

Construction of Perfect Periodic Binary Sequences for Radar Applications

Tara Dutt Bhatt

Department of Electronics and Communication Engineering, Mahatma Gandhi Institute of Technology, Hyderabad, Telangana, India.

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ABSTRACT: Periodic binary sequences with good autocorrelation properties are widely used in communications and continuous wave (CW) pulse compression radar systems. To achieve high range-resolution in pulse compression radar systems, long binary sequences are required. This paper presents the construction of perfect periodic binary sequences with larger lengths and higher energy efficiency. The applications and suitability of such sequences for pulse compression radars are demonstrated. Both amplitude-symmetrical and asymmetrical sequences up to lengths N=149 are presented. This construction method can be extended even for higher lengths also.

Keywords: Perfect periodic autocorrelation, pulse compression, radar, Legendre sequences, on-off keying, energy efficiency.

Abbreviations: PACF, periodic autocorrelation function; CW, continuous wave; SNR, signal to noise ratio; AACF, aperiodic autocorrelation function; NCPC, Non-coherent Pulse Compression; OOK, on-off keying; PPAC, perfect periodic autocorrelation; ZCZ, zero-correlation zone.

I. INTRODUCTION

Signals with impulse-like autocorrelation function, which can disappear off-peak sidelobes in between main peaks are called perfect periodic sequences [1-2]. Perfect periodic sequences find applications in many areas of engineering. Some of the applications are synchronization, transform coding, channel coding, synthetic aperture imaging, communications, measurements and radars. In radar systems, to enhance the signal to noise ratio (SNR) and range resolution, the transmitted pulse is modulated either in phase or frequency and received signal is correlated with the replica of transmitted signal using matched filter. Improved SNR and high resolution are achieved at the cost of sidelobes at the output of matched filter. which are undesirable. This needs a continuous search for the design of optimum radar signals. Due to ease of implementation, binary or ternary sequences are widely used for modulation of transmitted signal in pulse compression radar systems. The important parameter for the design of pulse compression sequences is that the sequences must have good auto correlation property. In addition to good autocorrelation property, for the detection of targets at longer ranges the reflected echo signal must have sufficient energy. Therefore, for radar applications we need the signals that exhibit both the properties. Generally, sequences with ideal aperiodic autocorrelation function (AACF) do not exist [2]. On the other hand, construction of perfect periodic sequences is possible, which can be extensively used in continuous wave (CW) radars [3-4]. The properties and suitability of such signals are studied in [5-8]. Therefore, the focus of the work is to design the binary or biphase sequences which have higher energy efficiency and perfect periodic autocorrelation. The construction method is based on the modification of Legendre and M-sequences. Such sequences are referred as

'synthesized sequences' and can be designed for all prime numbers with some modifications which is explained in section III.

II. PROPERTIES OF PERIODIC SEQUENCES

Let s_i (n) be a real sequences of length N and its periodic repetition with period N is represented by $\hat{s}_i(n)$. The autocorrelation and cross-correlation functions of such periodic sequences can be given by:

$$R_{ii}(\tau) = \sum_{n=0}^{N-1} s_i(n) \,\hat{s}_i^{*}(n+\tau) \tag{1}$$

$$R_{ij}(\tau) = \sum_{n=0}^{N-1} s_i(n) \,\hat{s}_j^{*}(n+\tau)$$
⁽²⁾

$$R_{ii}(\tau) = \begin{cases} E, \text{ for } \tau = 0\\ 0, \text{ for } \tau \neq 0 \end{cases}$$
(3)

In the region $0 \le \tau \le N$.

where $\hat{s}_i (n + \tau) = s_i((n + \tau) \mod N)$, and $\tau = mt_b$ is time delay and t_b is duration of single bit. Eqn. (1) is representing the autocorrelation function and Eqn. (2) gives the cross-correlation function of such periodic sequences. Eqn. (1) describes that P number of received pulses are correlated with Q number of reference pulse where P > Q. (In all correlation figs. In this paper P = 5, and Q = 1, considered).

