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Rotational Effect on Rayleigh Waves in Transversely Isotropic Medium with Two Temperature in Three Phase Lag Model

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ABSTRACT: The present study is concerned with Rayleigh wave propagation in homogenous transversely isotropic medium. Effect of rotation on Rayleigh waves in thermo-elastic half space in the presence of magnetic field at two temperature is discussed in context of Three-phase-Lag (TPL) Model . The expression for displacement components, stresses and temperature distribution are obtained using Normal Mode Analysis and closed form of frequency equation is derived, particular cases for thermally insulated and isothermal are discussed. Effect of rotation on attenuation coefficient and Rayleigh wave velocity with respect to frequency and wave number are presented graphically.

Keywords: Attenuation coefficient, Frequency, Isotropic, Normal Mode Analysis, Phase velocity, Rayleigh waves.

I. INTRODUCTION

It is well known fact that in classical Coupled Theory (CT) of thermo-elasticity concluded that if thermal disturbance is applied to material that conducting heat, then effects of disturbance can be instantaneously felt infinitely far away from its source and heat governing equation is in parabolic form emits thermal signals of infinite speed. To overcome this short coming of classical theory, in the last few decades, various theories have been developed. Lord-Shulman [1] developed a theory that discard the hypothesis proposed by classical coupled theory and formulated the generalized theory of thermo-elasticity in which the coupling between strain and temperature that resulted in hyperbolic equation. Green-Lindsay [2] developed another form of generalised theory , that uses entropy production inequality in constitutive relations. Green-Naghadi [3-5] formulated three models of generalized thermo-elasticity of homogenous isotropic material. In GN-I theory constitutive relations in terms of both linear and non-linear theories.GN-II developed the thermo-elastic theory without energy dissipation and GN-III developed the theory with energy dissipation. Bromwich [6] studied the propagation of shocks in the compressible material which has different elastic constant from outer body to inner body where gravity has not been taken into consideration. Kolsky [7] investigated the mechanical pulses in polymer using condenser microphone on one side and a small detonator on other side. The shape of pulses had been predicted using Fourier analysis on the condition that frequency range is known. Nowinski [8] studied thermo-elasticity graphically as well as analytically. Puri and Cowin [9] analysed the material with voids for the propagation of harmonic waves and discovered the dilation waves, carrying volume fraction and these waves are dissipative and dispersive and attenuated to each other. Chandrasekharaiah [10] studied the relevant literature of existing theories of thermo-elasticities and derived the governing equations. Hawaa and Neyfeh [11] investigated anisotropic plate's layers for harmonic waves propagation in context of generalized thermo-elasticity and each layers have degree of symmetry in terms of thermo-elasticity. Abd-Alla [12] studied the influence of initial stress and gravity on the orthotropic medium . Sharma *et al.*, [13] studied the propagation of thermo-elastic waves in the condition of stress free for homogenous isotropic plates and derived frequency equation for isothermal and thermally insulated surfaces. Choudhri [14] formulated Three-Phase-Lag (TPL) model for thermo-elasticity in which heat flux vector, thermal displacement and gradient are the key components in constitutive relations. Singh *et al.*, [15] studied the transversely isotropic medium in the presence of initial stress and magnetic field for the Rayleigh wave propagation. Shaw and Mukhopadahay [16] analyzed micro polar isotropic solid for the Rayleigh wave proliferation in context Three-Phase-Lag model and obtained frequency equation for both thermally insulated and isothermal surfaces. Biswas *et al.,* [17] analyzed homogenous orthotropic thermo-elastic medium for Rayleigh wave propagation in context of the Three-Phase-Lag model. Chandrasekhariah [18] reviewed the hyperbolic thermo-elasticity that includes temperature dependent and thermal relaxation components. Tzou [19] proposed the relationship of temperature gradient and heat flux vector to show the important behaviour of diffusion, wave and phonon-electron interactions.

Buchwald [20] studied the isotropic media for the Rayleigh wave propagation in which free plane is parallel or normal to the symmetry of direction of rotation. Biswas *et al.*, [21] studied the influence of rotation in the presence of magnetic field for the propagation of Rayleigh waves in thermo-elastic isotropic medium and discussed the path of particles in waves. Schoenberg and Censor [22] studied the harmonic waves in elastic medium with rotation in which centripetal and corolis accelerations included in the equations of motion of rotating media . Chen and Gurtin [23] constructed the theory that involved non-simple material at two different temperature assumed the condition that entropy, internal energy, heat flux and thermodynamic temperature depends upon conductive temperature's present value. Chen *et al.*, [24] developed the theory of thermo-elasticity involving two temperature conductive ($\emptyset$) and thermodynamic (T) .This theory introduced material constant *a** if this parameter tends to zero then conductive temperature is equal to thermodynamic temperature and hence transformed into classical theory. Warren and Chen [25] studied the wave propagation within the framework of two-temperature theory .Youseef [26] constructed a theory of generalized thermo-elasticity by considering the hypothesis that heat supply in elastic bodies depends upon conductive and thermodynamic temperature, which are at different level and derived the equation for homogenous isotropic body in terms of two temperature, Youseef [27] formulated a new theory of thermo elasticity of two temperature without energy dissipation and by taking into consideration GN-II model a general uniqueness theorem has been developed without energy dissipation. Lotfy and Wafaa [28] studied the effect of rotation on the homogenous isotropic half space in context of two temperature theory. Singh [29] analysed the homogenous isotropic Thermo elastic medium at two temperature in the context of Lord-Shulman Theory. Maganaet.al [30] investigated the stability of Taylor series approximation for phase Lag at two temperature. Ezzat *et al.*, [31] formulated the new model of thermo elasticity involving two temperature with time delay and Kernel Function and proved the Taylor Theorem for memory dependent derivatives. Makhopadhyay and Prasad [32] analysed the harmonic wave propagation in rotating homogenous isotropic elastic medium with angular velocity in terms of two temperature thermo-elasticity. Kumar and Mukhopadhyay [33] analysed the medium with cavity at two temperature Green Lindsay theory under the influence of temperature dependent thermal conductivity. Makhopadhyay *et al.*, [34] discussed the two temperature linear thermo elasticity in the frame work Hilbert Space Ibarahim and Youssef [35] developed a new model of thermo elasticity depend upon fractional order of strain and derived system of differential equations for governing equations having one temperature and two temperature.

Since the nineteenth century, Geophysicists shown keen interest in surface waves produced during the seismic movement. The most destructive surface waves are Rayleigh surface waves that transmits seismic range phase velocities along stress free surface and amplitude continuously decaying exponentially with increase in depth. In case of stratified semi-infinite medium, these waves are dispersive, otherwise these are non-dispersive.

In the present study, an effort has been made to study the influence of rotation in the presence of magnetic field for the Rayleigh surface wave propagation in transversely isotropic medium at two temperature in the context of Three Phase Lag (TPL) Model. Frequency equation has been derived for particular cases of Isothermal and thermally insulated. Effects of variation of rotation in the presence of magnetic field on attenuation coefficient on Rayleigh wave velocity has been demonstrated graphically. Comparison of attenuation coefficient and Rayleigh wave velocity for various thermo-elastic models with respect to wave number has been discussed.

