International Journal on Emerging Technologies **11**(1): 364-369(2020)

ISSN No. (Print): 0975-8364
ISSN No. (Online): 2249-3255

Disturbance Observer Based Adaptive Sliding Mode Hybrid Projective Compound Synchronization

Ayub Khan[1], Nasreen[2], Harindri Chaudhary[2], Pushali Trikha[2] and Lone Seth Jahanzaib[2]
[1]Professor, Department of Mathematics, Jamia Millia Islamia, Delhi, India.
[2]Research Scholar, Department of Mathematics, Jamia Millia Islamia, Delhi, India.

(Corresponding author: Lone Seth Jahanzaib)
(Received 04 November 2019, Revised 06 January 2020, Accepted 12 January 2020)
(Published by Research Trend, Website: www.researchtrend.net)

ABSTRACT: In this paper, hybrid projective compound synchronization using disturbance observer based adaptive sliding mode control technique has been performed in presence of unknown bounded disturbances. The unknown disturbances are estimated using fractional order disturbance observer. Numerical simulations have been performed using MATLAB which verify the efficacy of our theoretical results. The obtained results have been compared with prior published literature. To design controllers in the presence of disturbances is a very challenging task but controllers have been designed successfully and results have been compared which show better results. It may have a great application across many areas like secure communication, image encryption etc.

Keywords: Compound Synchronization, Unknown Bounded Disturbance, Hybrid Projective Synchronization, Adaptive Sliding Mode Control, Fractional order disturbance observer.

I. INTRODUCTION

Chaos Theory is an emerging field of research owing to its growing applications in various fields of science and engineering. Chaotic systems possessing the unique property of showing extreme high sensitivity to initial conditions and parameter values are being considered suitable in areas of secure communication, image encryption, control theory etc. Also since the fractional order system of equations are shown to represent real life situations more accurately as compared to their integer counterpart, therefore fractional calculus is turning to be a prime tool for research worldwide. Many systems such as electromagnetic waves, viscoelastic systems, dielectric polarization are all known to have fractional order dynamics.

Ever since chaos was observed for the first time in fractional ordered Lorenz system by Grigorenko & Grigorenko, chaos control and synchronization Luo (2009), Pecora & Carroll (1990), Zhang *et al.*, (2009) [12, 13, 21] of fractional ordered chaotic systems became active research areas. Many synchronization techniques Khan & Bhat (2016) [7], Singh *et al.*, (2016) [16], Khan (2017) [8], Khan *et al.*, (2017) [9], He *et al.*, (2018) [5], Dongmo *et al.*, (2018) [3], Das and Yadav (2017) [2], Zhang *et al.*, (2017) [22], Wei *et al.*, (2014) [19], Khan & Tyagi (2017) [11], Wu *et al.*, (2019) [23], Skardal *et al.*, (2017) [24], such as Active Control, Parameter Estimation Method, Tracking Control Method, Sliding Mode Control Method have been developed and applied on fractional ordered chaotic systems. Yang *et al.*, (2011) [20], Podlubny (1998) [14], Hilfer (2000) [6], Hilfer *et al.*, (2000) [1], Tavassoli *et al.*, (2013) [18]. However sliding mode control method (Fang and Hou 2016) [4] is supposed to be the best because of its high robustness and rapid convergence.

In this manuscript, disturbance observer based adaptive sliding mode hybrid projective compound synchronization is studied among modified fractional order jerk system in presence of unknown bounded disturbances by suitably designing a FODO and using sliding mode control technique to achieve the synchronization. The disturbance observer here helps approximate the unknown disturbances.

The rest of the article is arranged as:

Section II begins with preliminaries. Section III gives the system description on which numerical simulations have been performed. Section IV designs the FODO based adaptive sliding mode compound synchronization to estimate the disturbances. Section V achieves the desired synchronization. Section VI contains the numerical simulations and discussions. Section VII compares our obtained results with previous literature. Section VIII concludes the article.

II. PRELIMINARIES

We here state a few preliminaries that will be used throughout the paper:

Definition 1: The Caputo's derivative of fractional order 'p' on function f(t) is given by:

$$cD_t^p f(y) = \frac{1}{\gamma(n-p)} \int_c^y \frac{f^n(x)}{(y-x)^{p-n+1}} dx$$

where $n-1 < p < n$ and $\gamma(p) = \int_0^\infty x^{p-1} e^{-x} dx$ is the Gamma function.

Property 1: The Caputo's fractional derivative is zero when f(t)=constant.

Property 2: The Caputo's fractional derivative satisfies the linear property:

$D^q[af(t) + bg(t)] = aD^q f(t) + bD^q g(t)$, where a and b are constants.

Lemma 1: Suppose $\Phi \in R$ be a continuously derivable function and $0 < q < 1$. Then, for any time $t \geq t_0$

$$\frac{1}{2} D^q \Phi(t)^2 \leq \Phi(t) D^q \Phi(t)$$

Lemma 2: For the equation

$$D^q K(t) \leq -b_0 K(t) + b_1$$

there exists a constant $t_1 > 0$ for which for all $t \epsilon (t_1, \infty)$ satisfies the condition

$$\|K(t)\| \leq \frac{2b_1}{b_0}$$

where $K(t)$ is state variable of the system and b_0, b_1 are non-negative constants.

Assumption: The Caputo's derivative of the unknown external disturbances are considered to be bounded throughout the paper i.e. $|D^q \Phi_i(t)| \leq |a_i|$ where $\Phi_i(t)$ are unknown external disturbances and $a_i > 0$ are positive constants.

