International Journal on Emerging Technologies **11**(2): 610-617(2020)

ISSN No. (Print): 0975-8364
ISSN No. (Online): 2249-3255

Dusty Time Fractional MHD Flow of a Newtonian Fluid through a Cylindrical Tube

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(Received 31 December 2019, Revised 25 February 2020, Accepted 27 February 2020)
(Published by Research Trend, Website: www.researchtrend.net)

ABSTRACT: In this paper, time fractional flow of a Newtonian fluid containing dust particles through a uniform cylindrical tube in presence of uniform magnetic field placed along meridian axis is discussed. The implication of time fractional order differential equations in flow problems and some benefits of fractional order differential equations are highlighted. The Adomian Decomposition Method (ADM) is used in the process to get an approximate solution to the proposed problem. The difference between the effects of fractional order and integer order of the differential equations and also, the effects of some important parameters on the flow system are discussed with the help of graphs and tables. The convergence of the method is also tested. It has been observed that the fractional order differential equation can reveal many things like the increase in dust particle velocity as magnetic field increases for fractional order derivatives, whereas, no noticeable change in dust particle velocity with change in magnetic field for integer order derivatives are observed, implying that fractional order differential equations are more sensitive in comparison to the classical/integer order differential equations.

Keywords: Time-fractional order Navier-Stokes equation, Adomian Decomposition Method (ADM), Magnetohydrodynamics (MHD).

Abbreviations: ADM, Adomian Decomposition Method; MHD, Magnetohydrodynamics.

I. INTRODUCTION

In the field of biomedical sciences including blood flow like behaviour of blood flow in presence of magnetic field etc. as well as in the field of mechanical engineering like coolant theory etc. fractional Navier-Stokes equations have played a very important role. There are several methods to solve fractional Navier-Stokes equations. But out of all those methods, Adomian decomposition method (ADM) is one of the best methods.

In this work, the famous Adomian decomposition method (ADM) is used to solve the proposed fractional differential equations. Momani and Odibat (2006) used ADM to obtain an analytical solution of time fractional Navier-Stokes equation [1]. Hashim (2006) applied the ADM to solve both linear and nonlinear boundary value problems for fourth order integro-differential equations [2]. Abassy (2010) introduced a refined version of ADM which he called it the improved ADM, he claimed that his improved method was better than the original ADM [3]. Sanchez Cano (2011) used the ADM together with some properties of nested integrals to obtain a solution to a class of non linear ordinary differential equations and a coupled system [4]. Hongjun *et al*., (2011) studied the mathematical model of laminar boundary layer problem in hydrodynamics compared to Navier-Stokes equation which is nothing but a second order nonlinear partial differential equations which is solved with analytical approximation by using ADM [5]. Abdelrazec and Pelinovsky (2011) proved the convergence of the ADM for an initial value problem, he checked the convergence rate of the ADM by applying in the nonlinear Schrodinger equation [6]. Duan *et al*., (2012) reviewed the ADM and its applications to fractional differential equations [7]. Bougoffa used the ADM for solving a moving boundary problem arising from the diffusion of oxygen in absorbing tissue [8]. Agom and Badmus (2015) studied the correlation of ADM and finite difference methods to solve non homogeneous boundary value problem [9]. El-Borai *et al*., (2015) used the ADM to solve a system of fractional partial differential equation which has numerous applications in many fields of science [10]. Nhawu *et al*., (2016) used the ADM for numerical solution of first order differential equations [11]. Hosseini and Nasabzadeh (2016) studied the convergence of ADM [12]. Gonzalez-Gaxiola and Bernal-Jaquez (2017) used ADM to solve the nonlinear partial differential equation representing a model for tumour growth under medical treatment [13]. Mungkasi & Dheno (2017) used the ADM to solve the gravity wave equations [14].

Further, the flow with dust particles is one of the most realistic and important phenomena in any flow system. Rukmangadachari (1979) studied the unsteady laminar flow of a dusty viscous incompressible fluid through the annular tube formed by two coaxial circular cylinders [15]. Maitra (1997) studied the unsteady dusty fluid flow through a circular cylinder with time varying pressure gradient [16]. Attia (2011) studied the magnetohydrodynamics (MHD) flow of a dusty fluid through a circular pipe considering the Hall effect [17]. Gireesha *et al*., (2012) studied the geometry of the unsteady motion of a dusty fluid through porous media in a uniform pipe [18]. Attia *et al*., (2014) studied the unsteady magnetohydrodynamics (MHD) flow and heat transfer of a dusty electrically conducting fluid between two infinite horizontal plates [19]. Kumar *et al.*, (2015) studied a fractional model of Navier-Stokes equation to study the unsteady flow of a viscous fluid [20]. Shah *et al.*, (2016) discussed the effects of fractional order and magnetic field on the blood flow in cylindrical domains [21]. Hamid *et al.*, (2019) also studied fractional order unsteady flow of nanofluids [22]. Kumar *et al.*, (2019) discussed system of time fractional linear and nonlinear differential equations [23]. Motivated by the above mentioned works, this paper is focused on solving a fractional order differential equations arising from a fluid flow in uniform cylindrical tube using ADM under certain initio-boundary conditions. Thus, the axisymmetric flow of a Newtonian fluid through a uniform cylindrical tube in presence of magnetic field is being studied in this work. Also, it is considered that the fluid contains impurities in the form of dust particles. A flow model for this situation in the form of fractional Navier-Stokes equation is developed. Since it is not possible to obtain the general solution of Navier-Stokes equation easily, so certain assumptions are considered and an approximate solution of our fractional Navier-Stokes equation is obtained. In doing so, Adomian Decomposition Method (ADM) is used, to obtain an approximate solution. After successfully checking the convergence of the solution, graphs of the different parameters involved are obtained and effects of different parameters are highlighted in a table. This paper aims at highlighting a more general case of dusty two phase fluid flow system as compared to that considered by Shah *et al.,* (2016) [21], by considering different time fractional orders for fluid flow and dust particle flow equations separately and also by considering the volume fraction of the dust particles. As a result, it can be clearly found that the dust particle flow pattern can be traced easily if fractional order value is considered.