II. GOVERNING EQUATIONS

Maxwell Equation are followed as

$$\nabla\times\vec{H}=\vec{J},\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t},\nabla.\vec{B}=0,\vec{B}=\mu_e\vec{H} \quad (1)$$

Maxwell Stress

$$\overline{\sigma_{ij}}=\mu_e(H_ih_j+H_jh_i-(\vec{H}.\overline{h})\delta_{ij}) \quad (2)$$

In deformable media modified Ohm's is written as

$$\vec{J}=\sigma[\vec{E}+\frac{\partial\vec{u}}{\partial t}\times\vec{B}] \quad (3)$$

The Strain-Displacement Relation:

$$e_{ij}=\frac{1}{2}(u_{i,j}+u_{j,i}) \quad (4)$$

The Energy equation

$$-q_{i,i}=\rho T_0\dot{S} \quad (5)$$

The Energy-Strain-Temperature relation

$$\rho S=\frac{\rho c_E}{T_0}\Theta+\beta_{ij}e_{ij} \quad (6)$$

The Stress- Displacement – Temperature are given as

$$\sigma_{11}=c_{11}u_{1,1}+c_{13}u_{3,3}-\beta\Theta$$

$$\sigma_{33}=c_{13}u_{1,1}+c_{11}u_{3,3}-\beta\Theta$$

$$\sigma_{13}=c_{44}[u_{1,3}+u_{3,1}] \quad (7)$$

The Two- Temperature relation

$$\Theta=T-a^*(T_{,11}+T_{,33}) \quad (8)$$

The dynamical equations of elastic medium under rotation in the effect of Lorentz force

$$\sigma_{11,1}+\sigma_{31,3}+F_{x_1}=\rho(\ddot{u}_1+\Omega^2u_1+2\Omega\dot{u}_3)$$

$$\sigma_{13,1}+\sigma_{33,3}+F_{x_3}=\rho(\ddot{u}_3-\Omega^2u_3-2\Omega\dot{u}_1) \quad (9)$$

The Modified Fourier Law under Three Phase Lag Model

$$-K_{ij}\left[1+\tau_T\frac{\partial}{\partial t}\right]T_{,ij}-K_{ij}^*\left[1+\tau_v\frac{\partial}{\partial t}\right]v_{,ij}=\left[1+\tau_q\frac{\partial}{\partial t}+\frac{\tau_q^2}{2}\frac{\partial^2}{\partial t^2}\right]q_{i,i} \quad (10)$$

where $\frac{\partial v_{,ij}}{\partial t}=T_{,ij}$

III. NOTATIONS

σ = Conductivity of Material

μ_e = magnetic permeability

k = Wave number

a* = Parameter of two temperature

T_0 = Body's Reference Temperature

$\vec{E}$ = Electric field

Θ = Conductive Temperature

c = Rayleigh wave's phase velocity

$\vec{H} = \vec{H}_0 + \vec{h}$ where $\vec{H}_0$ initial magnetic field and $\vec{h}$ perturbed magnetic field and $\vec{H}$ is magnetic field

$\sigma_{11}, \sigma_{33}, \sigma_{13}$ Stress Tensor Components

K_1 and K_3 = components of thermal conductivity

K_1^* and K_3^* = components of material constant characteristic of elastic Solid,

c_e = Specific heat at constant strain

β_1, β_3 = thermal Modulus

IV. FORMULATION OF THE PROBLEM

Consider an isotropic thermo-elastic medium which have $x_3 = 0$ is at stress free surface. Assume that the plane strain problem parallel to $x_1\, x_3$ plane, propagation of Rayleigh wave is along the direction of x_1 axis in unstressed and unstrained medium and the displacement vector $\bar{u} = (u_1, 0, u_3)$, constant magnetic field vector $H_0 = (0, H_0, 0)$

$$c_{11}u_{1,11} + c_{44}u_{1,33} + (c_{13} + c_{44})u_{3.13} - \beta_1\Theta,_1 + \mu_e H_0^2(u_{1,11} + u_{3,13}) = \rho(\ddot{u}_1 + \Omega^2 u_1 + 2\Omega\dot{u}_3) \quad (11)$$

$$(c_{44} + c_{13})u_{1,13} + c_{44}u_{3,11} + c_{33}u_{3,33} - \beta_3\Theta,_3 + \mu_e H_0^2(u_{1,13} + u_{3,33}) = \rho(\ddot{u}_3 - \Omega^2 u_3 - 2\Omega\dot{u}_1) \quad (12)$$

$$K_1\left[1 + \tau_T \frac{\partial}{\partial t}\right]\frac{\partial T_{,11}}{\partial t} + K_3\left[1 + \tau_T \frac{\partial}{\partial t}\right]\frac{\partial T_{,33}}{\partial t} + K_1^*\left[1 + \tau_v \frac{\partial}{\partial t}\right]T_{,11} + K_3^*\left[1 + \tau_v \frac{\partial}{\partial t}\right]T_{,33} = \left[1 + \tau_q + \frac{\tau_q^2}{2}\frac{\partial^2}{\partial t^2}\right]\frac{\partial^2}{\partial t^2}\left[\rho c_e\Theta + T_0(\beta_1 u_{1,1} + \beta_3 u_{3,3})\right] \quad (13)$$

$$K_1 = K_{11}, K_3 = K_{33}, K_1^* = K_{11}^*, K_3^* = K_{33}^*$$

V. BOUNDARY CONDITIONS

The Mechanical and thermal boundary conditions at thermally stress free surface are as follows:

(a) Tangential stress component vanished

$$\sigma_{13} + \bar{\sigma}_{13} = 0 \quad (14)$$

(b) Normal stress component vanished

$$\sigma_{33} + \bar{\sigma}_{33} = 0 \quad (15)$$

(c) Thermal conditions $q_3 + m\Theta = 0$ For thermally insulated surface $m \to 0$ and for isothermal surface $m \to \infty$, $\overline{\sigma_{33}}$ and $\overline{\sigma_{13}}$ are Maxwell stress Tensor (16)

VI. SOLUTION OF THE PROBLEM

The relation between displacement components u_1, u_3 and displacement potentials $\phi(x_1, x_3, t), \psi(x_1, x_3, t)$ is as follows:

$$u_1 = \frac{\partial\phi}{\partial x_1} - \frac{\partial\psi}{\partial x_3}; u_3 = \frac{\partial\phi}{\partial x_3} + \frac{\partial\psi}{\partial x_1}$$

Substitute the value of u_1 and u_3 in Eqns. (11-13) we obtain the following equations

$$c_{11}\phi_{,11} + (c_{13} + 2c_{44})\phi_{,33} - \beta_1\theta + \mu_e H_0^2(\phi_{,11} + \phi_{,33}) = \rho(\ddot{\phi} - \Omega^2\phi + 2\Omega\dot{\psi}) \quad (17)$$

$$-c_{11}\psi_{,11} - c_{44}\psi_{,33} + (c_{13} + c_{44})\psi_{,11} + \mu_e H_0^2(-\psi_{,11} + \psi_{,33}) = \rho(-\ddot{\psi} - \Omega^2\psi + 2\Omega\dot{\phi}) \quad (18)$$

$$(c_{44} + c_{13})\phi_{,11} + c_{44}\phi_{,11} + c_{33}\phi_{,33} - \beta_3\theta + \mu_e H_0^2(\phi_{,11} + \phi_{,33}) = \rho(\ddot{\phi} - \Omega^2\phi + 2\Omega\dot{\psi}) \quad (19)$$

$$(c_{33} - c_{44} - c_{13})\psi_{,33} + c_{44}\psi_{,11} = \rho(\ddot{\psi} - \Omega^2\psi + 2\Omega\dot{\phi}) \quad (20)$$

$$K_1\left[1 + \tau_T \frac{\partial}{\partial t}\right]\frac{\partial T_{,11}}{\partial t} + K_3\left[1 + \tau_T \frac{\partial}{\partial t}\right]\frac{\partial T_{,33}}{\partial t} + K_1^*\left[1 + \tau_v \frac{\partial}{\partial t}\right]T_{,11} + K_3^*\left[1 + \tau_v \frac{\partial}{\partial t}\right]T_{,33} = \left[1 + \tau_q + \frac{\tau_q^2}{2}\frac{\partial^2}{\partial t^2}\right]\frac{\partial^2}{\partial t^2}\begin{bmatrix}\rho c_e\theta + T_0(\beta_1(\phi_{,11} - \psi_{,13}) \\ + \beta_3(\phi_{,33} + \psi_{,31}))\end{bmatrix} \quad (21)$$

only Eqns. (17), (20) and (21) are considered to be the solutions that satisfy the boundry conditions..

