2} - \frac{t}{v} + \frac{2}{v^2} \right]_0^\infty = 3! v^6. \]

Therefore, \( J[t^3] = 3! v^6 \) and hence
\[ J^{-1}[v^6] = \frac{t^3}{3!}. \]

In the general case, if \( n > 0 \)

We can derive \( J[t^n] = n! \sqrt{v} \sqrt{n+3} \) and hence
\[ J^{-1}[v^{n+3}] = -\frac{t^n}{n!}. \]

Let \( f(t) = e^{at} \).

By the definition of J-transform
\[ J[f(t)] = v^2 \int_0^\infty f(t) e^{-vt} dt \]
\[ J[e^{at}] = v^2 \int_0^\infty e^{at} e^{-vt} dt = v^2 \left[ \frac{1}{v-a} \right]_0^\infty = \frac{v^3}{1-a/v}. \]

Therefore, \( J[e^{at}] = \frac{v^3}{1-a/v} \) and hence
\[ J^{-1}\left[ \frac{v^3}{1-a/v} \right] = e^{at}. \]

Let \( f(t) = \sin at \).

By the definition of J-transform
\[ J[f(t)] = v^2 \int_0^\infty f(t) e^{-vt} dt \]
\[ J[\sin at] = \frac{v^3}{1+a^2/v^2} \]

We know that \( \sin at = \frac{e^{iat} - e^{-iat}}{2i} \).

Apply J-transform on both sides
\[ J[\sin at] = \frac{1}{2i} J[e^{iat}] - \frac{1}{2i} J[e^{-iat}] \]
\[ = \frac{1}{2i} \left[ \frac{v^3}{1-ia} \right] - \frac{1}{2i} \left[ \frac{v^3}{1+ia} \right] = \frac{a v^4}{1+a^2/v^2}. \]

Therefore, \( J[\sin at] = \frac{a v^4}{1+a^2/v^2} \) and hence
\[ J^{-1}\left[ \frac{a v^4}{1+a^2/v^2} \right] = \sin at. \]

Let \( f(t) = \cos at \).

By the definition of J-transform
\[ J[f(t)] = v^2 \int_0^\infty f(t) e^{-vt} dt \]
\[ J[\cos at] = \frac{v^3}{1+a^2/v^2} \]

We know that \( \cos at = \frac{e^{iat} + e^{-iat}}{2} \).

Apply J-transform on both sides
\[ J[\cos at] = \frac{1}{2} J[e^{iat}] + \frac{1}{2} J[e^{-iat}] \]
\[ = \frac{1}{2} \left[ \frac{v^3}{1-ia} \right] + \frac{1}{2} \left[ \frac{v^3}{1+ia} \right] = \frac{v^3}{1+a^2/v^2}. \]

Therefore, \( J[\cos at] = \frac{v^3}{1+a^2/v^2} \) and hence
\[ J^{-1}\left[ \frac{v^3}{1+a^2/v^2} \right] = \cos at. \]

Let \( f(t) = \sinh at \).

By the definition of J-transform
\[ J[f(t)] = v^2 \int_0^\infty f(t) e^{-vt} dt \]
\[ J[\sinh at] = v^2 \int_0^\infty e^{at} e^{-vt} dt = v^2 \left[ \frac{1}{v-a} \right]_0^\infty = \frac{v^3}{1-a/v}. \]

Therefore, \( J[\sinh at] = \frac{v^3}{1-a/v} \) and hence
\[ J^{-1}\left[ \frac{v^3}{1-a/v} \right] = \sinh at. \]
We know that 

\[ \sinh t = \frac{e^{at} - e^{-at}}{2}. \]

Apply J-transform on both sides

\[ J[\sinh t] = \frac{1}{2} J[e^{at}] - \frac{1}{2} J[e^{-at}] \]

\[ = \frac{1}{2} \left[ \frac{v^3}{1 -av} \right] - \frac{1}{2} \left[ \frac{v^3}{1 + av} \right] = \frac{a v^4}{1-a^2v^2}. \]