III. SYSTEM DESCRIPTION

The modified fractional order jerk chaotic system is given by:

$$D^q x_{11}(t) = x_{12}$$
$$D^q x_{12}(t) = x_{13}$$
$$D^q x_{13}(t) = -b_1 x_{11} - x_{12} - b_2 x_{13} - f_3\big(x(t)\big) \quad (1)$$

where $b_1 = 1.5, b_2 = 0.35$ and $f_3(x(t))$ is defined by

$$f_3\big(x(t)\big) = \tfrac{1}{2}(v_1 - v_2)(|x_{11} + 1| - |x_{11} - 1|) + v_2 x_{11} \quad (2)$$

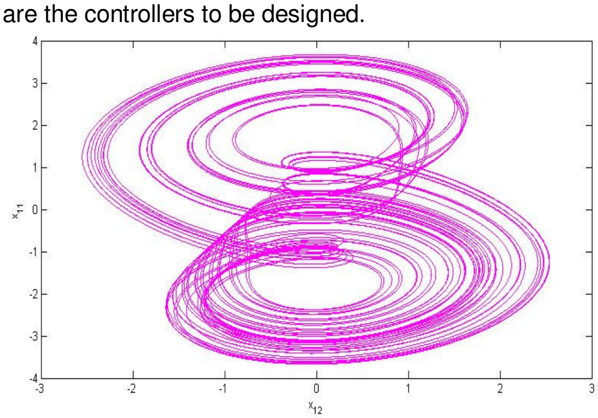
where $v_1 = 2.5, v_2 = 0.5$

For hybrid projective compound synchronization we need one scaling drive system and two base drive systems as in (1). The state variables of the master systems respectively are x_{1i}, x_{2i}, x_{3i} for $i = 1,2,3$ and initial conditions as $(1,1,1), (1.2, 0.6, 0.5), (1.3, 0.5, 0.4)$ respectively.

Next we consider the slave system as the modified fractional order jerk system in presence of external disturbances as:

$$D^q y_{11}(t) = y_{12} + \sin t + u_1$$
$$D^q y_{12}(t) = y_{13} + 5 \sin 4t + u_2 \quad (3)$$
$$D^q y_{13}(t) = -b_1 y_{11} - y_{12} - b_2 y_{13} - \frac{1}{2}(v_1 - v_2)(|y_{11} + 1| - |y_{11} - 1|) - v_2 y_{11} + 0.5 \sin 5t + u_3$$

where y_{1i} are the state variables of the system having initial condition as $(1.4, 0.4, 0.3)$, $\Phi_1 = \sin t, \Phi_2 = 5 \sin 4t, \Phi_3 = 0.5 \sin 5t$ are the disturbances, u_1, u_2, u_3 are the controllers to be designed.



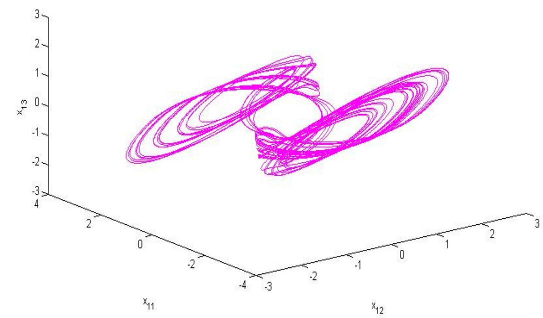


Fig. 1. Phase portrait of modified fractional-order Jerk system in different planes.

IV. PROBLEM FORMULATION

We first design a FODO based adaptive sliding m mode compound synchronization scheme to achieve the desired synchronization between four modified jerk system. We design the scheme for a non-linear FODO to estimate the unknown bounded disturbances present in the slave system. This scheme helps to increase the robustness of the system performance. We have proposed here a subsidiary variable $\Theta_i, i = 1,2,3$ for scheming the non-linear FODO of fractional order.

Therefore we have:

$$\Theta_1(t) = \Phi_1(t) - \sigma_1 y_{11}(t)$$
$$\Theta_2(t) = \Phi_2(t) - \sigma_2 y_{12}(t) \quad (4)$$
$$\Theta_3(t) = \Phi_3(t) - \sigma_3 y_{13}(t)$$

where $\sigma_1, \sigma_2, \sigma_3 > 0$ are positive constants to be determined. Considering Caputo's derivative of the above system and using (3), we get

$$D^{q1}\Theta_1(t) = D^{q1}\Phi_1(t) - \sigma_1(y_{12} + \Theta_1 + \sigma_1 y_{11}) - \sigma_1 u_1$$
$$D^{q2}\Theta_2(t) = D^{q2}\Phi_2(t) - \sigma_2(y_{13} + \Theta_2 + \sigma_2 y_{12}) - \sigma_2 u_2$$
$$D^{q3}\Theta_3(t) = D^{q3}\Phi_3(t) - \sigma_3\big(-b_1 y_{11} - y_{12} - b_2 - y_{13} - \tfrac{1}{2}(v_1 - v_2)(|y_{11}(t) + 1| - |y_{11}(t) - 1|) - v_2 y_{11}(t) + \Theta_3 + \sigma_3 y_{13}(t)\big) - \sigma_3 u_3 \quad (5)$$