A. Riemann-Liouville (R-L) Fractional Integral

Before defining R-L fractional integral, we first give the following definition.

Definition: A real function $\boldsymbol{g(t)}, \boldsymbol{t} > 0$ is said to be in the space $\boldsymbol{C_\lambda}, \boldsymbol{\lambda} \in \boldsymbol{R}$ *(*set of real nos.) if there exists a real number $\boldsymbol{q}\,(> \boldsymbol{\lambda})$ such that $\boldsymbol{g(t)} = \boldsymbol{t^q g_1(t)}$, where $\boldsymbol{g_1(t)} \in \boldsymbol{C}[\boldsymbol{0}, \infty)$ and is said to be in the space $\mathbf{C}_\lambda^\mathbf{n}$ *iff* $\boldsymbol{g^{(n)}} \in \boldsymbol{C_\lambda}$, $\boldsymbol{n} \in \boldsymbol{N}$ *(*set of natural nos.).

R-L fractional integral: The Riemann-Liouville fractional integral operator of order $\boldsymbol{\beta} \geq \boldsymbol{0}$, of a function $\boldsymbol{g} \in \boldsymbol{C_\lambda}$, $\boldsymbol{\lambda} \geq -\mathbf{1}$, is defined as

$$\boldsymbol{J^\beta g(t)} = \frac{\mathbf{1}}{\boldsymbol{\Gamma(\beta)}}\int_{\mathbf{0}}^{\boldsymbol{t}} \boldsymbol{(t-z)^{\beta-1} g(z) dz}, \quad \boldsymbol{\beta} > 0, t > 0$$

$$\boldsymbol{J^0 g(t)} = \boldsymbol{g(t)}$$

B. Caputo Fractional Derivative

The fractional derivative of a function $g(t)$ in Caputo sense is defined as

$$D^\beta g(t) = J^{p-\beta} D^p g(t)$$
$$= \frac{1}{\Gamma(p-\beta)}\int_0^t (t-z)^{p-\beta-1} g^{(p)}(z) dz$$

for $p-1 < \beta \leq p,\ p \in N,\ t > 0,\ g \in C_{-1}^p$.

Properties: *Let* $f, g \in C_\lambda^p, \lambda \geq -1, c, d \in R, p-1 < \alpha \leq p, p \in N, D^\alpha f(t)$ *and* $D^\alpha g(t)$ *exist, then*

1. $D^\alpha\big(cf(t) + dg(t)\big) = cD^\alpha f(t) + dD^\alpha g(t)$
2. $D^\alpha J^\alpha f(t) = f(t)$
3. $J^\alpha D^\alpha f(t) = f(t) - \sum_{k=0}^{p-1} f^{(k)}(0)\frac{t^k}{k!}$, $t > 0$

C. Basic equations used

The basic equations for the flow of an incompressible fluid in a porous cylindrical domain is given by

$$\frac{D^\alpha \vec{q}}{Dt^\alpha} = -\frac{1}{\rho}\nabla p + \nu\nabla^2\vec{q} - \frac{\vec{J}\times\vec{B}}{1-\phi_p}\vec{q} + \frac{\kappa N_0}{1-\phi_p}\left(\vec{q} - \vec{q_p}\right) - \frac{\nu}{(1-\phi_p)K}\vec{q}$$

$$m\frac{D^\beta \vec{q}_p}{Dt^\beta} = \kappa\left(\vec{q} - \vec{q}_p\right)$$

where $\vec{J} = \sigma\left(\vec{E} + \vec{q}\times\vec{B}\right)$, $\vec{J}$ is current density, σ is electrical conductivity, $\vec{E}$ is electricfield, $\vec{B}$ is magnetic induction, $\vec{q}$ is the fluid velocity, $\vec{q}_p$ is the dust particle velocity, ν is kinematic coefficient of viscosity, ϕ_p is volume fraction of the dust particle, $\rho$is the density of the fluid particle, $p$is the pressure, $t$is the time, $\alpha$and $\beta$are time fractional derivatives, K is the medium's permeability, N_0 is the number density of dust particle, $\kappa$is the Stoke's resistance.

In our case, $\frac{\partial}{\partial\theta} \equiv 0$ (due to axi-symmetric flow) and also the velocity components along radial and meridian direction are also taken to be zero due to axial flow consideration i.e. if $q = (q_r, q_\theta, q_z)$ then $q_r = q_\theta = 0$ and thus, we have $q = q_z(r, z, t)$.

D. Fundamental assumptions

The following assumptions are considered in this work:

– The fluid under consideration is an incompressible Newtonian fluid.

– The flow is considered axisymmetric as well as axial in nature.

– Only the time fractional derivative of Navier-Stokes equations is taken into consideration.

– The magnetic field is applied along the meridian axis i.e. for our case $B = (0, B_0, 0)$ where $B_0$is the component of magnetic field in meridian axis, considered as uniform.

– We consider the voltage difference along the rear ends of the tube to be very low and as a result, the electric field is neglected i.e. for our case $\vec{E} = 0$.

E. Analysis of ADM

To understand ADM, let us see a general description of the problem as follows. First of all, let us consider a differential equation

$Fu(t) = g(t)$

where F represents a general nonlinear ordinary or partial differential operator with both linear and nonlinear terms. The linear term is decomposed into $L + R$, where L is invertible and R is the remainder of the linear operator. Also, we may take L as the highest order derivative so as to avoid difficult integrations involving complicated Green's functions. Thus, the equation may be written as

$$Lu + Ru + Nu = g \qquad (1)$$

where Nu represents the nonlinear terms. Solving for Nu, we get

$$Lu = g - Ru - Nu \qquad (2)$$

Since L is invertible, so its inverse L^{-1} exist and therefore, operating L^{-1}on (2), we obtain

$$L^{-1}Lu = L^{-1}g - L^{-1}Ru - L^{-1}Nu \qquad (3)$$

If (3) represents an initial value problem, the L^{-1} will denote a definite integral from t_0 to t. If L is a second order operator then L^{-1} will represent a two fold integration operator and we get,