Therefore, \( J[\sinh t] = \frac{a v^4}{1-a^2v^2} \) and hence

\[ J^{-1}\left[ \frac{a v^4}{1-a^2v^2} \right] = \sinh t. \]

Let \( f(t) = \cosh t. \)

By the definition of J-transform

\[ J[f(t)] = \int_{0}^{\infty} f(t) e^{-vt} dt. \]

We know that 

\[ \cosh t = \frac{e^{at} + e^{-at}}{2}. \]

Apply J-transform on both sides

\[ J[\cosh t] = \frac{1}{2} J[e^{at}] + \frac{1}{2} J[e^{-at}] \]

\[ = \frac{1}{2} \left[ \frac{v^3}{1 -av} \right] + \frac{1}{2} \left[ \frac{v^3}{1 + av} \right] = \frac{v^3}{1-a^2v^2}. \]

Therefore, \( J[\cosh t] = \frac{v^3}{1-a^2v^2} \) and hence

\[ J^{-1}\left[ \frac{v^3}{1-a^2v^2} \right] = \cosh at. \]

**B. Linearity property of J-transform**

If \( J[f(t)] = F(v) \) and \( J[g(t)] = G(v) \) then

\[ J[a f(t) + b g(t)] = a J[f(t)] + b J[g(t)] = a F(v) + b G(v). \]

Where \( a \) and \( b \) are arbitrary constants.

**Proof:** From the definition of a J-transform we have

\[ J[f(t)] = \int_{0}^{\infty} f(t) e^{-vt} dt. \]

Using the linearity properties of integration, we have

\[ J[a f(t) + b g(t)] = \int_{0}^{\infty} \left[ a f(t) + b g(t) \right] e^{-vt} dt \]

\[ = \int_{0}^{\infty} a f(t) e^{-vt} dt + \int_{0}^{\infty} b g(t) e^{-vt} dt \]

\[ = a J[f(t)] + b J[g(t)] = a F(v) + b G(v) . \]

Therefore, \( J[a f(t) + b g(t)] = a F(v) + b G(v) . \)

In particular, using the definition (1) and property (B) we solve an example as

\[ J[5e^{4t} + 3\sin2t] = 5J[e^{4t}] + 3J[\sin2t] \]

\[ = 5 \cdot \frac{v^3}{1-4v} + 3 \left( \frac{4v}{1+2^2v^2} \right) \]

\[ = 5 \cdot \frac{v^3}{1-4v} + \frac{6v^4}{1+4v^2} \]

\[ J[5e^{4t} + 3\sin2t] = \frac{5v^3}{1-4v} + \frac{6v^4}{1+4v^2} . \]

### III. J-TRANSFORM APPLICATION IN CRYPTOGRAPHY

**A. Encryption Algorithm Steps**

– Assign every alphabet in the plain text message as ASCII values.

– Next, the plain text message is organized as a finite sequence of numbers based on the above conversion.

– Apply J-transform of the polynomial considered as \( g(t) = F(t)e^{bt} \). Here \( F \) takes the ASCII values of the plain text message.

– Find \( r_i \) such that \( r_i \equiv M_i \mod 500 \) where \( 1 \leq i \leq n \). Here \( M_i \) are the coefficients of \( J \{ g(t) \} \) given in the above polynomial.

– Determine a new finite sequence of remainders \( r_1, r_2, \ldots, r_n \) using the above step.

– The ASCII values of \( r_1, r_2, \ldots, r_n \) will be the encrypted message and the set of quotients are taken as a key \( c_i \), \( i = 1, 2, 3, \ldots, n \).

**B. Decryption Algorithm Steps**

– Convert the given secret message to original text by using the given key \( c_i \) for \( i = 1, 2, 3, \ldots, n \) and prepare the cipher text in the corresponding finite sequence of numbers \( r_1, r_2, \ldots, r_n \).

