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Dynamical Behavior of Prey-Predator System with Alternative Food for Predator

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ABSTRACT: In this present work, a model on predator-prey system is proposed and analyzed. It is considered that both the prey and predator species are harvested at different harvesting rates. There are considerable evidences that there exist alternative food source for the predator population. It has been shown that the dynamics of prey-predator is largely affected by the presence of alternative food for predator population. Conditions for the existence and stability of all feasible equilibrium points are obtained. Furthermore, it has been shown that addition of alternative food resource and controlled harvesting makes a positive impact on the stability of non-zero equilibrium point. Uniform persistence of the system is also discussed. Numerical simulations are given to justify the obtained theoretical results.

Keywords: Carrying capacity, Harvesting, Predator, Prey, Predation rate

I. INTRODUCTION

The study of population dynamics is one of the most studied branch of mathematical biology. Mathematical models have been developed to prevent the extinction of the species due to several reasons like exploitation of resources, uncontrolled harvesting, excess predation etc [1-2, 6, 14-18]. Moreover it has been observed that presence of alternative food makes a positive impact on the predator population. The fact is justified by Abrams and Routh [3] where a theoretical model is established as model of additional tropic levels. Sometimes, alternative prey consume natural resources for their growth, therefore due to less accessibility of resources, number of target prey decreased. Baalen *et al.* [4], Rijn *et al.* [5] observed that, presence of alternative food resource for predator population ultimately decreases the focal prey population. Harwood and Obryeki [7], Holt and Lowton [8], Wootton [9] observed that presence of alternative food for predator does not always make positive impact on the predation of modeled prey species. Considerable work is done by Spencer and Collie [10] where they acknowledged the verity by taking intra-specific competition in predatory fish. The existence of alternative food source can increase predator abundance when modeled prey abundance is low. Srinivasu *et. al.* [11] also considered the quality of food provided in a two dimensional prey predator system. In this continuation, significant impact of alternative food source in exploited prey-predator system has been studied extensively by Pahari *et. al.* [12] and Kar *et. al.* [13]. Ganguly [19] considered a prey-predator model where additional food is available for the predator species. In his paper, only prey species has been harvested at a constant rate. In the present paper, we considered that both the prey and predator species have been harvested by different harvesting agencies. The combined effect of harvesting and additional food available for the predator species has been studied extensively.

II. FORMULATION OF THE MODEL

Let us consider a two species prey predator model with harvesting of both the species. The prey population follows logistic growth with '*r*' as intrinsic growth rate and 'K' as the carrying capacity of the environment. The prey and predator species are also subjected to harvesting with harvesting attempt E_1 and E_2 correspondingly. Let γ_1 and γ_2 be the catch ability coefficients of prey and predator species. *'d'* is natural mortality rate for the predator species. We assumed that predator takes *'A'* part from the focal prey and *1-A* part from the alternative food source. If *A*=0, then it means that predator species becomes independent of available focal prey species, which is neither relevant nor revealing. Similarly, if *A*=1, then it means that there is no role of alternative food source in predator growth. So to study extensively the dynamics of harvested prey predator system, it is quite natural to assume that $0 < A < 1$.

Based on the above assumptions, we write the following equations for our eco- epidemiological model.

$$\frac{dx}{dt} = rx\left(1-\frac{x}{k}\right) - \alpha Axy - \gamma_1 E_1 x$$

$$\frac{dy}{dt} = \alpha\beta Axy + (1-A)y - dy - \gamma_2 E_2 y \qquad (1)$$

$$\left(0 \le x(0) \le k,\ 0 \le y(0)\right)$$

III. EQUILIBRIUM POINTS

The system can have following different equilibriums

(a) The trivial equilibrium point E_0 (0,0), where both prey and predator populations extinct.