$L^{-1}Lu = u - u(t_0) - (t - t_0)u'(t_0)$

We use indefinite integrations for boundary value problems and the constants are evaluated from the given boundary conditions. And thus (3) gives

$$u = A + Bt + L^{-1}g - L^{-1}Ru - L^{-1}Nu \quad (4)$$

The decomposition method assumes the series solution of u in the form

$$u = \sum_{n=0}^{\infty} u_n \quad (5)$$

where, the first term of the series $u_0$represents $A + Bt + L^{-1}g$. The nonlinear part Nuwill be decomposed in the form $Nu = \sum_{n=0}^{\infty} A_n$ (6)

where $A_n$are called the Adomian polynomials generated for each nonlinearity so that A_0 depends only on u_0 A_1 depends only on u_0 and u_1, A_2 depends on u_0, u_1, u_2 and etc.The Adomian polynomials can be obtained from the formula

$$A_n = \frac{1}{n!}\frac{d^n}{dt^n}\left[h\left(\sum_{n=0}^{\infty} \mu^n u_n\right)\right]_{\mu=0},\ n = 0,1,2, \quad (7)$$

Thus, we obtain

$$A_0 = h(u_0)$$

$$A_1 = u_1 \frac{d}{dt} h(u_0)$$

$$A_2 = u_2 \frac{d}{dt} h(u_0) + \frac{1}{2!} u_1^2 \frac{d^2}{dt^2} h(u_0)$$

$$A_3 = u_3 \frac{d}{dt} h(u_0) + u_1 u_2 \frac{d^2}{dt^2} h(u_0) + \frac{1}{3!} u_1^3 \frac{d^3}{dt^3} h(u_0)$$

and so on.

where h is the non linear term in (1).

Thus, Eqn. (4) can be written as

$$\sum_{n=0}^{\infty} u_n = u_0 - L^{-1}R\sum_{n=0}^{\infty} u_n - L^{-1}\sum_{n=0}^{\infty} A_n \quad (8)$$

Consequently, we can write

$$u_1 = -L^{-1}Ru_0 - L^{-1}A_0$$
$$u_2 = -L^{-1}Ru_1 - L^{-1}A_1 \quad (9)$$
$$u_{n+1} = -L^{-1}Ru_n - L^{-1}A_n$$

So, in practice, the n^{th} term approximation of the solution is given by

$$\psi_n = \sum_{k=0}^{n-1} u_k \quad (10)$$

where $u(x,t) = \lim_{n\to\infty} \psi_n(x,t) = \sum_{k=0}^{\infty} u_k(x,t)$ (11)

Result: Let N be an operator from a Hilbert space H into H and u be the exact solution of (1). $\sum_{n=0}^{\infty} u_n$ which is obtained by (9) converges to u if there exists $0 \le \alpha < 1$ such that $\|u_{k+1}\| \le \alpha\|u_k\|$ where $\|.\|$ is the supremum norm. [12].

Definition: For every $n \in N \cup \{0\}$, we define [12]

$$\alpha_n = \begin{cases} \dfrac{\|u_{n+1}\|}{\|u_n\|}, & \|u_n\| \neq 0 \\ 0, & \|u_n\| = 0 \end{cases}$$

II. FORMULATION OF THE PROBLEM

The schematic representation of the flow system is shown in the following figure. We have considered a uniform cylindrical tube of radius $r$with z −axis as the axis of the cylinder.

We have also considered that the fluid which is allowed to pass through it contains dust particles and a uniform magnetic field is also applied along the meridian axis. The volume fraction of the dust particles ϕ_p are also taken into consideration.

On the basis of the above construction, we define a model of Newtonian fluid flow with impure elements treated as dust particles, through a uniform cylindrical tube in the presence of a uniform magnetic field.

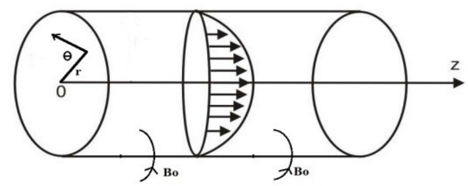

Fig. 1. Schematic diagram of the flow system.

We consider the flow to be taking place in the axial direction of the tube and the magnetic field being applied perpendicular to the direction of the flow.

$$\rho(1-\phi_p)\frac{\partial u^*}{\partial t^*} = (1-\phi_p)\left[-\frac{\partial p^*}{\partial z^*} + \frac{\mu}{r^*}\frac{\partial}{\partial r^*}\left(r^*\frac{\partial u^*}{\partial r^*}\right)\right] - \sigma B_0^2 u^* - \kappa N_0(u^* - u_p^*) \quad (12)$$

$$mN_0\frac{\partial u_p^*}{\partial t^*} = \kappa N_0(u^* - u_p^*) \quad (13)$$

We define the following non-dimensional quantities as

$$r = \frac{r^*}{r_0},\ u = \frac{u^*}{u_R},\ u_p = \frac{u_p^*}{u_R},\ t = \frac{t^*}{t_R},\ p = \frac{p^* t_R}{\rho u_R},\ z = \frac{z^*}{r_0}$$

where r_0 is the reference radius, u_R is the reference velocity, t_R is the reference time.

Therefore, from Eqns. (12) and (13) we get the non-dimensional set of equations

$$\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial z} + \frac{1}{Re}\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) - \frac{H_a^2}{1-\phi_p}u - \frac{\delta\eta}{1-\phi_p}(u - u_p) \quad (14)$$

$$\frac{\partial u_p}{\partial t} = \eta(u - u_p) \quad (15)$$

where $Re = \frac{\rho r_0^2}{\mu t_R}$, $H_a^2 = \frac{\sigma B_0^2 t_R}{\rho}$, $\delta = \frac{N_0 m}{\rho}$, $\eta = \frac{\kappa t_R}{m}$.