VII. NORMAL MODE ANALYSIS

Consider the harmonic wave propagation along x_1 direction and the solution of above Eqns.(17), (20) and (21) in the following form:

$$\phi = F(x_3)e^{ik(x_1 - ct)}$$
$$\psi = G(x_3)e^{ik(x_1 - ct)} \quad (22)$$
$$T = H(x_3)e^{ik(x_1 - ct)}$$

Substituting the value of Eqn. (22) in Eqns. (17), (20-21)

$$[(c_{13} + 2c_{44} + \mu_e H_0^2)D^2 - (c_{11} + \mu_e H_0^2)k^2 + \rho k^2c^2 + \rho\Omega^2]F + 2ikc\rho\Omega G - \beta_1(1 + a^*k^2 - a^*D^2)H = 0 \quad (23)$$

$$[(c_{33} - c_{44} - c_{13})D^2 + (\rho k^2c^2 + \rho\Omega^2 - c_{44}k^2)]G - 2ikc\rho\Omega F = 0 \quad (24)$$

$$[\{ik^3ck_1\tau_1 - k^2\tau_2 k_1^* + \rho c_e(1 + a^*k^2 - a^*D^2)k^2c^2\} + (k_3^*\tau_2 - ik_3kc\tau_1)D^2]H + (T_0\beta_3D^2 - T_0\beta_1k^2)k^2c^2F + (ik\beta_3T_0 - ik\beta_1T_0)k^2c^2DG = 0 \quad (25)$$

where $D^2 = \frac{d^2}{dz^2}$, $\tau_5 = 1 - ikc\tau_q - \frac{k^2c^2\tau_q^2}{2}$

$\tau_4 = 1 - ikc\tau_v$, $\tau_3 = 1 - ikc\tau_T$, $\tau_2 = \frac{\tau_4}{\tau_5}$

$\tau_1 = \frac{\tau_3}{\tau_5}$

solving the Eqns. (23-25) for we get the following relation

$$(D^6 + RD^4 + SD^2 + T)(F(x_3), G(x_3), H(x_3)) = 0$$

which can be written as

$$(D^2 - r_1^2)(D^2 - r_2^2)(D^2 - r_3^2)(F, G, H) = 0 \quad (26)$$

and its characteristic Equation can be written as

$$\lambda^6 + R\lambda^4 + S\lambda^2 + T = 0 \quad (27)$$

where r_1, r_2 and r_3 are the positive solutions of above characteristic Eqn. (27)

$$r_1 = \sqrt{\frac{1}{3}[2j\sin(f) - R]}$$

$$r_2 = \sqrt{\frac{1}{3}\left[-R - j(\sqrt{3}\cos f + \sin f)\right]}$$

$$r_3 = \sqrt{\frac{1}{3}\left[-R + j(\sqrt{3}\cos f + \sin f)\right]}$$

where $j = \sqrt{R^2 - 3S}$, $f = \frac{\sin^{-1} d}{3}$ and

$$d = \frac{2R^3 - 9RS + 27T}{2j^3}$$

$$R = \left[\frac{a^* b_7 b_{13}(b_1 - b_2)k^4}{(b_7 - b_6)\left[a^*\{(b_1+b_3)b_{13} + b_{12}\beta_1\} - (b_1+b_3)(b_9 - b_{10})\right]}\right]$$
$$+\left[\frac{(b_6 - b_7)\{(b_1+b_3)(b_{15} + b_5 b_{13} - b_8)\}k^2}{(b_7 - b_6)\left[a^*\{(b_1+b_3)b_{13} + b_{12}\beta_1\} - (b_1+b_3)(b_9 - b_{10})\right]}\right]$$
$$+\left[\frac{\{(b_2+b_3)(b_{10} - b_9) - a^* b_4 b_{13} + a^* b_{11}\beta_1\}k^2}{(b_7 - b_6)\left[a^*\{(b_1+b_3)b_{13} + b_{12}\beta_1\} - (b_1+b_3)(b_9 - b_{10})\right]}\right]$$
$$+\left[\frac{\{b_7(b_1+b_3)(b_{10} - b_9)k^2 + a^* b_7 b_{12}\beta_1\}k^2}{(b_7 - b_6)\left[a^*\{(b_1+b_3)b_{13} + b_{12}\beta_1\} - (b_1+b_3)(b_9 - b_{10})\right]}\right]$$
$$+\left[\frac{b_4(b_9 - b_{10})(b_6 - b_7 + b_1 + b_3)}{(b_7 - b_6)\left[a^*\{(b_1+b_3)b_{13} + b_{12}\beta_1\} - (b_1+b_3)(b_9 - b_{10})\right]}\right]$$
$$+\left[\frac{\beta_1 b_5 b_{12}(b_6 - b_7) - a^* b_4 b_{12}\beta_1}{(b_7 - b_6)\left[a^*\{(b_1+b_3)b_{13} + b_{12}\beta_1\} - (b_1+b_3)(b_9 - b_{10})\right]}\right]$$

$$S = \left[\frac{(b_1+b_3)\{(b_{15} - b_8 + b_5 b_{13})(b_4 - b_7 k^2)k^2\}}{(b_7 - b_6)\left[a^*\{(b_1+b_3)b_{13} + b_{12}\beta_1\} - (b_1+b_3)(b_9 - b_{10})\right]}\right]$$
$$+\left[\frac{(b_2+b_3)\{b_5 b_{13}(b_7 - b_6)\}k^4}{(b_7 - b_6)\left[a^*\{(b_1+b_3)b_{13} + b_{12}\beta_1\} - (b_1+b_3)(b_9 - b_{10})\right]}\right]$$
$$+\left[\frac{[(b_6 - b_7)(b_8 - b_{15}) + b_7(b_9 - b_{10}) + a^* b_4 b_{13}]k^4}{(b_7 - b_6)\left[a^*\{(b_1+b_3)b_{13} + b_{12}\beta_1\} - (b_1+b_3)(b_9 - b_{10})\right]}\right]$$
$$+\left[\frac{b_4(b_{10} - b_9)k^2 - a^* b_7 b_{13}k^6}{(b_7 - b_6)\left[a^*\{(b_1+b_3)b_{13} + b_{12}\beta_1\} - (b_1+b_3)(b_9 - b_{10})\right]}\right]$$
$$+\left[\frac{b_4(b_6 - b_7)(b_{15} - b_8 + b_5 b_{13})k^2}{(b_7 - b_6)\left[a^*\{(b_1+b_3)b_{13} + b_{12}\beta_1\} - (b_1+b_3)(b_9 - b_{10})\right]}\right]$$
$$-\left[\frac{a^* b_4^2 b_{13}k^2 + b_4(b_{10} - b_9)(b_7 k^2 - b_4) - a^* b_7 b_{11}\beta_1 k^4}{(b_7 - b_6)\left[a^*\{(b_1+b_3)b_{13} + b_{12}\beta_1\} - (b_1+b_3)(b_9 - b_{10})\right]}\right]$$
$$+\left[\frac{\beta_1 b_5 b_{11}(b_7 - b_6)k^2 + (a^* b_4 b_{11}\beta_1 - \beta_1 b_5 b_7 b_{12} - a^* b_{13} b_{14}^2)k^2}{(b_7 - b_6)\left[a^*\{(b_1+b_3)b_{13} + b_{12}\beta_1\} - (b_1+b_3)(b_9 - b_{10})\right]}\right]$$
$$+\left[\frac{ik\beta_1 a^* b_{14}(b_{11} - b_{12}) + \beta_1 b_5 b_4 b_{12} + b_{14}^2(b_9 - b_{10})}{(b_7 - b_6)\left[a^*\{(b_1+b_3)b_{13} + b_{12}\beta_1\} - (b_1+b_3)(b_9 - b_{10})\right]}\right]$$

$$T = \left[\frac{(b_7 k^2 - b_4)(b_{15} - b_8 + b_5 b_{13})[(b_2+b_3)k^4 - b_4 k^2]}{(b_7 - b_6)\left[a^*\{(b_1+b_3)b_{13} + b_{12}\beta_1\} - (b_1+b_3)(b_9 - b_{10})\right]}\right]$$
$$+\left[\frac{b_{14}^2[b_{15} - b_8 + b_5 b_{13}]k^2}{(b_7 - b_6)\left[a^*\{(b_1+b_3)b_{13} + b_{12}\beta_1\} - (b_1+b_3)(b_9 - b_{10})\right]}\right]$$
$$+\left[\frac{ik\beta_1 b_5 b_{14}(b_{12} - b_{11}) + \beta_1 b_5 b_{11}(b_7 k^2 - b_4)k^2}{(b_7 - b_6)\left[a^*\{(b_1+b_3)b_{13} + b_{12}\beta_1\} - (b_1+b_3)(b_9 - b_{10})\right]}\right]$$

where

$$b_1 = c_{13} + 2c_{44};\ b_2 = c_{11};\ b_3 = \mu_e H_0^2;$$
$$b_4 = \rho k^2 c^2 + \rho\Omega^2;\ b_5 = 1 + a^* k^2;\ b_6 = c_{33} - c_{13};$$
$$b_7 = c_{44};\ b_8 = k_1^*\tau_2;\ b_9 = k_3^*\tau_2;\ b_{10} = iK_3 kc\tau_1;$$
$$b_{11} = T_0\beta_1 k^2 c^2;\ b_{12} = T_0\beta_3 k^2 c^2;\ b_{13} = \rho c_e c^2;$$
$$b_{14} = 2ikc\rho\Omega;\ b_{15} = iK_1 c\tau_1$$