The estimates of $\Theta_i(t) (i = 1,2,3)$ are:

$$D^{q1}\widehat{\Theta}_1(t) = -\sigma_1\big(y_{12}(t) + \sigma_1 y_{11}(t)\big) - \sigma_1 \widehat{\Theta}_1(t) - \sigma_1 u_1$$
$$D^{q2}\widehat{\Theta}_2(t) = -\sigma_2(y_{13}(t) + \sigma_2 y_{12}(t)) - \sigma_2 \widehat{\Theta}_2(t) - \sigma_2 u_2 \quad (6)$$
$$D^{q3}\widehat{\Theta}_3(t) = -\sigma_3(-b_1 y_{11}(t) - y_{12}(t) - b_2 y_{13}(t) - \frac{1}{2}(v_1 - v_2)(|y_{11}(t) + 1| - |y_{11}(t) - 1|) - v_2 y_{11}(t) + \sigma_3 y_{13}(t)) - \sigma_3 \widehat{\Theta}_3(t) - \sigma_3 u_3$$

where $\widehat{\Theta}_i(t)$ is the estimate of Θ_i

From (4) we have:

$$\widehat{\Phi}_1(t) = \widehat{\Theta}_1(t) + \sigma_1 y_{11}(t)$$
$$\widehat{\Phi}_2(t) = \widehat{\Theta}_2(t) + \sigma_2 y_{12}(t) \quad (7)$$
$$\widehat{\Phi}_3(t) = \widehat{\Theta}_3(t) + \sigma_3 y_{13}(t)$$

Error of the disturbance estimates can be stated as

$\widetilde{\Phi}_i(t) = \Phi_i - \widehat{\Phi}_i, i = 1,2,3.$

From Eqn. (4), we have

$$\widetilde{\Theta}_i(t) = \Theta_i(t) - \widehat{\Theta}_i(t) = \Phi_i(t) - \widehat{\Phi}_i(t) = \widetilde{\Phi}_i(t) \quad (8)$$

Using Eqns. (3) and (8), the Caputo's fractional derivatives in $\widetilde{\Theta}_i(t), (i = 1,2,3)$ can be written as:

$$D^{q_i}\widetilde{\Theta}_1(t) = -\sigma_1 \widetilde{\Theta}_1(t) + D^{q_i}\Phi_1(t)$$
$$D^{q_i}\widetilde{\Theta}_2(t) = -\sigma_2 \widetilde{\Theta}_2(t) + D^{q_i}\Phi_2(t) \quad (9)$$
$$D^{q_i}\widetilde{\Theta}_3(t) = -\sigma_3 \widetilde{\Theta}_3(t) + D^{q_i}\Phi_3(t)$$

To analyze the convergence of approximation disturbance error $\widetilde{\Phi}_i(t), (i = 1,2,3)$ we consider the Lyapunov function as:

$$V_{\Phi_i}(t) = \frac{1}{2}\widetilde{\Phi}_i^2(t) = \frac{1}{2}\widetilde{\Theta}_i^2(t), \forall\, i = 1,2,3 \quad (10)$$

Using Lemma 1, the Caputo's derivative of $V_{\Phi_i}(t)$ can be written as:

$$D^{q_i}V_{\Phi_i}(t) < \widetilde{\Theta}_i(t)D^{q_i}\widetilde{\Theta}_i(t) \quad (11)$$

After substituting (9) in (11), we get the following:

$$D^{q_i}V_{\Phi_i}(t) \le \widetilde{\Theta}_i(t)\left(-\sigma_i\widetilde{\Theta}_i(t) + D^{q_i}\Phi_i(t)\right)$$
$$\le -\sigma_i\left(\widetilde{\Theta}_i(t)\right)^2 + \widetilde{\Theta}_i(t)D^{q_i}\Phi_i(t) \quad (12)$$

Using Assumption 1 in Eqn. (12), we obtain:

$$D^{q_i}V_{\Phi_i}(t) \le -\sigma_i\widetilde{\Theta}_i^2(t) + \frac{1}{2}\widetilde{\Theta}_i^2(t) + \frac{1}{2}\zeta_i^2$$
$$\le -\left(\sigma_i - \frac{1}{2}\right)\widetilde{\Theta}_i^2(t) + \frac{1}{2}\zeta_i^2 \quad (13)$$
$$= C_0V_{\Theta_i}(t) + C_1$$

where $C_0 = 2\sigma_i - 1$ and $C_1 = \frac{1}{2}\zeta_i^2$.

If $\sigma_i > \frac{1}{2}$, then from (13) and Lemma 2, we have the following:

$$\left|V_{\Phi_i}(t)\right| \le \frac{\zeta_i^2}{2(\sigma_1 - 0.5)} \quad (14)$$

which implies

$$\left|\widetilde{\Phi}_i(t)\right| \le \sqrt{\frac{\zeta_i^2}{\sigma_1 - 0.5}} \quad (15)$$

From Eqn. (15), we have that the disturbance estimation error $\widetilde{\Phi}_i(t)$ is bounded above. Thus, for the external disturbances $\widetilde{\Phi}_i(t), (i = 1,2,3)$, the disturbance approximation error $|\widetilde{\Phi}_i(t)|$ satisfies $\left|\widetilde{\Phi}_i(t)\right| \le k_i$, where $k_i > 0$ is unknown positive constant. In reality, it is very difficult to determine the upper bounds $\left|\widetilde{\Phi}_i(t)\right|$ and therefore we introduce the estimated value $\widetilde{k}_i$ of $k_i (i = 1,2,3)$. Thus from the above analysis, the disturbance estimated error of the Modified Jerk system is bounded using the designed nonlinear FODO.