In this paper, we will consider time-fractional derivative i.e. we will consider the order of partial derivative with respect to time to be $0 < \alpha, \beta \le 1$ and also, we consider the pressure gradient to be constant throughout the flow and so, in general, the time-fractional derivative form of Eqns. (14) and (15) with constant pressure gradient are given by

$$\frac{\partial^\alpha u}{\partial t^\alpha} = P + \frac{1}{Re}\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) - \frac{H_a^2}{1-\phi_p}u - \frac{\delta\eta}{1-\phi_p}(u - u_p) \quad (16)$$

$$\frac{\partial^\beta u_p}{\partial t^\beta} = \eta(u - u_p) \quad (17)$$

where $u$is the fluid velocity vector, $u_p$is the dust particle velocity vector, $Re$is the Reynold's number,H_a^2 is the square of Hartmann number (magnetic field parameter), $P$is the constant pressure gradient, $t$is the time, r is the radius of the tube, $\phi_p$is the volume fraction of the dust particle, η and δ are material constant parameters, $\alpha$and $\beta$are time-fractional order parameters. Now, we will find solutions of (16) and (17) with respect to some suitable initial conditions which will be divided into two cases.

Case I. Initial conditions $u(r,0) = a(1-r^2) = u_p(r,0)$ where $r \in [0,1]$ and $a > 0$.

We try to solve (16) and (17) subject to the initial conditions $u(r,0) = a(1-r^2) = u_p(r,0)$ where $r \in [0,1]$.

Using Riemann-Liouville integral operator on (16) and applying Adomian decomposition series solution, we obtain an approximate solution as

$$u_1 = B_1\frac{t^\alpha}{\Gamma(\alpha+1)} + C_1\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}$$

$$u_2 = B_2\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + C_2\frac{t^{3\alpha}}{\Gamma(3\alpha+1)} + D_2\frac{t^{2\alpha+\beta}}{\Gamma(2\alpha+\beta+1)}$$

$$u_3 = B_3\frac{t^{3\alpha}}{\Gamma(3\alpha+1)} + C_3\frac{t^{4\alpha}}{\Gamma(4\alpha+1)} + D_3\frac{t^{2\alpha+\beta}}{\Gamma(2\alpha+\beta+1)} + E_3\frac{t^{3\alpha+2\beta}}{\Gamma(3\alpha+2\beta+1)} + F_3\frac{t^{3\alpha+\beta}}{\Gamma(3\alpha+\beta+1)}$$

and so on.

And also $\alpha_0 = 0.494144 < 1$, $\alpha_1 = 0.107333 < 1$, $\alpha_2 = 0.0545257 < 1$and so on. (under some fixed values of the parameters).

According to the above result, we can conclude that our series solution obtained by ADM method must be convergent. Thus, we get the approximate solution of (16) as

$$u = u_0 + u_1 + u_2 + u_3 \tag{18}$$

Again, we get the following from (17)

$$u_{p_1} = B_{p_1}\frac{t^{\alpha+\beta}}{\Gamma(\alpha+\beta+1)}$$

$$u_{p_2} = B_{p_2}\frac{t^{\alpha+\beta}}{\Gamma(\alpha+\beta+1)} + C_{p_2}\frac{t^{2\alpha+\beta}}{\Gamma(2\alpha+\beta+1)} + D_{p_2}\frac{t^{\alpha+2\beta}}{\Gamma(\alpha+2\beta+1)}$$

$$u_{p_3} = B_{p_3}\frac{t^{2\alpha+\beta}}{\Gamma(2\alpha+\beta+1)} + C_{p_3}\frac{t^{3\alpha+\beta}}{\Gamma(3\alpha+\beta+1)} + D_{p_3}\frac{t^{2\alpha+2\beta}}{\Gamma(2\alpha+2\beta+1)} + E_{p_3}\frac{t^{\alpha+2\beta}}{\Gamma(\alpha+2\beta+1)} + F_{p_3}\frac{t^{\alpha+3\beta}}{\Gamma(\alpha+3\beta+1)}$$

and so on.

And also $\alpha_0' = 0.075 < 1$, $\alpha_1' = 0.817097 < 1$, $\alpha_2' = 0.965916 < 1$and so on. (under some fixed values of the parameters)

According to the above result, we can conclude that our series solution obtained by ADM method must be convergent. Thus, we get the approximate solution of (17) as

$$u_p = u_{p_0} + u_{p_1} + u_{p_2} + u_{p_3} \tag{19}$$

Case II. Initial conditions $\boldsymbol{u(r,0) = ar = u_p(r,0)}$ where $\boldsymbol{r} \in [\mathbf{0},\mathbf{1}]$and $\boldsymbol{a} > 0$.

We try to solve (16) and (17) subject to the initial condition $u(r,0) = ar = u_p(r,0)$ where $r \in [0,1]$.

Using Riemann-Liouville integral operator on (16) and applying Adomian decomposition series solution, we obtain an approximate solution as

$$u_1 = G_1\frac{t^{\alpha}}{\Gamma(\alpha+1)} + H_1\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}$$

$$u_2 = G_2\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + H_2\frac{t^{3\alpha}}{\Gamma(3\alpha+1)} + I_2\frac{t^{2\alpha+\beta}}{\Gamma(2\alpha+\beta+1)}$$

$$u_3 = G_3\frac{t^{3\alpha}}{\Gamma(3\alpha+1)} + H_3\frac{t^{4\alpha}}{\Gamma(4\alpha+1)} + I_3\frac{t^{2\alpha+\beta}}{\Gamma(2\alpha+\beta+1)} + J_3\frac{t^{3\alpha+\beta}}{\Gamma(3\alpha+\beta+1)} + K_3\frac{t^{2\alpha+2\beta}}{\Gamma(2\alpha+2\beta+1)}$$

and so on.