Thus the Eqn. (26) is bounded when $x_3 \to \infty$ can be written in other Words as follows

$$F(x_3) = \sum_{i=1}^{3} R_i \exp[-r_i x_3]$$
$$G(x_3) = \sum_{i=1}^{3} S_i \exp[-r_i x_3] \quad (28)$$
$$H(x_3) = \sum_{i=1}^{3} T_i \exp[-r_i x_3]$$

where R_i, S_i, T_i (i =1, 2, 3) are constants, using Eqn. (28) in (23-25), we obtained the following relations

$T_i = d_i R_i$ and $S_i = p_i R_i$

where $$p_i = \frac{b_{14}}{[(b_6 - b_7)r_i^2 + (b_4 - b_7)k^2]}$$

$$d_i = \frac{ik(b_{12} - b_{11})r_i p_i - (b_{12} r_i^2 - b_{11}k^2)}{[b_{15}k^2 - b_8 k^2 + b_{13}(b_5 - a^* r_i^2)k^2 + (b_9 - b_{10})r_i^2}$$

Hence, the solutions of Eqns. (17), (20) and (21) are given by

$$\phi = \sum_{i=1}^{3} R_i \exp[-r_i x_3 + ik(x_1 - ct)]$$
$$\psi = \sum_{i=1}^{3} p_i R_i \exp[-r_i x_3 + ik(x_1 - ct)] \quad (29)$$
$$T = \sum_{i=1}^{3} d_i R_i \exp[-r_i x_3 + ik(x_1 - ct)]$$

VIII. DERIVATION OF FREQUENCY EQUATION

The heat flux vector's normal component q_3 is associated to two temperature gradient $\frac{\partial\Theta}{\partial x_3}$ by the succeeding

$$q_3 = \left[\frac{-K_3(1 + \tau_T D' - K_3^*(1 + \tau_v D')}{D'(1 + \tau_q D' + \frac{\tau_q^2}{2}D'^2)}\right]\frac{\partial\Theta}{\partial x_3} \quad (30)$$

Where $D' = \frac{\partial}{\partial t}$; $\Theta = T - a^*(T_{,11} + T_{,33})$

The stress components in terms of thermo elastic potentials at two temperature are given by:

$$\sigma_{33} = c_{13}\phi_{,11} + c_{33}\phi_{,33} - (c_{13} - c_{33})\psi_{,13} - \beta_3\Theta$$

$$\overline{\sigma}_{33} = \mu_e H_0^{\,2}(\phi_{,11} + \phi_{,33})$$

Applying the condition $\sigma_{33} + \overline{\sigma}_{33} = 0$

Hence, we obtain

$$\sum_{i=1}^{3}\begin{bmatrix}(c_{33}+\mu_e H_0^2)r_i^2-(c_{13}+\mu_e H_0^2)k^2+(c_{13}-c_{33})ikr_ip_i \\ -\beta_3(b_5-a^*r_i^2)d_i\end{bmatrix}A_i=0 \quad (31)$$

$\sigma_{13} = c_{44}(2\phi_{,13} - \psi_{,33} + \psi_{,11})$ and $\overline{\sigma}_{13} = 0$

Applying the condition $\sigma_{13} + \overline{\sigma}_{13} = 0$

$$\sum_{i-1}^{3}\left[(r_i^{\,2} + k^2)p_i + 2ikr_i\right]A_i = 0 \quad (32)$$

Applying the condition $q_3 + m\Theta = 0$ hence, the equation obtained as:

$$\sum_{i=1}^{3}\left[(b_5 - a^* r_i^{\,2})(\alpha r_i + m)\right]d_i A_i = 0 \quad (33)$$

Eqns. (31-33) have non-trivial solution if

$$\left[(c_{33}+\mu_e H_0^2)r_1^2-(c_{13}+\mu_e H_0^2)k^2+(c_{13}-c_{33})ikr_1p_1-\beta_3(b_5-a^*r_1^2)d_1\right]$$

$$\begin{bmatrix}(p_2r_2^{\,2}+k^2p_2+2ikr_2)(b_5-a^*r_3^{\,2})(\alpha r_3+m)d_3 \\ -(p_3r_3^{\,2}+k^2p_3+2ikr_3)(b_5-a^*r_2^{\,2})(\alpha r_2+m)d_2\end{bmatrix}$$

$$+$$

$$\left[(c_{33}+\mu_e H_0^2)r_2^2-(c_{13}+\mu_e H_0^2)k^2+(c_{13}-c_{33})ikr_2p_2-\beta_3(b_5-a^*r_2^2)d_2\right]$$

$$\begin{bmatrix}(p_3r_3^{\,2}+k^2p_3+2ikr_3)(b_5-a^*r_1^{\,2})(\alpha r_1+m)d_1 \\ -(p_1r_1^{\,2}+k^2p_1+2ikr_1)(b_5-a^*r_3^{\,2})(\alpha r_3+m)d_3\end{bmatrix}$$

$$+$$

$$\left[(c_{33}+\mu_e H_0^2)r_3^2-(c_{13}+\mu_e H_0^2)k^2+(c_{13}-c_{33})ikr_3p_3-\beta_3(b_5-a^*r_3^2)d_3\right]$$

$$\begin{bmatrix}(p_1r_1^{\,2}+k^2p_1+2ikr_1)(b_5-a^*r_2^{\,2})(\alpha r_2+m)d_2 \\ -(p_2r_2^{\,2}+k^2p_2+2ikr_2)(b_5-a^*r_1^{\,2})(\alpha r_1+m)d_1\end{bmatrix}=0 \quad (34)$$

Eqn. (34) represent the Rayleigh wave's frequency Equation for orthotropic thermo-elastic half space at two temperature in context of TPL model

IX. PARTICULAR CASE

Case (1) Thermally Insulated Surface: By applying the boundary conditions $q_3 = 0$ on $x_3 = 0$ for Thermally Insulated surfaces, Eqn. (34) transformed to

$$\left[(c_{33}+\mu_e H_0^2)r_1^2-(c_{13}+\mu_e H_0^2)k^2+(c_{13}-c_{33})ikr_1p_1-\beta_3(b_5-a^*r_1^2)d_1\right]$$

$$\begin{bmatrix}(p_2r_2^{\,2}+k^2p_2+2ikr_2)(b_5-a^*r_3^{\,2})(r_3)d_3 \\ -(p_3r_3^{\,2}+k^2p_3+2ikr_3)(b_5-a^*r_2^{\,2})(r_2)d_2\end{bmatrix}$$

$$+$$

$$\left[(c_{33}+\mu_e H_0^2)r_2^2-(c_{13}+\mu_e H_0^2)k^2+(c_{13}-c_{33})ikr_2p_2-\beta_3(b_5-a^*r_2^2)d_2\right]$$

$$\begin{bmatrix}(p_3r_3^{\,2}+k^2p_3+2ikr_3)(b_5-a^*r_1^{\,2})(r_1)d_1 \\ -(p_1r_1^{\,2}+k^2p_1+2ikr_1)(b_5-a^*r_3^{\,2})(r_3)d_3\end{bmatrix}$$

$$+$$

$$\left[(c_{33}+\mu_e H_0^2)r_3^2-(c_{13}+\mu_e H_0^2)k^2+(c_{13}-c_{33})ikr_3p_3-\beta_3(b_5-a^*r_3^2)d_3\right]$$

$$\begin{bmatrix}(p_1r_1^{\,2}+k^2p_1+2ikr_1)(b_5-a^*r_2^{\,2})(r_2)d_2 \\ -(p_2r_2^{\,2}+k^2p_2+2ikr_2)(b_5-a^*r_1^{\,2})(r_1)d_1\end{bmatrix}=0 \quad (35)$$