V. ADAPTIVE SLIDING MODE HYBRID PROJECTIVE COMPOUND SYNCHRONIZATION

We now define the hybrid projective compound synchronization error between identical fractional order modified jerk systems in presence of unknown bounded disturbance

$$e_{11}(t) = y_{11} - \alpha_1x_{11}(x_{21} + x_{31})$$
$$e_{12}(t) = y_{12} - \alpha_2x_{12}(x_{22} + x_{32}) \quad (16)$$
$$e_{13}(t) = y_{13} - \alpha_3x_{13}(x_{23} + x_{33})$$

Therefore, the error dynamics can be written as:

$$D^\alpha e_{11}(t) = D^\alpha y_{11} - \alpha_1D^\alpha x_{11}(x_{21} + x_{31}) - \alpha_1x_{11}(D^\alpha x_{21} + D^\alpha x_{31})$$
$$D^\alpha e_{12}(t) = D^\alpha y_{12} - \alpha_2D^\alpha x_{12}(x_{22} + x_{32}) - \alpha_2x_{12}(D^\alpha x_{22} + D^\alpha x_{32}) \quad (17)$$
$$D^\alpha e_{13}(t) = D^\alpha y_{13} - \alpha_3D^\alpha x_{13}(x_{23} + x_{33}) - \alpha_3x_{13}(D^\alpha x_{23} + D^\alpha x_{33})$$

Substituting the values of the derivatives we get:

$$D^\alpha e_{11}(t) = e_{12} + (x_{22} + x_{32})(\alpha_2x_{12} - \alpha_1x_{11}) - \alpha_1x_{12}(x_{21} + x_{31}) + \sin t + u_1$$

$$D^\alpha e_{12}(t) = e_{13} + (x_{23} + x_{33})(\alpha_3x_{13} - \alpha_2x_{12}) - \alpha_2x_{13}(x_{22} + x_{32}) + 5\sin 4t + u_2$$

$$D^\alpha e_{13}(t) = -b_1e_{11} - b_1\alpha_1x_{11}(x_{21} + x_{31}) - e_{12} + \alpha_2x_{12}(x_{22} + x_{32}) - b_2\alpha_3(x_{23} + x_{33}) - \frac{1}{2}(v_1 - v_2)(|y_{11} + 1| - |y_{11} - 1|) - v_2y_{11} + b_1\alpha_3\big(x_{11}(x_{23} + x_{33}) + x_{13}(x_{21} + x_{31})\big) + \alpha_3\big(x_{12}(x_{23} + x_{33}) + x_{13}(x_{22} + x_{32})\big) + 2b_2\alpha_3x_{13}(x_{23} + x_{33}) + \frac{1}{2}(v_1 - v_2)(|x_{11} + 1| - |x_{11} - 1|)(x_{23} + x_{33}) + \alpha_3v_2x_{11}(x_{23} + x_{33}) + \frac{1}{2}(v_1 - v_2)(|x_{21} + 1| - |x_{21} - 1| + |x_{31} + 1| - |x_{31} - 1|)x_{13} + \alpha_3v_2x_{13}(x_{21} + x_{31}) + 0.5\sin 5t + u_3 \quad (18)$$

To study the stability of fractional order synchronization error dynamical system, chose a linear sliding mode surface as

$$s_i(t) = e_{1i}(t), i = 1,2,3 \quad (19)$$

Taking the fractional derivative of (19), we get

$$D^\alpha s_i(t) = D^\alpha e_{1i}(t), i = 1,2,3 \quad (20)$$

Using the adaptive sliding mode approach, we design the controller as:

$$u_1(t) = -e_{12} - (x_{22} + x_{32})(\alpha_2x_{12} - \alpha_1x_{11}) - \alpha_1x_{12}(x_{21} + x_{31}) - \eta_1s_1 - \widehat{K}_1\mathrm{sign}(s_1(t)) - \widehat{\Phi}_1$$
$$u_2(t) = -e_{13} + (x_{23} + x_{33})(\alpha_3x_{13} - \alpha_2x_{12}) - \alpha_2x_{13}(x_{22} + x_{32}) - \eta_2s_2 - \widehat{K}_2\mathrm{sign}(s_2(t)) - \widehat{\Phi}_2 \quad (21)$$
$$u_3(t) = b_1e_{11} + b_1\alpha_1x_{11}(x_{21} + x_{31}) + e_{12} + \alpha_2x_{12}(x_{22} + x_{32}) + b_2\alpha_3(x_{23} + x_{33}) + \frac{1}{2}(v_1 - v_2)(|y_{11} + 1| - |y_{11} - 1|) + v_2y_{11} - b_1\alpha_3\big(x_{11}(x_{23} + x_{33}) + x_{13}(x_{21} + x_{31})\big) - \alpha_3\big(x_{12}(x_{23} + x_{33}) + x_{13}(x_{22} + x_{32})\big) - 2b_2\alpha_3x_{13}(x_{23} + x_{33}) - \frac{1}{2}(v_1 - v_2)(|x_{11} + 1| - |x_{11} - 1|)(x_{23} + x_{33}) - \alpha_3v_2x_{11}(x_{23} + x_{33}) - \frac{1}{2}(v_1 - v_2)(|x_{21} + 1| - |x_{21} - 1| + |x_{31} + 1| - |x_{31} - 1|)x_{13} - \alpha_3v_2x_{13}(x_{21} + x_{31}) - \eta_3s_3 - \widehat{K}_3\mathrm{sign}(s_3(t)) - \widehat{\Phi}_3$$

where $sign(s) = \frac{|s|}{s}$ and $\eta_i > 0$ are constants. $\widehat{K}_i$ is estimated value of K_i ($\forall\, i = 1,2,3$) are uptaded by

$$D^\alpha\widehat{K}_1 = \gamma_1(|s_1(t)| - \widehat{K}_1)$$
$$D^\alpha\widehat{K}_2 = \gamma_2(|s_2(t)| - \widehat{K}_2) \quad (22)$$
$$D^\alpha\widehat{K}_3 = \gamma_3(|s_3(t)| - \widehat{K}_3)$$

where $\gamma_i > 0, i = 1,2,3$ are constants. Substituting the controllers we get the error dynamical system as

$$D^\alpha e_{11}(t) = -\eta_1s_1 - \widehat{K}_1\mathrm{sign}(s_1(t)) + \Phi_1 - \widehat{\Phi}_1$$
$$D^\alpha e_{12}(t) = -\eta_2s_2 - \widehat{K}_2\mathrm{sign}(s_2(t)) + \Phi_2 - \widehat{\Phi}_2 \quad (23)$$
$$D^\alpha e_{13}(t) = -\eta_3s_3 - \widehat{K}_3\mathrm{sign}(s_3(t)) + \Phi_3 - \widehat{\Phi}_3$$