(b) A prey free equilibrium point $E_1\ (x_1, 0)$, where $x_1 = \frac{K}{r}(r - \gamma_1 E_1)$ which exist

if $$E_1 < \frac{r}{\gamma_1} \qquad (2)$$
(c) An endemic positive equilibrium $E^*(x^*, y^*)$, where x^* and y^* are the solutions of the following equations:

$$r\left(1 - x^*/K\right) - \alpha A y^* - \gamma_1 E_1 = 0$$

$$\alpha\beta A x^* + (1-A) - d - \gamma_2 E_2 = 0$$

Now from the second equation, we have

$$x^* = \frac{d + \gamma_2 E_2 - 1 + A}{\alpha\beta A}$$

which exist if $A > 1 - d - \gamma_2 E_2$

Now substituting this value of x^* in first equation, we get

$$y^* = \frac{1}{\alpha A}\left[r - \gamma_1 E_1 - \frac{r}{K}\left(\frac{d + \gamma_2 E_2 - 1 + A}{\alpha\beta A}\right)\right]$$, exist if

$$A > \frac{r(d + \gamma_2 E_2)}{\alpha\beta K(r - \gamma_1 E_1)} \text{ and } r - \gamma_1 E_1 > 0 \qquad (3)$$

Theorem: The non zero interior equilibrium point $E^*(x^*, y^*)$ exists if

$$A > Max\left\{1 - d - \gamma_2 E_2, \frac{r(d + \gamma_2 E_2)}{\alpha\beta K(r - \gamma_1 E_1)}\right\} \text{and } E_1 < r/\gamma_1$$

IV. POSITIVITY AND UNIFORM BOUNDEDNESS

Lemma 1: All the solutions of the system (1) which initiate in R_+^2 will remain positive forever.

Proof: - The first equation of the system (1) can be written as

$$\frac{\mathrm{d}x}{\mathrm{x}} = \left(r\left(1 - \frac{x}{K}\right) - \alpha A y - \gamma_1 E_1\right) dt$$

Which is of the form

$$\frac{dx}{x} = \phi_1(x, y) dt$$

Where $\phi_1(x, y) = r\left(1 - \frac{x}{K}\right) - \alpha A y - \gamma_1 E_1$

Now integrating the above equation from *[0, t]*, we have

$$x(t) = x(0) e^{\int_0^t \phi_1(x,y)dt} > 0$$

Similarly second equation of the system (1) can be written as

$$\frac{\mathrm{d}y}{\mathrm{y}} = \left(\alpha\beta A x - d - \gamma_2 E_2 + (1-A)\right) dt$$

Which is of the form

$$\frac{dy}{y} = \phi_2(x, y) dt$$

Where $\phi_2(x, y) = \alpha\beta A x - d - \gamma_2 E_2 + (1-A)$

Integrating the above equation from *[0, t]*, we get

$$y(t) = y(0) e^{\int_0^t \phi_2(x,y)dt} > 0 \quad \forall t$$

Hence Proof.

Lemma 2: All the solutions of the system (1) will be in the region $R = \left[(x, y) \in R_+^2 : 0 \le x + y \le \mu/\lambda\right]$ as $t \to \infty$ for all positive initial values $(x(0), y(0)) \in R_+^2$ where $\lambda = \mathrm{Min}\{\gamma_1 E_1, d\}$ and $\mu = rx + (1-A)y$.

Proof: - We define a function *W* as follows

Let $W(t) = x(t) + y(t)$

So $$\frac{dW}{dt} = \frac{dx}{dt} + \frac{dy}{dt}$$

$$= rx - \frac{rx^2}{K} - \gamma_1 E_1 x - dy + (\beta - 1)\alpha A x y - \gamma_2 E_2 y + (1-A) y$$

$\le rx - \gamma_1 E_1 x - dy + (1-A) y$ as $\beta < 1$

$\le rx + (1-A) y - \lambda W$ where $\lambda = \mathrm{Min}\{\gamma_1 E_1, d\}$

So $\frac{dW}{dt} + \lambda W \le \mu$ where

$\mu = rx + (1-A) y > 0$ as $A < 1$

Applying the theory of Differential Inequality, we get

$$0 \le W(x, y) \le \frac{\mu}{\lambda}(1 - e^{-\lambda t}) + W\left(x(0), y(0)\right) e^{-\lambda t}$$

As $t \to \infty$, we get $0 \le W \le \frac{\mu}{\lambda}$

So all the solutions of the system (1) are uniformly bounded.

Lemma 3: The model system (1) under the assumptions cannot have any periodic solution in the interior of the positive quadrant of xy plane.

Proof: - Let $H(x, y) = \frac{1}{xy}$

Clearly $H(x, y)$ is positive in the interior of the positive quadrant of xy plane.