And also $\alpha_0 = 0.332178 < 1$, $\alpha_1 = 0.347567 < 1$, $\alpha_2 = 0.341919 < 1$ and so on (under some fixed values of the parameters)

According to the above result, we can conclude that our series solution obtained by ADM method must be convergent. Thus, we get the approximate solution of (16) as

$$u = u_0 + u_1 + u_2 + u_3 \tag{20}$$

Again, we get from (17)

$$u_{p_1} = G_{p_1}\frac{t^{\alpha+\beta}}{\Gamma(\alpha+\beta+1)}$$

$$u_{p_2} = G_{p_2}\frac{t^{\alpha+\beta}}{\Gamma(\alpha+\beta+1)} + H_{p_2}\frac{t^{2\alpha+\beta}}{\Gamma(2\alpha+\beta+1)} + I_{p_2}\frac{t^{\alpha+2\beta}}{\Gamma(\alpha+2\beta+1)}$$

$$u_{p_3} = G_{p_3}\frac{t^{2\alpha+\beta}}{\Gamma(2\alpha+\beta+1)} + H_{p_3}\frac{t^{3\alpha+\beta}}{\Gamma(3\alpha+\beta+1)} + I_{p_3}\frac{t^{2\alpha+2\beta}}{\Gamma(2\alpha+2\beta+1)} + J_{p_3}\frac{t^{\alpha+2\beta}}{\Gamma(\alpha+2\beta+1)} + K_{p_3}\frac{t^{\alpha+3\beta}}{\Gamma(\alpha+3\beta+1)}$$

and so on.

And also $\alpha_0' = 0 < 1$, $\alpha_1' = 0 < 1$, $\alpha_2' = 0.30156 < 1$ and so on.(under some fixed values of the parameters).

According to the Theorem (1), we can conclude that our series solution obtained by ADM method must be convergent. Thus, we get the approximate solution of (17) as

$$u_p = u_{p_0} + u_{p_1} + u_{p_2} + u_{p_3} \tag{21}$$

III. RESULTS AND DISCUSSION

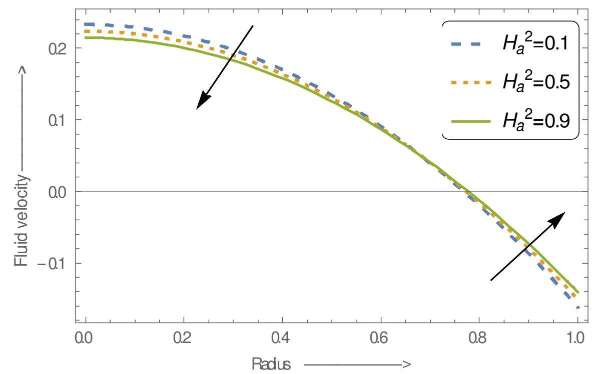

Fig. 2. u vs r (H_a^2 varies, ϕ_p = 0.03, P= 0.5, δ= 0.5, η= 0.6, a= 0.4, Re = 0.9, α= 0.9, β= 0.3, t= 0.1) from Eqns. (18) and (19).

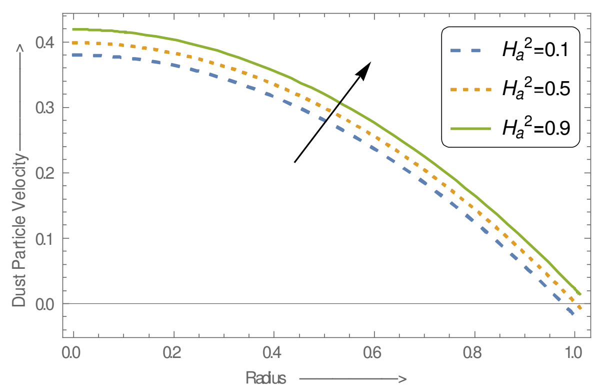

Fig. 3. u_p vs r (H_a^2 varies, ϕ_p= 0.03, P= 0.5, δ= 0.5, η= 0.6, a= 0.4, Re= 0.9, α= 0.9, β= 0.3, t= 0.1) from Eqns. (18) and (19).

In Fig. 2 and 3, the parametric influence of the magnetic field parameter (H_a^2) placed along the meridian axis direction to the flow is initially found to decrease the axial fluid velocity profiles upto r = 0.7 and thereafter, a minor form of increment of the velocity profile is observed to be in increasing trend though the velocity attains negative values near to the wall of the tube. On the other hand, as the dust particle constituents is electrically non conducting as such the negative influence of magnetic field on the dust particles is absent. This helps the tiny particles move without restriction imposed by the magnetic field.

So, an increasing trend of the dust particle velocity profile are clearly visible. Due to the presence of an uniform magnetic field along the meridian axis, a resistive force will act along the direction of flow which retards the axial fluid flow rate thereby decreasing the fluid velocity in the initial stage.

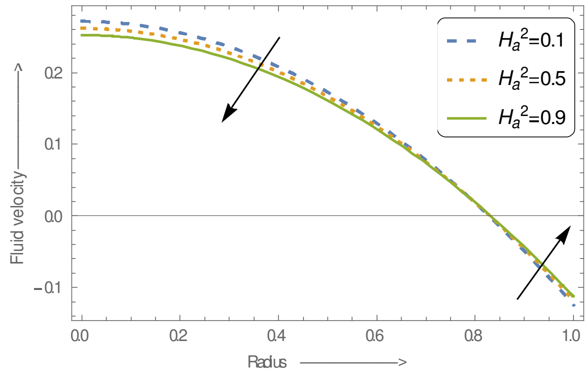

Fig. 4. u vs r (H_a^2 varies, ϕ_p= 0.03, P= 0.5, δ = 0.5, η= 0.6, a= 0.4, Re= 0.9, α= 1, β= 1, t = 0.1) from Eqns. (18) and (19).