The energy associated with the sequence is given by

$$E = \sum_{n=0}^{N-1} s_i^2(n)$$
 (4)

Luke proposed the methods of synthesizing perfect periodic autocorrelation sequences. It is also shown that such sequences have good energy efficiency. The techniques used to synthesize these sequences are computer search, modifying binary sequences, ternary sequences, M-sequences, Legendre sequences and product of two synthesized sequences [2]. By taking advantage of one of the important properties that is the magnitude of the perfect sequences is constant and it is given by

$$\widetilde{S}_{i}(k) = \sum_{n=0}^{N-1} \hat{s}_{i}(n) \exp(-j2\pi nk/N) \ 0 \le k < N$$
(5)
By setting 'k' to zero, Eqn. (5) becomes

$$\left|\tilde{S}_{i}(0)\right|^{2} = \left|\sum_{n=0}^{N-1} s_{i}(n)\right|^{2} = E$$
(6)

In his work [2], Luke tabulated the perfect sequences up to length 60 only, which in not enough for radar applications. Further he suggested construction of higher sequence lengths by using product of two sequences. However, the efficiency of synthesized sequences is degraded due to their non-uniform amplitude. In case of taking product of two non-uniform periodic sequences, the efficiency further decreases to $\eta = \eta_1 \eta_2$, where η_1 and η_2 are efficiencies two sequences. Formula for calculating η is:

$$\eta = \sum_{n=0}^{N-1} \frac{s^2(n)}{|s^2(n)|_{m=0}}$$
(7)

When these synthesized sequences are used for pulse compression radar applications, the energy efficiency of the transmitted signals must be high for detection of target present at long range and to achieve high range resolution, sequence length must be large. Hence, this paper is mainly focused on the design of sequences with perfect periodic autocorrelation having larger length and high energy efficiency.

III. CONSTRUCTION OF PERFECT PERIODIC BINARY SEQUENCES

M-sequences and Legendre sequences exhibit lowest periodic autocorrelation function (PACF) equal to $/Ri(\tau \neq$ 0) = 1/. The lpatov code [9] is also a code pairs exhibits perfect periodic autocorrelation (the cross correlation of the code pair) with minimal mismatch loss. But the construction method of reference code for lpatov pair is complicate. M-sequences are two valued binary codes {±1}, having code length N, which produces periodic auto-correlation of peak value equal to N and uniform sidelobes of value 1. Similarly, the Legendre codes also demonstrate a similar property that is the off-peak sidelobes are equal to 1. Fig. 1 shows the amplitude and autocorrelation function of M-sequence/Legendre sequence of code length 31. The ACF clearly show that the magnitude of sidelobes other than peak is constant and level is 1. Which is clearly revealed in Fig. 1 (b).



Fig. 1. M-sequence N=31 (a) amplitude of transmitted signal (b) autocorrelation function (sidelobes =1).



Fig. 2. Punctured M-sequence N=31 (a) amplitude of transmitted signal (b) cross-correlation function.

 Transmitted Signal:
 [1 0 0 1 0 0 1 0 0 0 0 1 1 1

 0 1 0 1 0 0 0 1 1 1 1 0 1 10 1 1

Fig. 2 shows that the amplitude of the transmitted signal, where '0' level shows no-transmission, and level '1' is transmission of signal. There are 15 '0s' and 16 '1s' in transmitted signal, represents that the duty cycle of transmitted signal is nearly 50%. These signals are commonly referred as punctured M-sequence or On-Off keying (OOK) sequences. The applications of OOK signals for non-coherent pulse compression (NCPC) radar is explained in [10-12]. The process of changing all '-1s' to '0' called puncturing [13-15]. Therefore, the energy efficiency of the transmitted signal N=31 is only 51. 6%. On the other hand, in Fig. 1(a) the amplitude of transmitted signal is constant and it is equal to 1. Hence, energy efficiency is 100%. Lei Xu and Qilian Liang [13, 14] also demonstrated the construction of punctured sequences which are similar to on-off keying sequence. Fig. 2(b) shows that the sidelobes of punctured M-sequence are zero. These zero sidelobes are achieved at the cost of sacrificing energy in transmitted signal. Another important point is that Msequences or Legendre sequences exhibit perfect periodic autocorrelation property (all sidelobes equal to zero) only when unipolar transmitted signal has '1's more than '0's by one element only. This verifies that the energy efficiency of such sequences is slightly greater than 50% and approaches 50% as sequences length is exceptionally large. When such sequences are used in radar applications, the probability of detection is reduced. This problem is dealt by using variable amplitude sequences, where in place of zero '0.739' is transmitted. Due to this, duty cycle in increased and in turn energy efficiency is increased to 78%.