Again let $h_1(x, y) = rx\left(1 - \frac{x}{K}\right) - \alpha A x y - \gamma_1 E_1 x$

and $h_2(x, y) = \alpha\beta A x y - dy - \gamma_2 E_2 y + (1-A) y$

Now $\Delta(x, y) = \frac{\partial}{\partial x}(h_1 H) + \frac{\partial}{\partial y}(h_2 H)$

$$= \frac{\partial}{\partial x}\left\{\frac{r}{y}\left(1 - \frac{x}{K}\right) - \alpha A - \frac{\gamma_1 E_1}{y}\right\} + \frac{\partial}{\partial y}\left\{\alpha\beta A - \frac{d}{x} - \frac{\gamma_2 E_2}{x} + \frac{1-A}{x}\right\}$$

$$= \frac{-r}{Ky} < 0 .$$

V. STABILITY ANALYSIS AND UNIFORM PERSISTENCE

The following theorems are direct consequences of linear stability analysis of the system (1)

Theorem 1: The trivial equilibrium $E_0(0,0)$ is locally stable if $E_1 > \frac{r}{\gamma_1}$ and $A > 1 - d - \gamma_2 E_2$

Proof: - The Eigen values for the equilibrium $E_0(0,0)$ are given by

$$\xi_1 = r - \gamma_1 E_1, \quad \xi_2 = -d - \gamma_2 E_2 + 1 - A$$

So $\xi_1, \xi_2 < 0$ iff $E_1 > \frac{r}{\gamma_1}$ and $A > 1 - d - \gamma_2 E_2 \qquad (4)$

Theorem 2: The prey free equilibrium $E_1(x_1, 0)$ if exist, is locally asymptotically stable for

$$E_2 > \frac{\alpha\beta AK(r-\gamma_1 E_1) - r(d-1+A)}{\gamma_2},$$

where $x_1 = \frac{K}{r}(r - \gamma_1 E_1)$

Proof: - The Eigen values for the equilibrium $E_1\ (x_1, 0)$ are given by

$$\xi_1 = -\frac{rx_1}{k} < 0, \quad \xi_2 = \alpha\beta A x_1 - d - \gamma_2 E_2 + 1 - A.$$

Clearly $\xi_1 < 0$.

Again $\xi_2 < 0$ iff

$$E_2 > \frac{\alpha\beta AK(r-\gamma_1 E_1) - r(d-1+A)}{\gamma_2}. \qquad (5)$$

Theorem 3: The positive endemic equilibrium $E^*\left(x^*, y^*\right)$ is locally asymptotically stable for all parametric values.

Proof:-The Eigen values for $E^*\left(x^*, y^*\right)$ are given by the equation $\xi^2 + \frac{rx^*}{k}\xi + \alpha^2\beta A^2 x^* y^* = 0$

Here ξ_1, ξ_2 are roots of above equation having the entire coefficients positive. So ξ_1, ξ_2 are clearly negative. Hence $E^*\left(x^*, y^*\right)$ is always locally stable.

Theorem 4: The model system (1) is uniformly persistent if the following conditions hold.

$$r\left(1 - \frac{x}{K}\right) - \alpha A y - \gamma_1 E_1 > 0, \quad \alpha\beta A x - d - \gamma_2 E_2 + (1 - A) > 0$$

Proof: Consider the Lyapunov function of the form $\sigma(x, y) = x^p y^q$, where *p* and *q* are assumed to be positive constants.

Obviously $\sigma(x, y)$ is a C^1 positive function defined in R_+^2 and $\sigma(x, y) \to 0$ if one of $x, y \to 0$.

Consequently, we get $H(x, y) = \frac{\sigma'(x, y)}{\sigma(x, y)}$

$$\frac{p_1}{x}\frac{dx}{dt} + \frac{p_2}{y}\frac{dy}{dt}$$

$$= p_1\left(r\left(1 - \frac{x}{K}\right) - \alpha A y - \gamma_1 E_1\right) + p_2\left(\alpha\beta A x - d - \gamma_2 E_2 + (1 - A)\right)$$

Now, we have proved in lemma that there is no periodic orbit in boundary planes, then for any initial point in R_+^2, the only possible H-limit set in the boundary planes of the system (1) is the equilibrium point $E^*\left(x^*, y^*\right)$. Thus the system (1) is uniformly persistent if $H(E^*) > 0$ at each of these points. So $H(E^*) > 0$ for any positive constants *p* and *q* if the following conditions hold.

$$r\left(1 - \frac{x}{K}\right) - \alpha A y - \gamma_1 E_1 > 0, \quad \alpha\beta A x - d - \gamma_2 E_2 + (1 - A) > 0$$

Then strictly positive solution of the system (1) doesn't have any H-limit set in the boundary planes. Hence the system (1) is uniformly persistent.