Case (2) Isothermal Surface: By applying boundary condition T = 0 on $x_3 = 0$ for isothermal surfaces, Eqn. (34) transformed to

$$\left[(c_{33}+\mu_e H_0^2)r_1^2-(c_{13}+\mu_e H_0^2)k^2+(c_{13}-c_{33})ikr_1p_1-\beta_3(b_5-a^*r_1^2)d_1\right]$$

$$\begin{bmatrix}(p_2r_2^{\,2}+k^2p_2+2ikr_2)(b_5-a^*r_3^{\,2})d_3 \\ -(p_3r_3^{\,2}+k^2p_3+2ikr_3)(b_5-a^*r_2^{\,2})d_2\end{bmatrix}$$

$$+$$

$$\left[(c_{33}+\mu_e H_0^2)r_2^2-(c_{13}+\mu_e H_0^2)k^2+(c_{13}-c_{33})ikr_2p_2-\beta_3(b_5-a^*r_2^2)d_2\right]$$

$$\begin{bmatrix}(p_3r_3^{\,2}+k^2p_3+2ikr_3)(b_5-a^*r_1^{\,2})d_1 \\ -(p_1r_1^{\,2}+k^2p_1+2ikr_1)(b_5-a^*r_3^{\,2})d_3\end{bmatrix}$$

$$+$$

$$\left[(c_{33}+\mu_e H_0^2)r_3^2-(c_{13}+\mu_e H_0^2)k^2+(c_{13}-c_{33})ikr_3p_3-\beta_3(b_5-a^*r_3^2)d_3\right]$$

$$\begin{bmatrix}(p_1r_1^{\,2}+k^2p_1+2ikr_1)(b_5-a^*r_2^{\,2})d_2 \\ -(p_2r_2^{\,2}+k^2p_2+2ikr_2)(b_5-a^*r_1^{\,2})d_1\end{bmatrix}=0 \quad (36)$$

Case (3) Rayleigh wave's frequency equation in isotropic Elastic half-space: The frequency of orthotropic elastic half space is obtained as follows:

$$2(c_{33}-c_{13})\sqrt{\left(\frac{\rho c^2-c_{44}}{c_{33}-c_{44}-c_{13}}\right)\left(\frac{\rho c^2-c_{11}}{2c_{44}+c_{13}}\right)} = \left(\frac{c_{33}(\rho c^2-c_{11})}{c_{13}+2c_{44}}+c_{13}\right)\left(\frac{\rho c^2-c_{44}}{c_{33}-c_{44}-c_{13}}-1\right) \quad (37)$$

Substitute the value $c_{33}, c_{11}, c_{13}, c_{44}$ in Eqn. (27)

$$c_{33} = c_{11} = 2\mu + \lambda; c_{13} = \lambda; c_{44} = \mu$$

$$\left(2-\frac{c^2}{c_2^{\,2}}\right)^2 = 4\sqrt{\left(1-\frac{c^2}{c_2^{\,2}}\right)\left(1-\frac{c^2}{c_1^{\,2}}\right)} \quad (38)$$

Hence the resulting Eqn. (38) represents Rayleigh wave frequency equation in isotropic elastic half space.

where $c_2^{\,2} = \dfrac{\mu}{\rho}$ and $c_1^{\,2} = \dfrac{2\mu+\lambda}{\rho}$

Case (4) When we put $\tau_q = \tau_T = 0$ and $K_1^* = K_3^* = 0$ Eqn. (34) The reduced frequency equation is similar for the case of theory of classical coupled thermo elasticity and this result comply with the result obtained by Singh *et al.*, [28]

Case (5) When we take $\tau_q = \tau_T = 0$ and $\tau_q \neq 0$ in and $K_1^* = K_3^* = 0$ the Eqn. (34) reduced to frequency equation of LS Model.

Eqn. (34) transform to frequency equation of GN Model Type-III when substitute $\tau_q = \tau_T = \tau_v = 0$

X. NUMERICAL RESULTS AND DISCUSSION

Generally phase velocity ω and wave number (k) are considered as complex quantities.

If it is assumed that $c^{-1} = B^{-1} + i\omega^{-1}Q$

The wave number can be represented as $k = M + iQ$ Here $M = \frac{\omega}{B}$ in which B and Q are real. The exponent term in Eqn. (31) becomes $iM(x - Bt) - Qx$. Here B represents the propagation speed, attenuation coefficient represents Q and angular frequency of waves represented by ω

It is assumed that the succeeding values of compatible parameters used for numerical calculations as per for transversely isotropic material Hawwa and Nayfeh [11] as follows:

$c_{11} = 128\ \text{MPa}$	$T_0 = 298\ \text{K}$
$c_{13} = 6\ \text{MPa}$	$\rho = 8.836\times10^3 \text{kg/m}^3$
$c_{33} = 32\ \text{MPa}$	$K_1 = 100\text{W/mks}$
$c_{44} = 18\ \text{MPa}$	$K_3 = 25\text{W/mks}$
$c_e = 2\times10^{-4}\text{J/kg}$	$\beta = 0.04/\text{m}^2\text{k}$
$K_1^* = 17\text{W/mks}$	$K_3^* = 21\ \text{W/mks}$
$\tau_q = 2\times10^{-7}\text{s}$	$\tau_T = 1.5\times10^{-7}\ \text{s}$

$\tau_v = 1\times10^{-8}\ s$

Effect of variation of rotation in the presence of magnetic field on wave velocity and attenuation coefficient of Rayleigh waves with respect to wave number and frequency has been represented graphically at material parameter *a** = 0.2. For numerical purpose we consider $\mu_e = 1.2\ \text{Hm}^{-1}$, comparison of Rayleigh wave velocity and attenuation co-efficient for Three-Phase-Lag (TPL) Model, GN-Model and LS Model with respect to wave number has analytically calculated and graphically represented.

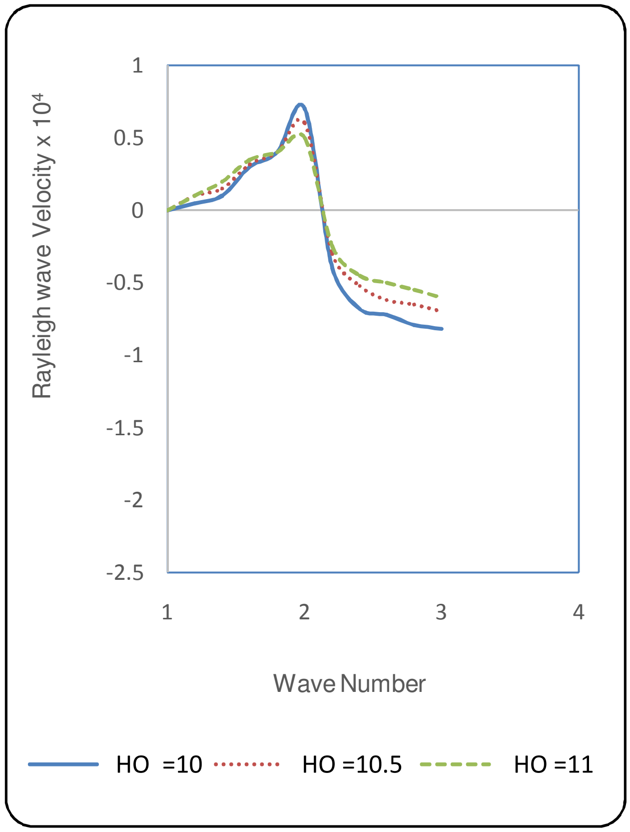

Fig. 1. Effect of variation of magnetic field on Rayleigh wave velocity for thermally insulated surfaces w.r.t. wave number.