The sliding surface $s_i(t)$ is stable and bounded for the designed controllers:

$$|s_1(t)| \le B \quad (24)$$

where B is an unknown constant parameter.

Using Eqn. (19) and (24), we get the error system as bounded and stable.

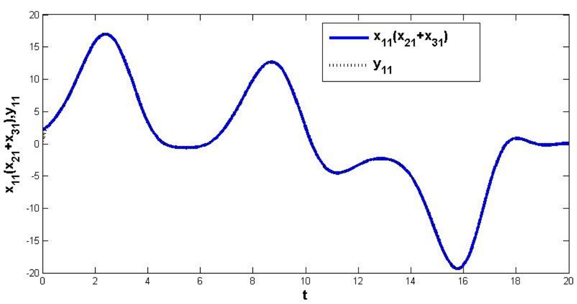
$$e_{1i}(t) \le B, i = 1,2,3 \tag{25}$$

We now summarize the above in the form of the following theorem:

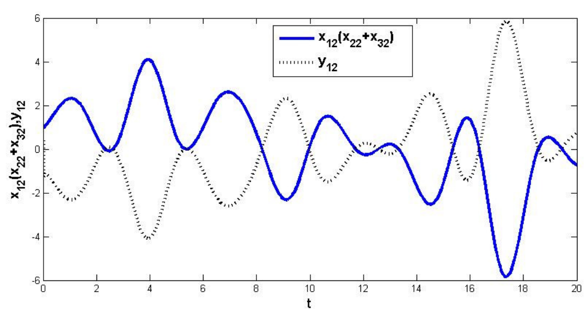
Theorem 1: For the hybrid projective compound synchronization error system (18) with $0 < \alpha_i < 1$, if the sliding mode surface is designed according to (19) and external unknown bounded disturbance is approximated by using the scheme non-linear FODO (6) and (7). Then, HPCS error is bounded and stable under the adaptive sliding mode control scheme as (21) and (22).

Proof: The Lyapunov function $V(t)$ is selected for the convergence of synchronization error $e(t)$ as:

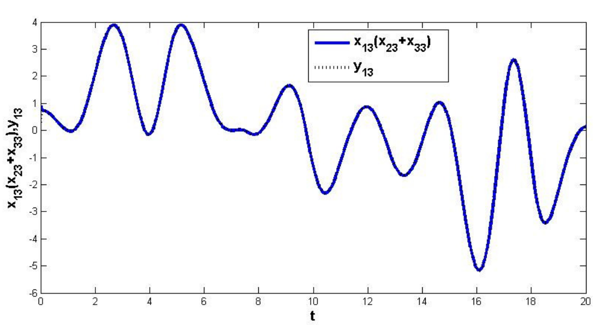
$$V(t) = \sum_{i=1}^{3} \frac{1}{2} s_i(t)^2 + \sum_{i=1}^{3} \frac{1}{2} \widetilde{\Phi}_i(t)^2 \sum_{i=1}^{3} \frac{1}{2} \left(\frac{1}{\sqrt{\gamma_i}} (\widehat{K}_i - K_i)\right)^2 \tag{26}$$



(a)



(b)



(c)

Fig. 2. Synchronized trajectories.

Using property 2 in Eqn. (26), we get

$$D^{\alpha_i} V(t) = \frac{1}{2}\left(\sum_{i=1}^{3} D^{\alpha_i} s_i(t)^2 + \sum_{i=1}^{3} D^{\alpha_i} \widetilde{\Phi}_i(t)^2 \sum_{i=1}^{3} D^{\alpha_i} \left(\frac{1}{\sqrt{\gamma_i}} (\widehat{K}_i - K_i)\right)^2\right) \tag{27}$$

using $\widetilde{K}_i = \widehat{K}_i - K_i$ and Lemma 1 in Eqn. (27) can be written as

$$D^{\alpha_i} V(t) \le \sum_{i=1}^{3} \frac{1}{2} s_i(t) D^{\alpha_i} s_i(t) + \sum_{i=1}^{3} \frac{1}{2} D^{\alpha_i} \widetilde{\Phi}_i^{\,2}(t) + \sum_{i=1}^{3} \frac{1}{\sqrt{\gamma_i}} \widetilde{K}_i D^{\alpha_i} \left(\frac{1}{\sqrt{\gamma_i}} \widetilde{K}_i\right) \tag{28}$$

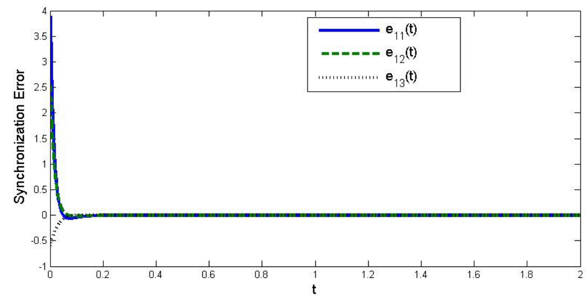


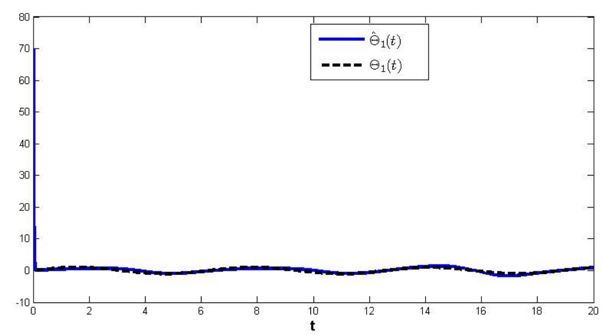
Fig. 3. Synchronization Error.