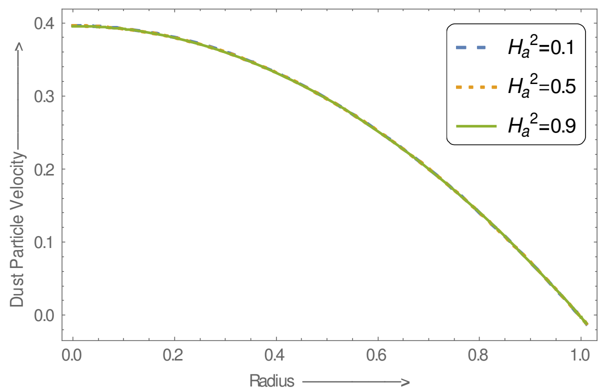

Fig. 5. $u_p$vs r (H_a^2 varies, ϕ_p= 0.03, P= 0.5, δ = 0.5, η = 0.6, a = 0.4, Re = 0.9, α = 1, β = 1, t = 0.1) from Eqns. (18) and (19).

Fig. 4 and 5 shows that the change in magnetic field(H_a^2) does not show any significant change in the dust particle velocity if we take the time fractional order parameters α = β = 1 even though decreasing and increasing trend is seen for fluid velocity.

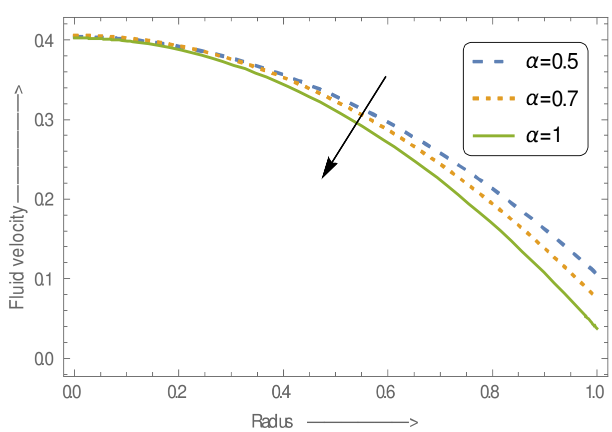

Fig. 6. u vs r (α varies, ϕ_p= 0.03, P = 0.5, δ= 0.5, η = 0.6, a = 0.4, Re = 0.9, H_a^2 = 0.9, β = 0.5, t = 0.1) from equations (18) and (19).

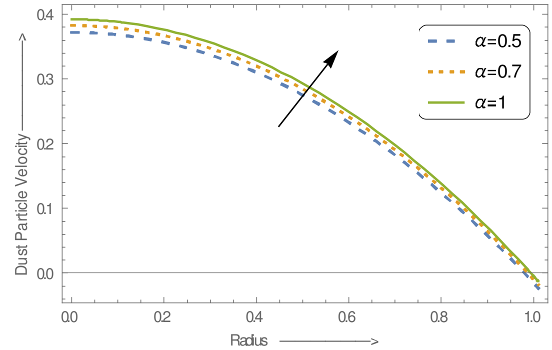

Fig. 7. u_p vs r (α varies, ϕ_p=0.03, P=0.5, δ=0.5, η=0.6, a=0.4, Re=0.9, H_a^2=0.9, β=0.5, t=0.1) from Eqns. (18) and (19).

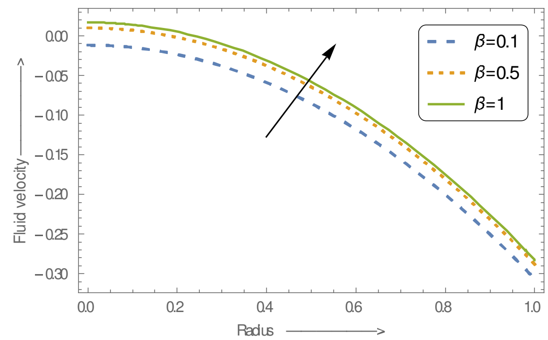

Fig. 8. u vs r (βvaries, ϕ_p=0.03, P=0.5, δ=0.5, η=0.6, a=0.4, Re=0.9, H_a^2=0.9, α=0.5, t=0.1) from Eqns. (18) and (19).

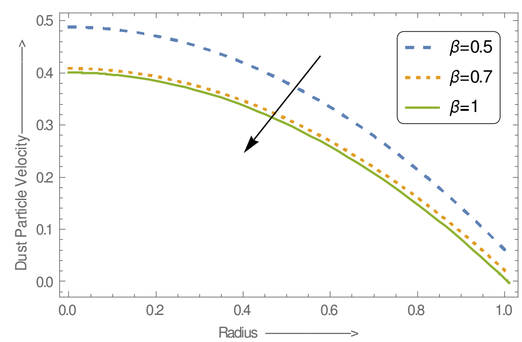

Fig. 9. u_p vs r (β varies, ϕ_p= 0.03, P = 0.5, δ = 0.5, η = 0.6, a = 0.4, Re = 200, H_a^2 = 0.9, α = 0.5, t = 0.1) from Eqns. (18) and (19).

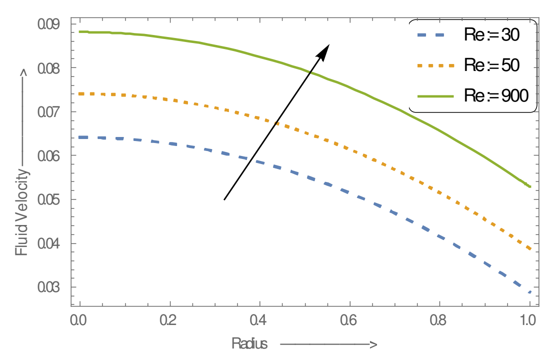

Fig. 10. u vs r (Re varies, ϕ_p= 0.03, P = 0.5, δ = 0.5, η = 0.6, a = 0.4, $H_a{}^2$ = 0.9, α = 0.5, β = 0.9, t = 0.1) from Eqns. (18) and (19).

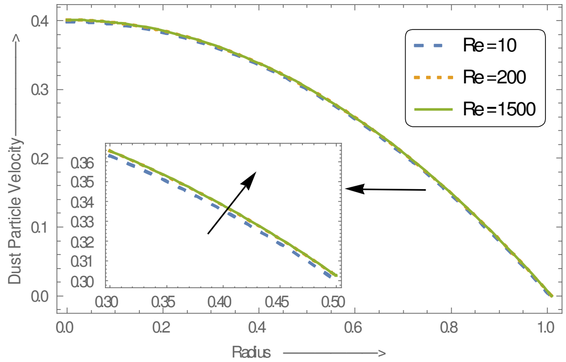

Fig. 11. u_p vs r (Re varies, ϕ_p=0.03, P=0.5, δ=0.5, η=0.6, a=0.4, H_a^2=0.9, α = 0.5, β = 0.9, t = 0.1) from Eqns. (18) and (19).