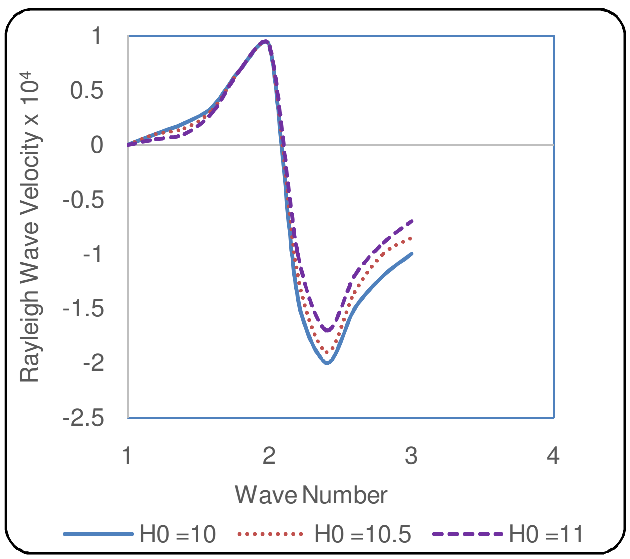

Fig. 2. Effect of variation of magnetic field on Rayleigh wave velocity for Isothermal surfaces w.r.t. wave number.

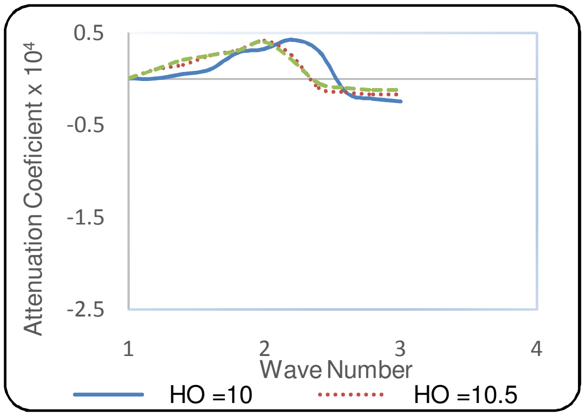

Fig. 3. Effect of variation of magnetic field on attenuation coefficient for thermally Insulated surfaces w.r.t. wave number.

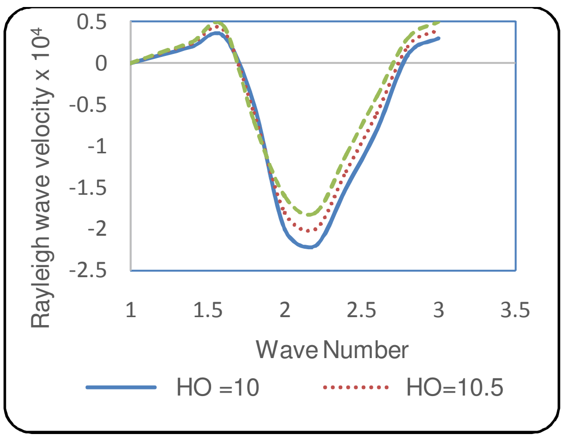

Fig. 4. Effect of variation of magnetic field on attenuation coefficient for isothermal surfaces w.r.t. wave number.

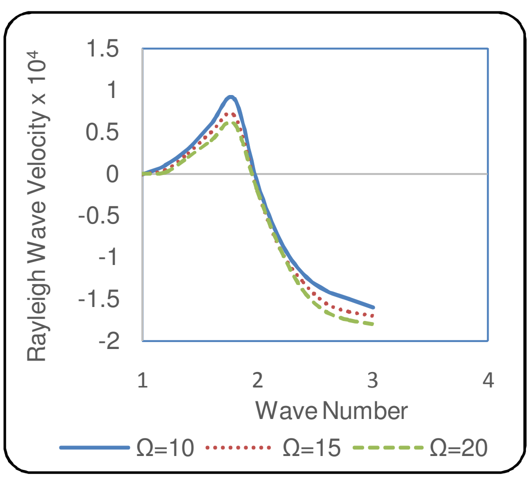

Fig. 5. Effect of variation of rotation on Rayleigh wave velocity for thermally Insulated surfaces w.r.t. wave number.

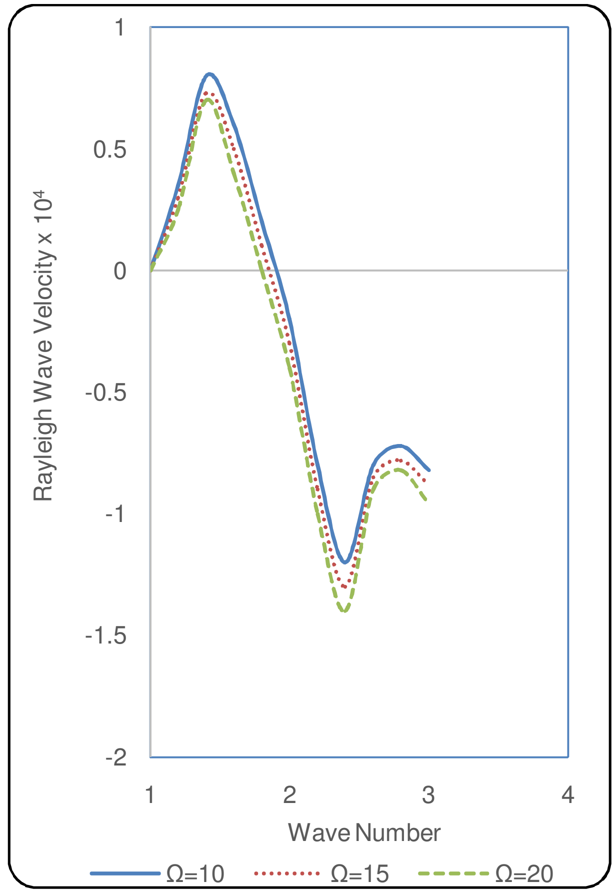

Fig. 6. Effect of variation of rotation on Rayleigh wave velocity for isothermal surfaces w.r.t. wave number.

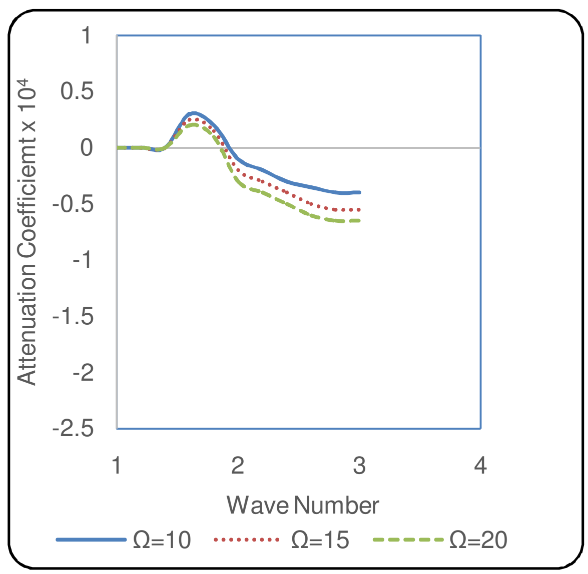

Fig. 7. Effect of variation of rotation on attenuation coefficient for thermally insulated surfaces w.r.t. wave number.

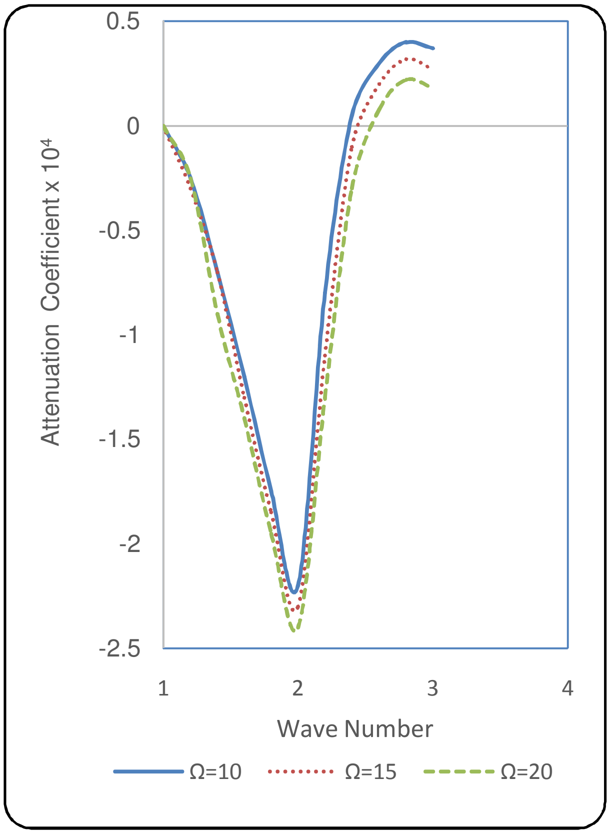

Fig. 8. Effect of variation on attenuation coefficient for isothermal surfaces w.r.t. wave number.