– To generate the original message consider the congruence \( M_i \equiv 500c_i + r_i \) where \( i = 1, 2, 3, \ldots, n \).

– Determine the values of \( M_1, M_2, \ldots, M_n \) using the above step.

– Apply the inverse J-transform to \( J \{ g(t) \} \).

– Arrange the coefficient of the polynomial \( g(t) \) as a finite sequence.

– Convert the number of finite sequence to alphabets by using ASCII values.

Hence, in this process, we get the original plain text message.

### IV. PROPOSED METHODOLOGY

Let us start with a plain text having the message "ENVIRONMENT".

**A. Encryption procedure**

Using Step1 of Encryption Algorithm, assign every alphabet in the plain text message in ASCII values as \( E = 69, N = 78, V = 86, I = 73, R = 82, O = 79, N = 78, M = 77 \), \( E = 69, N = 78 \).

Using Step2 of Encryption Algorithm, the plain text message is organized as a finite sequence of ASCII values as say
Using Step 3 of Encryption Algorithm, Consider the standard expansion of an exponential,
\[ e^{bt} = 1 + \frac{bt}{1!} + \frac{b^2t^2}{2!} + \frac{b^3t^3}{3!} + \frac{b^4t^4}{4!} + \frac{b^5t^5}{5!} + \frac{b^6t^6}{6!} + \cdots \]

Then we get,
\[ t^2e^{bt} = t^2 + \frac{b^3t^3}{1!} + \frac{b^4t^4}{2!} + \frac{b^5t^5}{3!} + \frac{b^6t^6}{4!} + \frac{b^7t^7}{5!} + \frac{b^8t^8}{6!} + \cdots \]

Using Step 3 of Encryption Algorithm, Consider the polynomial
\[ g(t) = F(t)^2 e^{2t} \]
\[ = 69t^2 + 27t^3 + 88t^4 + 24t^5 + 132t^6 + 72t^7 + 8t^8 + 77t^9 + 69t^{10} \]
\[ + 79t^{11} + \cdots \]

Apply J-transform on both sides
\[ J[g(t)] = J[F(t)^2 t] \]
\[ = [69t^2 + 27t^3 + 88t^4 + 24t^5 + 132t^6 + 72t^7 + 8t^8 + 77t^9 + 69t^{10} \]
\[ + 79t^{11} + \cdots ] \]

Using Step 4 of Decryption Algorithm, apply the inverse J-transform, we get
\[ J^{-1}[J[g(t)]] = 138v^5 + 3936v^6 + 4128v^7 + 11680v^8 + 39360v^9 + 106176v^{10} \]
\[ + 279552 v^{11} + 709632 v^{12} + 1589760 v^{13} + 4392960 v^{14} \]
\[ + 11354112 v^{15} \]

Using Step 5 of Encryption Algorithm, determine a new finite sequence of remainders \( r_1, r_2, \ldots, r_n \) using the above step as follows
\[ r_1 \equiv 138 \mod 500 = 500(0) + 138 = 138 \]
\[ r_2 \equiv 936 \mod 500 = 500(1) + 436 = 436 \]
\[ r_3 \equiv 4128 \mod 500 = 500(8) + 128 = 128 \]
\[ r_4 \equiv 11680 \mod 500 = 500(23) + 180 = 180 \]
\[ r_5 \equiv 39360 \mod 500 = 500(78) + 360 = 360 \]
\[ r_6 \equiv 106176 \mod 500 = 500(212) + 176 = 176 \]
\[ r_7 \equiv 279552 \mod 500 = 500(559) + 52 = 52 \]
\[ r_8 \equiv 709632 \mod 500 = 500(1419) + 132 = 132 \]
\[ r_9 \equiv 1589760 \mod 500 = 500(3179) + 260 = 260 \]
\[ r_{10} \equiv 4392960 \mod 500 = 500(8785) + 460 = 460 \]
\[ r_{11} \equiv 11354112 \mod 500 = 500(22708) + 112 = 112 \]

Prepare the cipher text in the corresponding finite sequence related to the cipher text.