In Fig. 6 and 7, the physical influence of the fractional order of the time derivative for the fluid flow over axial fluid velocity and dust particle velocity have been demonstrated graphically.

It is observed that due to the increase in values of α, the axial fluid flow rate de-accelerates whereas the dust particle flow rate accelerates within the flow region which is in complete agreement with Shah *et al.,* [21] for our fluid velocity for time $t < 1$. In Fig. 8 and 9, the effects of time fractional order derivative for dust particle β on the axial fluid velocity as well as dust particle velocity has been highlightedwhich is also in complete agreement with Shah *et al.,* [21] for our dust particle velocity for time $t < 1$. An opposite phenomenon is detected in this case in comparison to the effects shown by α.In Fig. 10 and 11, the effects of Reynold's number on the axial fluid velocity and dust particle velocity is presented graphically. We observed that the axial fluid velocity as well as dust particle velocity tend to increase with the increase in Reynold's number. Due to increase in Reynold's number, the inertial forces suppress the internal friction of the flow, as a result, rate of flow of axial velocity as well as dust particle velocity accelerate. This is why the velocities are found to be in increasing trend.

Table 1: Effects of parameters on skin friction (τ_r) and volumetric flux(Q) from Eqns. (18) and (19).

H_a^2	δ	η	P	t	α	β	ϕ_p	$\tau_r = -\left(\frac{\partial u}{\partial r}\right)_{r=1}$	Q
0.1	0.5	0.6	0.5	0.1	0.4	0.5	0.03	0.767321	0.31963π
0.5	0.5	0.6	0.5	0.1	0.4	0.5	0.03	0.653496	0.27736π
0.9	0.5	0.6	0.5	0.1	0.4	0.5	0.03	0.552238	0.23019π
1	**0.6**	0.6	0.5	0.1	0.4	0.5	0.03	0.525983	0.21143π
1	**0.7**	0.6	0.5	0.1	0.4	0.5	0.03	0.524976	0.20594π
1	**0.8**	0.6	0.5	0.1	0.4	0.5	0.03	0.523608	0.20014π
1	0.5	**0.7**	0.5	0.1	0.4	0.5	0.03	0.524837	0.21162π
1	0.5	**0.8**	0.5	0.1	0.4	0.5	0.03	0.52243	0.20613π
1	0.5	**0.9**	0.5	0.1	0.4	0.5	0.03	0.519406	0.200104π
1	0.5	0.6	**2**	0.1	0.4	0.5	0.03	0.526628	0.63916π
1	0.5	0.6	**3**	0.1	0.4	0.5	0.03	0.526628	0.92085π
1	0.5	0.6	**4**	0.1	0.4	0.5	0.03	0.526628	1.20255π
1	0.5	0.6	2	**0.2**	0.4	0.5	0.03	0.428111	$0.61285\ \pi$
1	0.5	0.6	2	**0.3**	0.4	0.5	0.03	0.334236	0.49363π
1	0.5	0.6	2	**0.4**	0.4	0.5	0.03	0.238271	0.30821π
1	0.5	0.6	2	0.5	**0.5**	0.5	0.03	0.259924	0.46524π
1	0.5	0.6	2	0.5	**0.6**	0.5	0.03	0.343448	0.68159π
1	0.5	0.6	2	0.5	**0.7**	0.5	0.03	0.398774	0.78438π
1	0.5	0.6	2	0.5	0.4	**0.6**	0.03	0.145246	0.09276π
1	0.5	0.6	2	0.5	0.4	**0.7**	0.03	0.151229	0.11341π
1	0.5	0.6	2	0.5	0.4	**0.8**	0.03	0.156619	0.13021π
1	0.5	0.6	2	0.5	0.4	0.5	**0.06**	0.0990837	-0.16767π
1	0.5	0.6	2	0.5	0.4	0.5	**0.07**	0.0847656	-0.18312π
1	0.5	0.6	2	0.5	0.4	0.5	**0.08**	0.0698304	-0.19921π

In the table, the parametric influences of some physical parameters like pressure gradient, magnetic field parameters (square of the Hartman number H_a^2), material constant parameter for particle concentration $\delta$and particle mass η, the time fractional parameters for fluid and dust particles (αa nd β respectively) along with particle volume fraction $\phi_p$on the surface skin friction and volumetric flux are depicted taking $a = 0.4$ and $Re = 9$ throughout the table. It is clearly observed from the table that due to increase in parametric values of α and β, the skin friction is in increasing trend but the increment of H_a^2 reduces the skin friction. The volumetric flux is in decreasing pattern as H_a^2 increases which is obvious as the rise in magnetic field produces a resistive force in the form of Lorentz force against the fluid flow. Also, increment in α and β gives increment to volumetric flux, which clearly shows that fractional order αandβ play important role in the fluid flow. Again, due to an increment in particle volume fractionϕ_p, the axial fluid flow rate is disrupted due to maximum surface area covered by the dust particles, as a result, less amount of fluid per unit area per unit time will get discharged which results in decreasing the volumetric flux rate of flow. Hence, due to less volumetric rate of flow, the effect of surface friction by the discharged fluid particle will be less at the wall resulting in decreasing skin friction. It is interesting to observe that the fluctuation in value of P (negative pressure gradient) has no impact on the surface friction though the increment in value of P (negative pressure gradient) which is nothing but decrease in pressure implies that the increment in volumetric flow rate which is also obvious. With increase in time t the volumetric flux decreases which is obvious as the axial fluid flow decreases with time. With increment in particle concentration $\delta$and particle mass η, the volumetric flux decreases which is also practically true as the concentration and mass act as a resistance to the axial fluid flow.