VI. NUMERICAL SIMULATIONS

Numerical simulations have been carried out to study the dynamics of the proposed 2-D model (1). Consider the following set of parametric values:

$$\begin{aligned} &r = 0.1,\ k = 20,\ \alpha = 0.1,\ d = 0.15,\ \beta = 0.5, \\ &A = 0.6,\ \gamma_1 = 0.2,\ E_1 = 0.9,\ \gamma_2 = 0.5,\ E_2 = 0.8, \end{aligned} \qquad (6)$$

The system (1) has equilibrium point $E_0\left(0,0\right)$ for the data set (6). It is locally asymptotically stable by Theorem (1), as the computed value of E_1 is so large and the value of *A* is so adjusted that the equation (4) is satisfied (Fig.1).

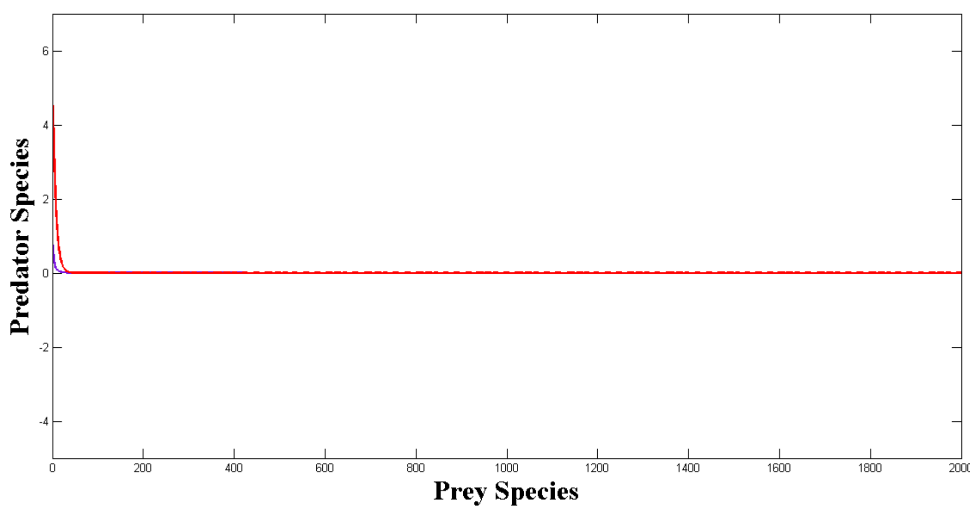

Fig. 1. Time series plot of the model equation (1) for the stability of equilibrium point E_0(0,0).

Again Consider the following set of parametric values:

$$\begin{aligned} &r = 0.1,\ k = 20,\ \alpha = 0.1,\ d = 0.15,\ \beta = 0.5, \\ &A = 0.5,\ \gamma_1 = 0.2,\ E_1 = 0.45,\ \gamma_2 = 0.5,\ E_2 = 0.8, \end{aligned} \qquad (7)$$

We had taken all the parametric values same as in data set (6) except the values of E_1 and *'A'* are so adjusted that the equations (2) and (5) are satisfied. System (1) has equilibrium point $E_0(2,0)$. It is locally asymptotically stable by Theorem 2. So it is observed that if prey species has been subjected to reasonable harvesting, then it can survive in the environment (Fig. 2).