Two major issues which are important in the design of waveforms for pulse compression radars are: (i) for

better detection probability, the received signal must contain sufficient energy because matched filter is used in radar receivers. The signal-to-noise ratio at the output of the matched filter is given by $2E/N_0$. In this expression, E denotes the energy contained by received signal. Therefore, it clearly shows that to detect the desired target at long range, the reflected echo signal must contain sufficient energy. To receive sufficient energy in reflected echo signal, high energy signals are to be transmitted.(ii) To achieve high range resolution, compression ratio must be high which is **directly** proportional to the sequence length N. In case of biphase or polyphase sequences compression ratio is equal to N, where N is number of bits in transmitted pulse.

Considering these two factors, this paper proposes the construction of amplitude-symmetrical binary sequences and amplitude-asymmetrical binary sequences, which have perfect periodic autocorrelation (PPAC) properties, higher energy efficiency and large sequence lengths by using Legendre sequences, modified Legendre sequences and M-sequences. The major advantage of using Legendre sequences is that these sequences can be designed for all prime numbers. Whereas Msequences are designed on for the lengths $N=2^{n}-1$, where n= 2, 3, For example; between sequence lengths of 1000 and 10,000, only four M-sequences are available (1023, 2047, 4095 and 8191). On the other hand, there are 519 Legendre sequence between 1000 and 10,000 [10]. Fig. 3(a) depicts the amplitude of the synthesized binary transmitted pulse whereas Fig. 3(b) shows the autocorrelation function of the same. In this case the transmitted and reference signals are same; hence we refer it as autocorrelation function. To demonstrate the resolution property of binary synthesized sequences, the amplitude of the transmitted signal and its autocorrelation function is shown in Fig. 4.





Table 1 shows the synthesized binary sequences up lengths 149. This table includes, all prime numbers from N=3 to N=149 and also includes lengths 4, 15 and 63 which are not prime. In case of all Legendre sequences, which satisfy $N \equiv 3(mod4)$, and N is prime number, the value of 'b' can be calculated as:

$$b = \frac{-1}{1 + \frac{2}{\sqrt{N+1}}} \tag{8}$$

The other Legendre sequences that satisfy the condition $N \equiv 1 \pmod{4}$, and N is prime number are referred as 'modified Legendre sequences'. In such sequences zero appears in leading edge. These sequences are modified by using search method and modified in such a way that these sequences have good periodic autocorrelation property. It is observed that in all such sequences, given

in Table 1, the sidelobe suppression in zero correlation zone (i.e. off-peak sidelobes) is better than-77dB. Fig. 4(b) clearly shows that the impulse like autocorrelation function will resolve closely spaced targets and this resolution depends upon the compression of the pulse. To enhance this resolution capability, sequences with higher lengths must be used. The resolution capability of sequence length N=149 can be understood by comparing Fig. 3(b) and 4(b). The width of the autocorrelation peak is like impulse in Fig. 4(b), whereas in Fig. 3(b) width of the autocorrelation peak is more. Therefore, higher sequence lengths are appropriate for high-resolution radar applications.



Fig. 4. Synthesized binary sequence N=149 (a) amplitude of transmitted signal (b) Autocorrelation function.

Гэ	h	ما	1	
a	D	Ie.		