IV. CONCLUSION

A time-fractional incompressible Newtonian viscous fluid through a circular tube with dust particle immersion, under an influence of a uniform magnetic field along meridian axis is considered. The Adomian decomposition method (ADM) is used to find an approximate solution to our proposed time fractional order differential equation. It is observed that the presence of magnetic field parameter (H_a^2) and time-fractional parameter (α) reduces the velocity flow rate of fluid but boost up the dust particle flow rate. An exactly opposite phenomenon is observed for time-fractional parameter (β). The fluid velocity as well as dust particle velocity are found to increase due to increase in values of Reynold's number (Re). Due to increase in values of material constant parameters δ and η, the volumetric flux decreases. Again, due to increase in adverse pressure gradient (P), volumetric flux increases. Thus, we have observed the way magnetic field affects a dusty fluid flow phenomenon. It means, the more the intensity of magnetic field, the slower will be the fluid flow and vice versa. Most importantly, it is observed that the change in dust particle velocities due to magnetic field occur only when we take non integer values of time fractional orders $\alpha$and β. But if we take $\alpha = \beta = 1$ then no change is observed in the dust particle velocity due to magnetic field. Thus, we can conclude that the time fractional orders also play a very important role in the fractional order fluid flow.

ACKNOWLEDGEMENT

The first author (H Imo Mani Singha) would also like to acknowledge the Council of Scientific and Industrial Research (CSIR), Government of India for providing financial support vide letter no. 09/747(0013)/2018-EMR-I to accomplish this work.

Conflict of Interest. The authors declare no conflict of interest associated with this work.

APPENDIX

- $B_1 = -\frac{4a}{Re} - \frac{a(1-r^2)H_a^2}{1-\phi_p}$
- $C_1 = -\frac{P}{1-\phi_p}(H_a^2 + \delta\eta)$
- $B_2 = \frac{4aH_a^2}{Re(1-\phi_p)} - \frac{(H_a^2+\delta\eta)B_1}{1-\phi_p}$
- $C_2 = -\frac{C_1}{1-\phi_p}(H_a^2 + \delta\eta)$
- $D_2 = \frac{\delta\eta B_{p_1}}{1-\phi_p}$
- $B_3 = -\frac{4aH_a^2(H_a^2+\delta\eta)}{Re(1-\phi_p)^2} - \frac{(H_a^2+\delta\eta)B_2}{1-\phi_p}$
- $C_3 = -\frac{C_2}{1-\phi_p}(H_a^2 + \delta\eta)$
- $D_3 = \frac{\delta\eta B_{p_2}}{1-\phi_p}$
- $\mathrm{E}_3 = \frac{\delta\eta D_{p_2}}{1-\phi_p}$
- $F_3 = -\frac{D_2(H_a^2+\delta\eta)}{1-\phi_p} + \frac{\delta\eta C_{p_2}}{1-\phi_p}$
- $B_{p_1} = P\eta$
- $B_{p_2} = B_1\eta$
- $C_{p_2} = C_1\eta$
- $D_{p_2} = -B_{p_1}\eta$
- $B_{p_3} = B_2\eta$
- $C_{p_3} = C_2\eta$
- $D_{p_3} = (D_2 - C_{p_2})\eta$
- $E_{p_3} = -B_{p_2}\eta$
- $F_{p_3} = -D_{p_2}\eta$
- $G_1 = \frac{a}{Re\times r} - \frac{H_a^2 ar}{1-\phi_p}$
- $H_1 = -\frac{H_a^2 P}{1-\phi_p} - \frac{\delta\eta P}{1-\phi_p}$
- $G_2 = \frac{2a}{Re^2 r^3} + \left(\frac{1}{Re\times r}\left(-\frac{a}{Re\times r^2} - \frac{H_a^2 a}{1-\phi_p}\right)\right) - \frac{H_a^2 G_1}{1-\phi_p} - \frac{\delta\eta G_1}{1-\phi_p}$
- $H_2 = -\frac{H_a^2 H_1}{1-\phi_p} - \frac{\delta\eta H_1}{1-\phi_p}$
- $I_2 = \frac{\delta\eta G_{p_1}}{1-\phi_p}$
- $G_3 = \frac{1}{Re}\left\{\frac{12a}{Re^2 r^5} - \frac{2H_a^2 a}{(1-\phi_p)r^3} - \frac{(H_a^2+\delta\eta)2a}{Re\,(1-\phi_p)r^3} - \frac{3a}{Re^2 r^5} + \frac{H_a^2 a}{Re(1-\phi_p)r^3} - \frac{H_a^2+\delta\eta}{1-\phi_p}\left(-\frac{a}{Re\times r^3} - \frac{H_a^2 a}{(1-\phi_p)r}\right)\right\} - \frac{H_a^2 G_2}{1-\phi_p} - \frac{\delta\eta G_2}{1-\phi_p}$
- $H_3 = -\frac{H_a^2 H_2}{1-\phi_p} - \frac{\delta\eta H_2}{1-\phi_p}$
- $I_3 = \frac{\delta\eta G_{p_2}}{1-\phi_p}$
- $J_3 = -\frac{H_a^2 I_2}{1-\phi_p} - \frac{\delta\eta(I_2 - H_{p_2})}{1-\phi_p}$
- $K_3 = \frac{\delta\eta I_{p_2}}{1-\phi_p}$
- $G_{p_1} = \eta P$
- $G_{p_2} = \eta G_1$
- $H_{p_2} = \eta H_1$
- $I_{p_2} = -G_{p_1}$
- $G_{p_3} = \eta G_2$
- $H_{p_3} = \eta H_2$
- $I_{p_3} = \eta\left(I_2 - H_{p_2}\right)$
- $J_{p_3} = -\eta G_{p_2}$
- $K_{p_3} = -\eta I_{p_2}$

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How to cite this article: Singha, H. I. M., and Sengupta, S. (2020). Dusty Time Fractional MHD Flow of a Newtonian Fluid through a Cylindrical Tube. *International Journal on Emerging Technologies*, *11*(2): 610–617.