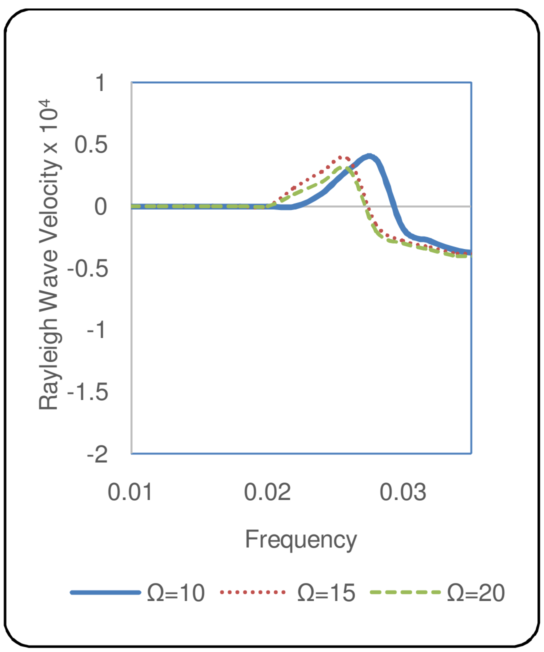

Fig. 9. Effect of variation of rotation on Rayleigh wave velocity for thermally insulated surfaces w.r.t. frequency.

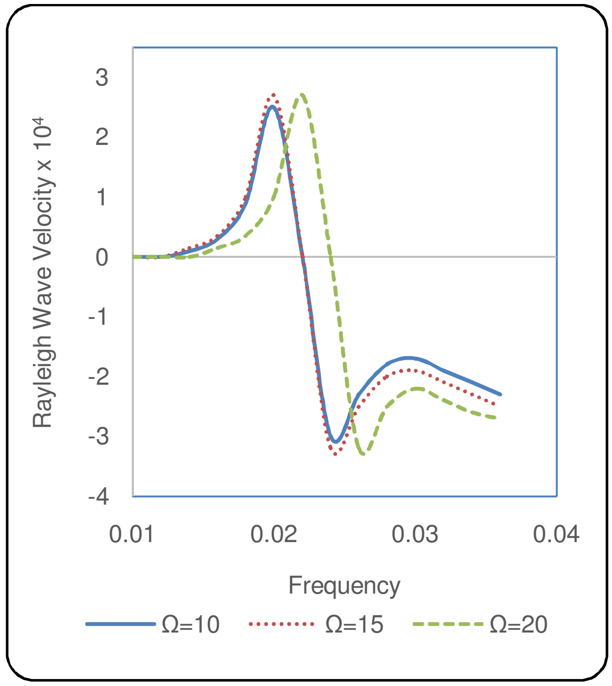

Fig. 10. Effect of variation of rotation on Rayleigh wave velocity for isothermal surfaces w.r.t. frequency.

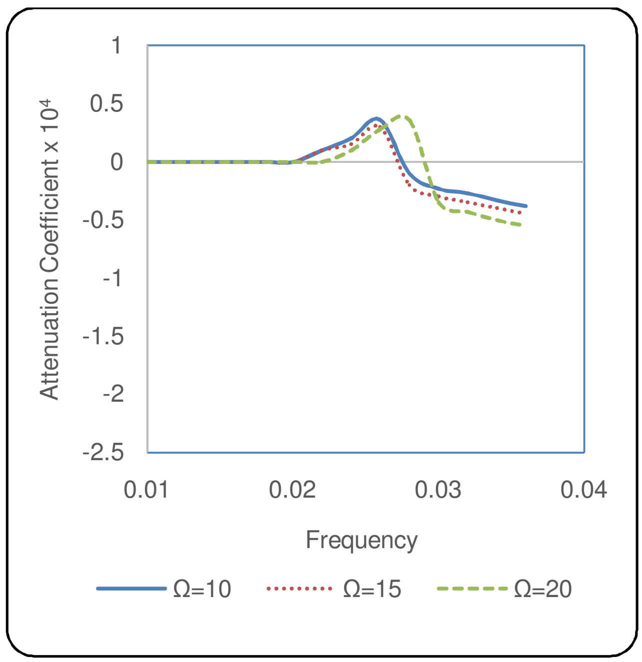

Fig. 11. Effect of variation of rotation on attenuation coefficient for thermally insulated surfaces w.r.t. frequency.

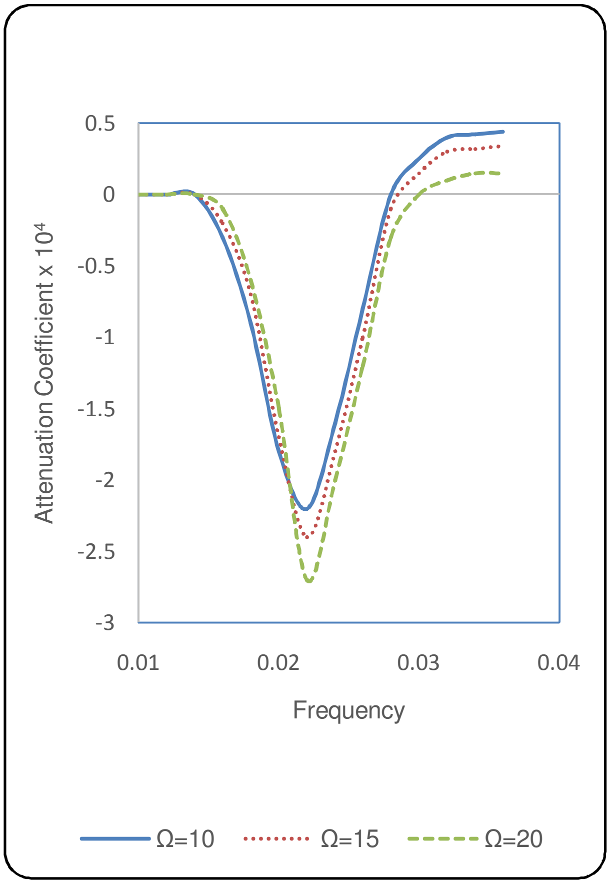

Fig. 12. Effect of variation of rotation on attenuation coefficient for isothermal surfaces w.r.t. frequency.

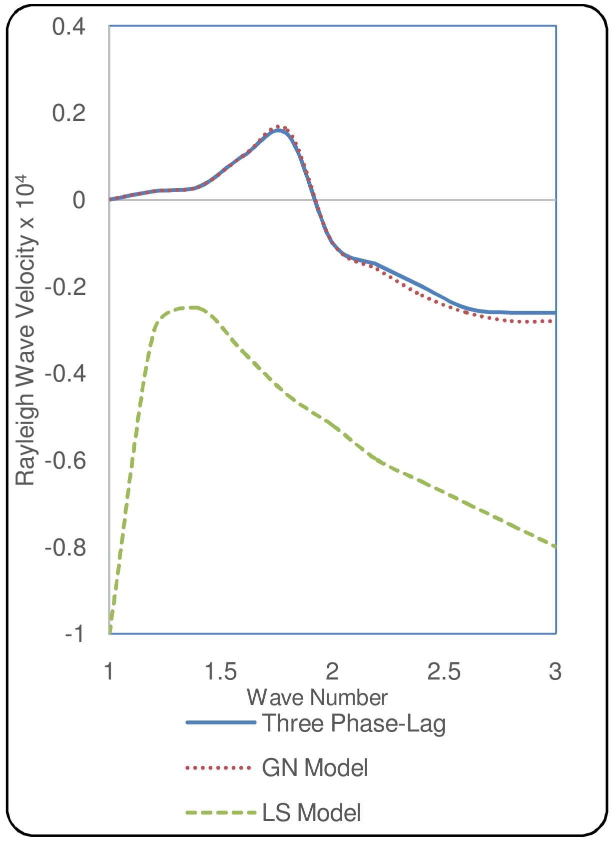

Fig. 13. Comparison of Rayleigh wave velocity with respect to wave Number for various thermoelastic Models.