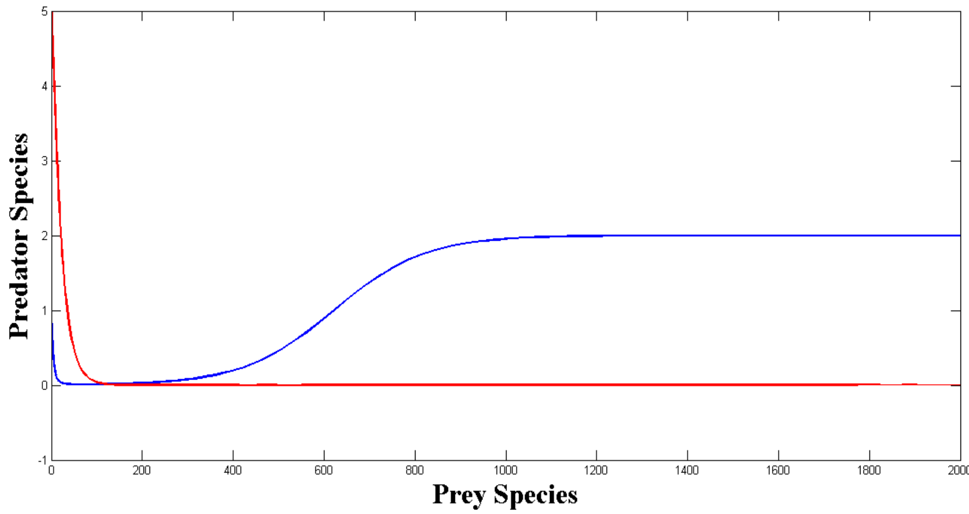

Fig. 2. Time series plot for the model equation (1) for the stability of equilibrium point $E_1(2,0)$.

Now if we take consider the parametric set of values

$$r = 0.1,\ \ k = 20,\ \ \alpha = 0.1,\ \ d = 0.15,\ \ \beta = 0.5,$$
$$A = 0.5,\ \gamma_1 = 0.2,\ E_1 = 0.45,\ \gamma_2 = 0.5,\ E_2 = 0.8, \tag{8}$$

System (1) has non-zero equilibrium point E* (5, 0.125) for the data set (8). The role of controlled harvesting is justified here. Further it has been shown that if the value of additional food resource for the predator population satisfies the equation (3), then both the prey and predator populations co-exist, which is good for our biodiversity (Fig. 3).

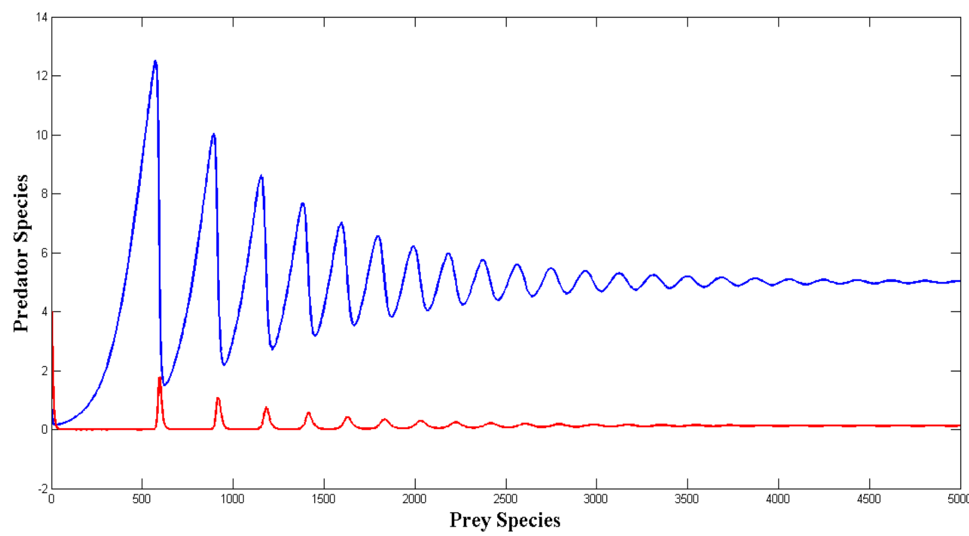

Fig. 3. Time series plot for the model equation (1) for the stability of positive endemic equilibrium point $E^{*}(5,0.125)$.

V. CONCLUSION

In this present work, a harvested model on predator-prey system is proposed and analyzed. The role of controlled harvesting has been justified here. It has been shown that existence and stability of non-zero equilibrium point depends upon the harvesting effort employed and the value of additional food resource available for the predator species. It has been observed that if the harvesting effort for prey species is such that E_1 = 0.9, then both the prey and predator species extinct from the environment, which is not good for the biodiversity. On decreasing the value of harvesting effort of prey to E_1 = 0.45 and keeping E_1 = 0.8, it has been observed that the equilibrium point E_1 = (2,0) exist and stable. So a reasonable harvesting of prey and predator species is helpful in preserving the prey population. Further it has been found that if E_1 = 0.45, E_2 = 0.8, A = 0.5, then both the prey and predator populations survive. Uniform persistence of the system is also discussed. Numerical simulations are given to justify the obtained theoretical results.

CONFLICT OF INTEREST STATEMENT

Authors declare that they have no conflict of interest.

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