Therefore, the original plain text message ‘ENVIRONMENT’ is changed into cipher text ‘эyCФ48Anрjп’ as the ASCII values of these remainders \( r_1, r_2, \ldots, r_{11} \) (obtained by pressing ALT key of the remainder values using the keyboard of the computer).

### B. Decryption Procedure

Using Step 1 of Decryption Algorithm, convert the given secret message to original text by using the given key \( c_i \) for \( i = 1, 2, 3, \ldots, n \) as 0, 1, 8, 23, 78, 212, 559, 1419, 3179, 8785, 22708. We observe that 138, 436, 128, 180, 360, 176, 52, 132, 260, 460, 112 numbers appear as in the form of finite sequence related to the cipher text.

Prepare the cipher text in the corresponding finite sequence of numbers \( r_1, r_2, \ldots, r_{11} \) as
\[ R_1 = 138, R_2 = 436, R_3 = 128, R_4 = 180, R_5 = 360, R_6 = 176, R_7 = 52, R_8 = 132, R_9 = 260, R_{10} = 460, R_{11} = 112 \]

Using Step 2 of Decryption Algorithm, to generate the original message considers the congruence
\[ M_i \equiv 500c_i + r_i \] where \( i = 1, 2, 3, \ldots, n \).

Using Step 3 of Decryption Algorithm, determine the values of \( M_1, M_2, \ldots, M_{11} \) using the above step.

Now by using Eqn. (2) we have
\[ J[g(t)] = 138v^5 + 3936v^6 + 4128v^7 + 11680v^8 + 39360v^9 + 106176v^{10} \]
\[ + 279552 v^{11} + 709632 v^{12} + 1589760 v^{13} + 4392960 v^{14} \]
\[ + 11354112 v^{15} \]

Using Step 4 of Decryption Algorithm, apply the inverse J-transform, we get
\[ g(t) = J^{-1}[J[g(t)]] = 138v^5 + 3936v^6 + 4128v^7 + 11680v^8 + 39360v^9 + 106176v^{10} \]
\[ + 279552 v^{11} + 709632 v^{12} + 1589760 v^{13} + 4392960 v^{14} \]
\[ + 11354112 v^{15} \]

Thus the original message is given by the ASCII values of these remainders 138, 436, 128, 180, 360, 176, 52, 132, 260, 460, 112 and the set of quotients taken in the key as \( c_1 = 0, c_2 = 1, c_3 = 8, c_4 = 23, c_5 = 78, c_6 = 212, c_7 = 559, c_8 = 1419, c_9 = 3179, c_{10} = 8785, c_{11} = 22708 \).
Using Step5 of Decryption Algorithm, arrange the coefficients of the polynomial $g(t)$ as a finite sequence such as $69, 78, 86, 73, 82, 79, 78, 77, 69, 78, 84$.

Using Step6 of Decryption Algorithm, convert the numbers of finite sequence to alphabets by using ASCII values, thereby we get the original text message “ENVIRONMENT.”

**V. CONCLUSION**

We introduced an efficient Laplace-type integral transform called the J-transform. A new integral transform J-transform of exponential functions using ASCII values is applied to encrypt and decrypt the given message. We presented some useful properties of this transform and its application in Cryptography. The algorithmic part is also simple. This procedure is allowed the plain text message in safety form as it involved large value of modulus (500). Thus, the process of encryption and decryption is more strengthened.

**VI. FUTURE SCOPE**

This transform can be significantly used like other integral transforms in solving ordinary and partial differential equations.

**Conflict of Interest.** The authors declare that there is no conflict of interests.

**REFERENCES**