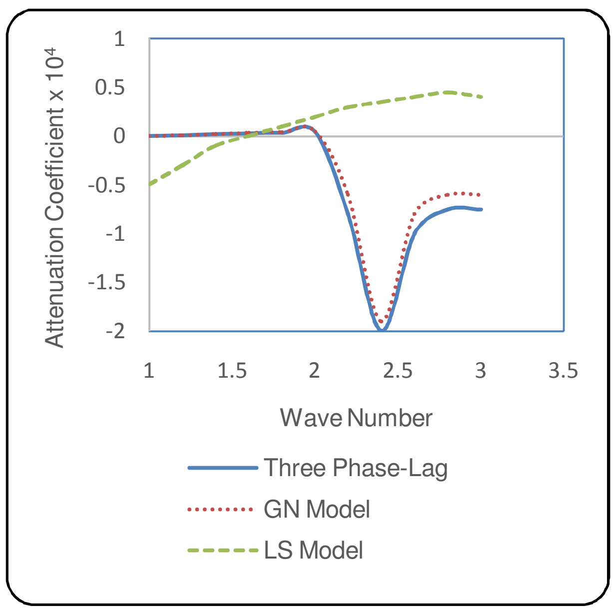

Fig. 14. Comparison of attenuation coefficient w.r.t. wave number for various thermoelastic models.

Figs. 1, 2 shows the effect of variation of magnetic field $(H_0 = 10; H_0 = 10.5; H_0 = 11)$ at material parameter at $a^* = 0.2$, on Rayleigh wave velocity with respect wave number. It has been observed that With the increase of Wave number the Rayleigh wave velocity decreased more sharply in case of Isothermal surfaces than insulated surfaces but if we consider the case of thermally insulated surfaces the effect of magnetic field shows flatten Curve about the line Rayleigh wave velocity at around zero but it is continuously going on decreasing but not as sharply.

Figs. 3, 4 shows the effect of variation of magnetic Field $(H_0 = 10; H_0 = 10.5; H_0 = 11)$ material parameter at $a^* = 0.2$ on attenuation with respect wave number, It has been observed that with the increase of wave number the attenuation Coefficient decreased more sharply in case of isothermal Surfaces than insulated Surfaces but if we Consider the case of thermally insulated surfaces the effect of magnetic field shows flatten Curve about the line Attenuation Coefficient at Zero

Figs. 5, 6 shows the effect of variation of rotation $(\Omega = 10 ; \Omega = 15 ; \Omega = 20)$ Material Parameter at a^* = 0.2, on Rayleigh wave velocity with respect wave number, it has been observed that with the increase of wave number the Rayleigh wave velocity decreased more sharply in case of isothermal surfaces than insulated surfaces but if we consider the case of Isothermal surfaces when wave number is about more than 2.5 there is increase in the Rayleigh wave velocity but it is opposite in the case of thermally insulated surfaces the wave velocity continuously decreasing with the wave number above around 2.5.

Figs. 7, 8 represent the effect of variation of rotation $(\Omega = 10 ; \Omega = 15 ; \Omega = 20)$ at Material Parameter a^* = 0.2, on attenuation coefficient with respect to wave number, it has been observed the with increase of wave number the attenuation coefficient decreased more sharply in case of Isothermal surfaces than insulated surfaces but if we consider the case of isothermal surfaces when wave number is about more than 2 there is Increase in the attenuation coefficient more sharply but it is opposite in the case of thermally insulated surfaces, the attenuation coefficient continuously decreasing with the wave number above around 2 shows more flatten curve around attenuation coefficient around zero.

Figs. 9, 10 shows the effect of variation of rotation $(\Omega = 10 ; \Omega = 15 ; \Omega = 20)$ at material parameter $a^* = 0.2$ for Rayleigh wave velocity with respect to frequency, it has been observed that with the increase of frequency, the Rayleigh wave velocity decreased more sharply in case of isothermal surfaces than insulated surfaces but if we consider the case of isothermal surfaces when frequency is about more than 0.025 there is increase in the Rayleigh wave velocity more sharply but it is opposite in the case of thermally insulated surfaces, the wave velocity continuously decreasing with the frequency above around 0.025 shows more flatten curve around Rayleigh wave velocity around zero but continuously decreasing.

Figs. 11, 12 shows the effect of variation of rotation $(\Omega = 10 ; \Omega = 15 ; \Omega = 20)$ at material parameter a^* = 0.2 on attenuation coefficient with respect to frequency. It has been noticed that with the increase of frequency the attenuation coefficient decreased more sharply in case of isothermal surfaces than insulated surfaces, but if we consider the case of isothermal surfaces when frequency is about more than 0.023, there is increase in the attenuation coefficient more sharply but it is opposite in the case of thermally insulated surfaces the wave velocity continuously decreasing with the frequency above around 0.027 shows more flatten curve around attenuation coefficient around zero but continuously decreasing.

Fig. 13 shows the comparison of Rayleigh Wave Velocity with respect to Wave Number for different Thermo-elastic Models (Three-Phase-Lag (TPL) Model, GN-Model, LS-Model) at material Parameter a^* = 0.2, it has been observed that in the both cases of (Three-Phase-Lag (TPL) Model, GN Model) Rayleigh wave velocity escalated with the surge of wave number up to 2 but as wave number increase more than 2, the Rayleigh wave velocity continuously going on decrease not sharply. As far as curve of LS Model Concerned it Rayleigh wave velocity firstly increase with the surge of Wave Number but it decreases sharply when wave number increase about 1.3 and curve of Three Phase Lag Model and GN Model almost Coincide with each Other.

Fig. 14 shows the comparison of variation of attenuation coefficient w.r.t. wave number for different thermo-elastic Models (Three-Phase-Lag (TPL) Model, GN-Model, LS-Model) at material Parameter a^* = 0.2, it has been observed that in both cases of (Three-Phase-Lag (TPL) Model, GN Model) the magnitude of attenuation coefficient declined sharply , with the surge of wave number in the range between 2 and 2.5 but as wave number increase more than 2.5, the attenuation coefficient continuously going on increasing sharply. In case of LS Model concerned, attenuation coefficient increase with the increase of wave number and the curves of Three Phase Lag Model and GN Model almost coincide with each other.

XI. CONCLUSION

The propagation of Rayleigh wave in the influence of rotation with two temperature in the purview of Three-Phase-Lag (TPL) has been investigated. The frequency equations for particular cases such as thermally insulated surfaces and isothermal surfaces has been derived. Effect of variation of rotation in the presence of magnetic field has been demonstrated graphically on the Rayleigh wave velocity and attenuation coefficient. Comparison of Three-Phase-Lag (TPL), GN Model, LS Model for Rayleigh wave velocity and attenuation coefficient with respect to wave number has been discussed. Based upon above numerical discussion and theoretical calculations, it can be concluded that if the value of rotation is increased, the magnitude of Rayleigh wave velocity and attenuation coefficient is decreased. And with the increase in magnetic field, the magnitude of Rayleigh wave velocity and attenuation coefficient is increased. In isothermal Surfaces, attenuation Coefficient and Rayleigh wave velocity shows sharp variation as compared to thermally insulated surfaces. Rayleigh wave velocity attains more value for Three-Phase-Lag (TPL) and GN Model as compared to LS Model. Both the Curves for Three-Phase-Lag (TPL) Model and GN Model coincide with each other. All these analytical calculations are theoretical one but this result can be helpful for researchers working in the field of Seismology and Geophysics.

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