On applying property 2 in the Eqn. (28), we obtain

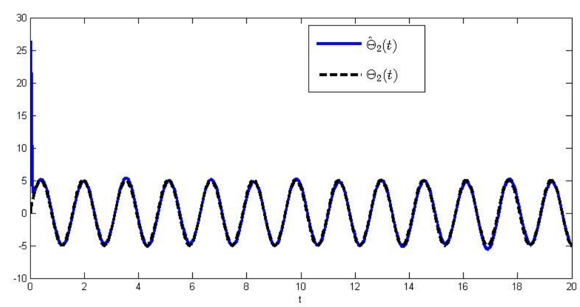
$$D^{\alpha_i} V(t) \le \sum_{i=1}^{3} s_i(t) D^{\alpha_i} s_i(t) + \sum_{i=1}^{3} \frac{1}{2} D^{\alpha_i} \widetilde{\Phi}_i(t)^2 + \sum_{i=1}^{3} \gamma_i^{-1} \widetilde{K}_i D^{\alpha_i} \widetilde{K}_i \tag{29}$$

Using (19) and substituting (18) into (29), we obtain

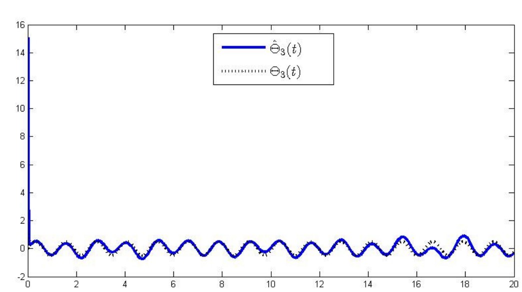
$$D^{\alpha_i} V(t) \le \sum_{i=1}^{3} s_i(t) \left(-\eta_i s_i + \widetilde{\Phi}_i(t) - \widehat{K}_i \mathrm{sign}(s_i(t))\right) + \sum_{i=1}^{3} \frac{1}{2} D^{\alpha_i} \widetilde{\Phi}_i^{\,2}(t) + \sum_{i=1}^{3} \gamma_i^{-1} \widetilde{K}_i D^{\alpha_i} \widetilde{K}_i \tag{30}$$



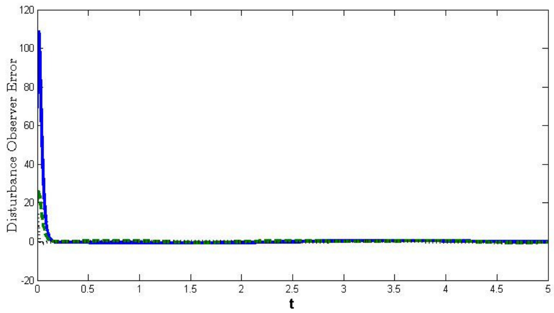
(a)



(b)



(c)



(d)

Fig. 4. Represents the Synchronized Disturbance Observers and its error is represented by (d).

Applying Property 1 and using $\widetilde{K}_i = \widehat{K}_i - K_i, (i = 1,2,3)$, we obtain

$$D^{\alpha_i}\widetilde{K}_i = D^{\alpha_i}\widehat{K}_i \quad (31)$$

where K_j is a constant parameter

Using (22), (30) and (31), we have

$$\sum_{i=1}^{3} \gamma_i^{-1}\widetilde{K}_i D^{\alpha_i}\widetilde{K}_i = \sum_{i=1}^{3} \widetilde{K}_i\left(|s_i(t)| - \widehat{K}_i\right)$$

$$= \sum_{i=1}^{3} \widetilde{K}_i|s_i(t)| - \sum_{i=1}^{3} \widetilde{K}_i\widehat{K}_i$$

$$\leq \sum_{i=1}^{3} \widetilde{K}_i|s_i(t)| - \frac{1}{2}\sum_{i=1}^{3} \widetilde{K}_i^{\,2} + \frac{1}{2}\sum_{i=1}^{3} K_i^{\,2} \quad (32)$$

After substituting (32) into (30), we get

$$D^{\alpha_i}V(t) \leq \sum_{i=1}^{3} s_i(t)\left(\eta_i s_i + \widetilde{\Phi}_i(t) - \widehat{K}_i \mathrm{sign}(s_i(t))\right) + \frac{1}{2}\sum_{i=1}^{3} D^{\alpha_i}\widetilde{\Phi}_i^{\,2}(t) + \sum_{i=1}^{3} \widetilde{K}_i|s_i(t)| - \frac{1}{2}\sum_{i=1}^{3} \widetilde{K}_i^{\,2} + \frac{1}{2}\sum_{i=1}^{3} K_i^{\,2} \quad (33)$$