N	η (in %)	Perfect periodic Binary Sequence with good Energy Efficiency η
3	75	1 b 1 (b=-0.5)
4	100	$1 \ 1 \ 1 \ b \ (b=-1)$
5	47.8	0.31 1 b b 1 (b = -0.382)
7	71.8	1 b b 1 b 1 1 (b=-0.586)
11	72.8	$1 \ b \ 1 \ b \ b \ b \ 1 \ 1 \ b \ 1 \ (b = -0.634)$
13	62	0.21 b 1 b b 1 1 1 b b 1 b (b = -0.58)
15	74	b b b 1 b b 1 1 b 1 b 1 1 1 (b = -0.667)
17	64.8	0.2 b b 1 b 1 1 1 b b 1 1 b b (b=-0.61)
19	75	1 b 1 1 b b b b 1 b 1 b 1 1 1 b b 1 (b = -0.691)
23	76.3	1 b b b b 1 b 1 b b 1 1 b b 1 1 b 1 b 1
29	71	0.16 b 1 1 b b b b 1 b 1 1 1 b 1 1 b 1 1 b b b b b 1 1 1 b 1 1 b 1 b 1 b b b b b 1 1 1 b 1 b 1 b
31	78	1 b b 1 b b 1 b b b b 1 1 1 b 1 b 1 b b b 1 1 1 b 1 1 b 1 1 (b=-0.739)
37	74	0.14 b 1 b b 1 1 b 1 b b b b 1 1 1 b 1 1 1 b 1 1 1 b 1 1 b 1 b b b b b 1 0 1 1 b b 1 b (b = -0.72)
41	74.8	0.135 b b 1 b b 1 1 b b b 1 1 1 1 1 b 1 b 1
43	80	1 b 1 1 b 1 b 1 1 b b b 1 b b b b b 1 1 1 b 1 b b b 1 1 1 1 1 b 1 1 1 b b 1 b 1
47	80.5	1 b b b b 1 b b b 1 1 b 1 b 1 b b b 1 1 b 1 1 b b 1 b b 1 1 1 b 1 1 b 1 1 b 1 1 b 1 1 b 1 1 1 b 1 1 1 b 1 1 1 (b = -0.776)
53	77.3	0.12 b 1 1 b 1 b b 1 b b 1 b 1 b b b 1 1 1 1 1 1 b b 1 1 b b 1 1 1 1 1 1 b b b 1 b 1