Eqn. (33) can be rewritten as

$$D^{\alpha_i}V(t) \leq -\sum_{j=1}^{3} \eta_i s_i^{\,2}(t) + \sum_{i=1}^{3} |s_i(t)|\widetilde{\Phi}_i + \sum_{i=1}^{3} \widetilde{K}_i|s_i(t)| - \sum_{i=1}^{3}\frac{1}{2}\widetilde{K}_i^{\,2} + \frac{1}{2}\sum_{i=1}^{3} K_i^{\,2} - \sum_{i=1}^{3} \widehat{K}_i|s_i(t)| + \frac{1}{2}\sum_{i=1}^{3} D^{\alpha_i}\widetilde{\Phi}_i^{\,2}(t) \quad (34)$$

Eqn. (34) can be written as follows using $|\widetilde{\Phi}_i(t)| < K_i$ and $\sum_{i=1}^{3} \widetilde{K}_i|s_i(t)| - \sum_{i=1}^{3} \widehat{K}_i|s_i(t)| = -\sum_{i=1}^{3} K_i|s_i(t)|$.

$$D^{\alpha_i}V(t) \leq -\sum_{j=1}^{3} \eta_i s_i^{\,2}(t) - \sum_{i=1}^{3}\frac{1}{2}\widetilde{K}_i^{\,2} + \sum_{i=1}^{3}\frac{1}{2}K_i^{\,2} + \frac{1}{2}\sum_{i=1}^{3} D^{\alpha_i}\widetilde{\Phi}_i^{\,2}(t) \quad (35)$$

From Eqn. (13) and (35), we have

$$D^{\alpha_i}V(t) \leq -\sum_{i=1}^{3} \eta_i s_i^{\,2}(t) - \sum_{i=1}^{3}\frac{1}{2}\widetilde{K}_i^{\,2} + \sum_{i=1}^{3}\frac{1}{2}K_i^{\,2} + \sum_{i=1}^{3} -\left(\sigma_i - \frac{1}{2}\right)\widetilde{\Phi}_i^{\,2} + \sum_{i=1}^{3}\frac{1}{2}\xi_i^{\,2}$$

$$\leq -C_2V(t) + C_3 \quad (36)$$

where $C_2 = \min(2\eta_i, 1, 2\eta_i - 1)$ and $C_3 = \sum_{i=1}^{3}\frac{1}{2}\xi_i^{\,2} + \sum_{i=1}^{3}\frac{1}{2}K_i^{\,2}$.

On selecting the value of parameters $\eta_i > 0$ and $\sigma_i > 0.5$, we have the error bounded. Using Lemma 2 in (36), we get

$$|V(t)| \leq \frac{2C_3}{C_2} = \frac{\sum_{i=1}^{3}\xi_i^{\,2} + \sum_{i=1}^{3}K_i^{\,2}}{C_2} \quad (37)$$

Eqn. (37) implies that

$$\|s(t)\| \leq \sqrt{\frac{2\left(\sum_{i=1}^{3}\xi_i^{\,2} + \sum_{i=1}^{3}K_i^{\,2}\right)}{C_2}} \quad (38)$$

From Eqns. (37) and (38), it is clear that the sliding surface $s_i(t)$ and synchronization error $e_i(t)$ are bounded as $t \to \infty$. Thus error dynamical system (18) is bounded and stable implying that the synchronization between master systems and slave system has been achieved.

VI. NUMERICAL SIMULATIONS AND DISCUSSIONS

For numerical simulations we have considered $\left(\widehat{\Theta}_1(0), \widehat{\Theta}_2(0), \widehat{\Theta}_3(0)\right) = (.1, .1, .1)$, $\left(\widehat{K}_1(0), \widehat{K}_2(0), \widehat{K}_3(0)\right) = (.1, .1, .1)$, the designed parameters, $(\sigma_1, \sigma_2, \sigma_3) = (50,50,50)$, $(\gamma_1, \gamma_2, \gamma_3) = (.1, .1, .1)$ and $(\eta_1, \eta_2, \eta_3) = (50,50,50)$. The disturbance here is taken as $\Phi_1 = \sin t$, $\Phi_2 = 5\sin 4t$, $\Phi_3 = 0.5\sin 5t$.

Fig. 2 shows the synchronized trajectories between different state variables. The synchronization errors converging to zero are shown in Fig. 3 for initial conditions $(e_{11}, e_{12}, e_{13}) = (3.9, 2.6, -0.6)$ and Fig. 4 shows the FODO results for disturbance observer result.

VII. COMPARISON OF GIVEN SYNCHRONIZATION WITH PREVIOUS PUBLISHED LITERATURE

Here adaptive sliding mode technique has been used to achieve the synchronization. When we compare our results with the previously published ones, we find that our results were much better and achieved synchronization error at t = 0.1 sec (approx.). In Khan and Trikha (2019) [10] author studies compound difference anti-synchronization and achieves the synchronization error at t = 5 sec. In Prajapati *et al.*, (2018) [15] author studies multi-switching compound synchronization error at t = 3 sec. In Sun *et al.*, (2019) [17] author studies modifies compound synchronization and achieves the synchronization error converge to zero at t = 2.5 sec. Hence we may conclude that our technique provided much more efficient and better results.

VIII. CONCLUSION

In this article hybrid projective compound synchronization has been achieved among four identical fractional order chaotic systems. We have constructed a non-linear FODO to estimate the unknown disturbance on the identical modified fractional order jerk system. A sliding mode control technique has been employed to achieve the desired synchronization in presence of external unknown bounded disturbance.

Finally on comparing our results with published literature, since our error trajectories synchronize in lesser time implying the superiority of our results.

Our future studies would include applying this synchronization technique in area of secure communication and image encryption.