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59	81.8	1 1 1 1 b 1 1 1 b b 1 b b 1 b 1 b 1 b 1
61	78.6	0.114 b 1 b b b 1 1 1 b 1 1 b b b b b 1 1 b b 1 b 1 b 1 b 1 b 1 b 1 1 1 1 1 b 1 b 1 1 b 1 b 1 1 b 1 b 1 1 b 1 b 1 1 b 1 b 1 b 1 b 1 b 1 b 1 b 1 b b b 1 b (b=-0.773)
63	82	b b b b b 1 b b b 1 1 b b b 1 b 1 b b 1 1 1 1 b 1 b b b 1 1 1 b b 1 b b 1 b 1 b 1 b b 1 b
67	82.6	1 b 1 1 b 1 b 1 1 b b 1 1 1 b b b b b 1 b 1 b b b b b b b 1 1 b 1 b 1 b 1 b b b 1 b b 1 1 1 1 b 1
71	83	1 b b b b b 1 b b 1 b 1 1 b b 1 b b 1 1 1 b b 1 b
73	80	0.105 b b b b 1 b 1 1 b 1 1 b 1 b b 1 1 1 b b b 1 b 1 1 1 b 1 b b b b 1 1 b 1 b b b b 1 1 b 1 b b b b 1 1 b 1 b 1 b 1 b 1 b 1 b 1 b 1 b b b b b b (b=-0.79)
79	83.6	1 b b 1 b b 1 1 b b b b 1 b 1 1 b 1 b 1
83	83.9	1 b 1 b b 1 b b b b 1 1 b b b 1 1 b b b 1 b 1 b b b b b b b b b 1 b 1 b b b b b 1 b 1 b b b 1 b 1 b b b 1 b 1 b b b 1 b b b 1 b b b 1 b b 1 b b b 1 b b 1 b b b 1 b b 1 b b 1 b b 1 b b 1 b b 1 b b 1 b b 1 b b 1 b b 1 b b b 1 b b b 1 b b b 1 b b 1 b b b 1 b b b 1 b b b 1 b b b 1 b b b 1 b b b 1 b b b 1 b b b 1 b b b b 1 b b b b b b b b b b b b b 1 b b b 1 b b b b 1 b b b b b b b b b b b b b 1 b b b b 1 b b b b 1 b
89	81.7	0.096 b b 1 b b 1 1 b b b b 1 1 1 1 b b b 1 b b b 1 b b 1 b b 1 b b 1 b b b 1 b b b 1 b b b 1 b b b b 1 b
97	82.4	0 = -0.808) 0.092 b b b b 1 b 1 b b 1 b b 1 1 1 b 1 b 1
		1 b 1 b b b b (b=-0.816) 0.09b 1 1 b b b 1 1 b b 1 b b 1 b b b b b b
101	82.8	1 b 1 b 1 b 1 1 b 1 b 1 b 1 b 1 1 1 1 1
103	85.1	1 b b 1 b 1 1 b b b 1 1 1 b b b b b b b
107	85.2	1 b 1 b b 1 1 1 1 b b b b b 1 b 1 1 b 1 1 1 b 1 b 1 b 1 b b 1 1 b b b b b 1 b b b 1 b 1 b b b 1 1 b b 1 b 1 b b 1 b 1 b b 1 b
109	83.3	0.087 b 1 b b b 1 b 1 b 1 1 b 1 1 b b 1 1 1 b b b 1 1 b b b b b b b 1 b 1 1 b b b 1 b 1 1 1 b 1 b b 1 b b 1 1 1 1
113	83.5	0.086 b b 1 b 1 1 b b b 1 b 1 b b b b 1 b 1
127	86	1 b b 1 b 1 1 1 b b 1 b 1 b 1 b b b b b
131	86.3	1 b 1 b b b 1 b 1 b 1 b b b 1 b b 1 1 1 b b 1 1 1 b b b b 1 1 1 1 b b b b 1 b 1 b 1 b b b b b b b b b b b b b b b b 1 1 1 1 1 1 1 b b 1 b 1 1 b b 1 b 1 1 b b 1 b 1 1 1 b b 1 b 1 1 1 b b 1 b 1 1 1 b
137	84.9	0.0785 b b 1 b 1 1 b b b 1 b 1 1 b b b b b b
139	86.6	1 b 1 1 b b b b 1 b 1 b 1 b 1 1 b 1 1 1 b 1 1 1 b b 1 1 b b b b 1 1 b b b b b b 1 1 b b b b b 1 b b b b b 1 b b b b b b b b b b b 1 b 1 1 1 1 b b b b b b b 1 b 1 1 1 1 b b 1 b 1 1 b 1 1 b b 1 b 1 1 b b 1 1 b b 1 1 1 1 b b 1 1 b b 1 1 1 1 b b 1 b b 1 b 1 b b 1 b 1 b b 1 b 1 b b 1 b 1 b b 1 b 1 b b 1 b 1 b b 1 b 1 b b 1 b 1 b b 1 b 1 b b 1 b 1 b b 1 b 1 b b 1 b 1 b b 1 b 1 b b 1 b 1 b b 1 b 1 b b 1 b 1 b b 1 b b 1 b b 1 b b 1 b 1 b b 1 b b 1 b
149	85.5	0.075 b 1 1 b b b b 1 b 1 1 1 1 1 1 b b 1 b b 1 b 1 b b b 1 b b b b 1 b 1 b b b 1 b 1 b 1 1 b 1 1 b 1 1 b 1 1 b b 1 1 b 1 1 b b 1 1 b b 1 1 b b 1 1 b b 1 1 b b 1 1 b b 1 1 b b 1 1 b b 1 1 b b 1 1 b b 1 1 b b b 1 1 b b b b 1 b b b b 1 b b b b 1 b b b b 1 b b b b 1 b b b b b 1 b b b b 1 b b b b 1 b b b b 1 b b b b 1 b b b b b 1 b b b b 1 b b b b 1 b b b b 1 b b b b 1 b b b b 1 b b b b 1 b b b b b 1 b b b b b 1 b b b b b 1 b b b b b 1 b b b b b 1 b b b b b 1 b b b b b 1 b b b b b 1 b b b b b b 1 b b b b b 1 b

IV. CONCLUSION

In this paper, binary periodic waveforms are studied for the application of phase coded radar systems that employs pulse train or CW waveforms. Majority of the PPAC sequences are achieved by modifying Legendre sequences and such sequences can be designed for all prime numbers. The applications and appropriateness of perfect periodic binary sequences for pulse compression radar is explained with the help of simulated results.

From Fig. 4, it can be understood that when we increase the sequence length, both energy efficiency and compression ratio increase which in-turn improve the detection probability as well as range resolution of the radar system. Additionally, the significant advantage of the designed binary sequences is that sidelobes are as low as zero within the zero-correlation zone (ZCZ) that is (N-1). Binary periodic sequences could be effectively applied in multi-target scenario where small targets near strong target goes undetected due to the presence of sidelobes.

V. FUTURE SCOPE

The performance of these sequences must be checked in real time applications.

Conflict of interest: The author declares that he has no conflict of interest.

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