ACKNOWLEDGMENT

The second and fourth author are funded by Junior Research Fellowship of University Grant Commission (Ref. No: 19/06/2016(i)EU-V) and C.S.I.R. (09/466(0189)/2017-EMR-I) respectively, India.

Conflict of Interest. Nil.

REFERENCES

[1]. Hilfer, R., Butzer, P. L., & Westphal, U. (2000). An introduction to fractional calculus. *Appl. Fract. Calc. Phys., World Scientific*, 1-85.

[2]. Das, S., & Yadav, V. K. (2017). Stability analysis, chaos control of fractional order Vallis and El-Nino systems and their synchronization. *IEEE/CAA Journal of Automatica Sinica*, *4*(1), 114-124.

[3]. Dongmo, E. D., Ojo, K. S., Woafo, P., & Njah, A. N. (2018). Difference synchronization of identical and nonidentical chaotic and hyperchaotic systems of different orders using active backstepping design. *Journal of Computational and Nonlinear Dynamics*, *13*(5), 1-9.

[4]. Fang, C. S., & Hou, Y. Y. (2016). Sliding mode controller design for chaos synchronization of Rikitake chaotic systems. In *2016 IEEE International Conference on Control and Robotics Engineering (ICCRE)* (pp. 1-4). IEEE.

[5]. He, J., Chen, F., & Lei, T. (2018). Fractional matrix and inverse matrix projective synchronization methods for synchronizing the disturbed fractional-order hyperchaotic system. *Mathematical Methods in the Applied Sciences*, *41*(16), 6907-6920.

[6]. Hilfer, R. (Ed.). (2000). *Applications of fractional calculus in physics*, *35*(12), 87-130. Singapore: World scientific.

[7]. Khan, A., & Bhat, M. A. (2016). Hybrid projective synchronization of fractional order chaotic systems with fractional order in the interval (1, 2). *Nonlinear Dyn Syst Theory*, *16*(4), 350-365.

[8]. Khan, A. (2017). Increased and reduced order synchronisations between 5D and 6D hyperchaotic systems. *Indian Journal of Industrial and Applied Mathematics*, *8*(1), 118-131.

[9]. Khan, A., Khattar, D., & Prajapati, N. (2017). Multiswitching compound antisynchronization of four chaotic systems. *Pramana*, *89*(6), 1488-1496.

[10]. Khan, A., & Trikha, P. (2019). Compound difference anti-synchronization between chaotic systems of integer and fractional order. *SN Applied Sciences*, *1*(7), 1-13.

[11]. Khan, A., & Tyagi, A. (2017). Fractional order disturbance observer based adaptive sliding mode synchronization of commensurate fractional order genesio-tesi system. *AEU- International Journal of Electronics and Communications, 82, 346-357.*

[12]. Luo, A. C. (2009). A theory for synchronization of dynamical systems. *Communications in Nonlinear Science and Numerical Simulation*, *14*(5), 1901-1951.

[13]. Pecora, L. M., & Carroll, T. L. (1990). Synchronization in chaotic systems. *Phys. Rev. Lett., 64*, 821-824.

[14]. Podlubny, I. (1998). *Fractional differential equations: an introduction to fractional derivatives, fractional differential equations, to methods of their solution and some of their applications*. Elsevier.

[15]. Prajapati, N., Khan, A., & Khattar, D. (2018). On multi switching compound synchronization of non identical chaotic systems. *Chinese Journal of Physics, 56*(4),1656-1666.

[16]. Singh, A. K., Yadav, V. K., & Das, S. (2016). Comparative study of synchronization methods of fractional order chaotic systems. *Nonlinear Engineering, 5*(3), 185-192.

[17]. Sun, J., Li, N., Wang, Y., & Wang, W. (2019). A novel chaotic system and its modified compound synchronization. *Fundamental Informaticae, 164*(2-3), 259-275.

[18]. Tavassoli, M. H., Tavassoli, A., & Rahimi, M. O. (2013). The geometric and physical interpretation of fractional order derivatives of polynomial functions. *Differential geometry-Dynamical systems*, *15*, 93-104.

[19]. Wei, Q., Wang, X. Y., & Hu, X. P. (2014). Hybrid function projective synchronization in complex dynamical networks. *AIP Advances*, *4*(2), 027128.

[20]. Yang, L., He, W., & Liu, X. (2011). Synchronization between a fractional order system and an integer order system. *Computers & Mathematics with Applications, 62*(12), 4708-4716.

[21]. Zhang, H. Liu, D., & Wang Z. (2009). Controlling chaos: suppression, synchronization and chaotification. *Springer Science & Business Media.*

[22]. Zhang, Q. Xiao, J. Zhang, X., & Cao, D. (2017). Dual projective synchronization between integer-order and fractional-order chaotic systems. *Optik-International Journal for Light and Electron Optics, 141,* 90-98

[23]. Wu, Z., Zhang, X., & Zhong, X. (2019). Generalized chaos synchronization circuit simulation and asymmetric image encryption. *IEEE Access*, *7*, 37989-38008.

[24]. Skardal, P. S., Sevilla-Escoboza, R., Vera-Avila, V., & Buldu, J. M. (2017). Optimal phase synchronization in networks of phase-coherent chaotic oscillators.Chaos: *An Interdisciplinary Journal of Nonlinear Science 27*(1), 1-9.

How to cite this article: Khan, Ayub, Nasreen, Chaudhary, Harindri, Trikha, Pushali and Jahanzaib, Lone Seth (2020). Disturbance Observer Based Adaptive Sliding Mode Hybrid Projective Compound Synchronization. *International Journal on Emerging Technologies*, *11*(1): 364–369.