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Fixed Point Theorems for Contractive Mapping in Fuzzy Soft Metric Space

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ABSTRACT: In this paper, we establish a fixed point theorem for multi-valued contractive mappings using Banach contraction principle in the setting of complete fuzzy soft metric space. Our results are the extensions of the result obtained by Sayed and Alahmari (Annals of Fuzzy Mathematics and Informatics, 15(1), pp. 73-87, 2018) to the case of complete fuzzy soft metric space. In order to validate our establish theorems we also provide some examples.

Keywords: Fixed point theorem, soft set, fuzzy set, fuzzy metric space, fuzzy soft metric space, multi-valued contractive maps.

I. INTRODUCTION

The development of fuzzy mathematics started with a presentation of the thought of fuzzy sets by Zadeh [25] in 1965 as another approach to speak to unclearness in regular day to day existence. The theory of soft sets started by Molodstov [23], which help to solve problems in all areas. After then, the properties and applications on this theorem have been studied by many authors [8, 9, 11, 18, 19]. Maji *et al.*, [17] initiated several operations in soft sets and has also coined fuzzy soft sets. Many interesting applications of soft set theory have been expanded by embedding the ideas of fuzzy sets [1, 2, 5, 7, 13, 17, 20]. In [3] Beaula *et al.,* were introduced a definition of the fuzzy soft metric space and also give the concept of fuzzy soft open balls and fuzzy soft closed balls.

The Banach contraction principle is certainly a classical result of modern analysis. This principle has been extended and generalized in different directions in metric spaces [21]. In (1988), Grabiec [13] initiated the study of the fixed point theory in fuzzy metric spaces. In (2002), Gregori and sapena [14] introduced new kind of contractive mappings in modified fuzzy metric spaces and proved a fuzzy version of Banach contraction principle [15, 21, 22, 24]. In this paper, we proved a fixed point theorem for multi-valued contractive mappings using Banach contraction principle in the setting of complete fuzzy soft metric space.

Theorem 1.1. Let (X,d) be a complete fuzzy metric space and let $T: X \to 2^X$ be a multivalued map such that Tx is a closed subset of X for all $x \in X$.

Let $J_b^x = \{y \in Tx: bd(x,y) \leq d(x,Tx)\}$, where $b \in (0,1)$.

If there exists a constant $c \in (0,1)$ such that for any $x \in X$, there exists $y \in J_b^x$ satisfying

$$d(x,Ty) \leq cd(x,y)$$

Then T has a fixed point in X, i.e., there exists $z \in X$ such that $z \in Tz$ provided $c < b$ and the function $d(x,Tx)$, $x \in X$ is lower semi-continuous.

Here we establish a Banach contraction fixed point theorem for contractive mappings in complete fuzzy soft metric spaces.

II. PRELIMINARIES AND DEFINITIONS

In this section we first give some basic definitions

Definition 2.1 [12] Let $(X, M, *)$ be a fuzzy metric space.

1. A sequence $\{x_n\}$ in X is called a Cauchy sequence if and only if $\lim_{n\to\infty} M\left(x_{n+p}, x_n, t\right) = 1$ for each $p > 0, t > 0$.
2. A sequence $\{x_n\}$ in X is converging to x in X if and only if $\lim_{n\to\infty} M\left(x_n, x, t\right) = 1$. A fuzzy metric space $(X, M, *)$ is said to be complete if and only if every Cauchy sequence in X is convergent in X.
3. A fuzzy metric space in which every sequence has a convergent subsequence is said to be compact.

Definition 2.2 [13] Let $(X, M, *)$ be a fuzzy metric space. Then, the mapping M is said to be continuous on $X \times X \times (0,\infty)$ if

$$\lim_{n\to\infty} M\left(x_{n+p}, y_n, t\right) = M(x,y,t)$$

When $\{(x_n, y_n, t_n)\}$ is a sequence in $X \times X \times (0,\infty)$ which converges to a point $(x,y,t) \in X \times X \times (0,\infty)$, *i.e.*,

$$\lim_{n\to\infty} M\left(x_{n+p}, x_n, t\right) = \lim_{n\to\infty} M\left(y_{n+p}, y_n, t\right) = 1 \text{ and } \lim_{n\to\infty} M\left(x, y, t_n\right) = M(x,y,t).$$

Definition 2.3 [16] Let $\mathcal{CB}(X)$ denote the set of all nonempty closed bounded subsets of X. Then for every $A, B, C \in \mathcal{CB}(X)$ and $t > 0$,

$M_{\overline{\nabla}}(A,B,t) = \min\{\min_{a\in A} M^{\overline{\nabla}}(a,B,t), \min_{b\in B} M^{\overline{\nabla}}(A,b,t)\}$

Where $M^{\overline{\nabla}}(C,y,t) = \max\{M(z,y,t): z \in C\}$.

Now, here we define fuzzy soft metric space by using soft points by the help of soft t-norm $\tilde{*}$ and give some properties of soft t-norms and fuzzy soft metric spaces. Let $\tilde{X}$ be a non-empty set and E be the nonempty set of parameters. Let $\tilde{X}$ be an absolute soft set, $SP(\tilde{X})$ be the collection of all soft points of $\tilde{X}$ and $\mathbb{R}(E^*)$ denote the set of all non-negative soft real numbers. Soft real numbers in the $[0,1]$ and $(0,\infty)$ are indicated by $[0,1](E)$ and $(0,\infty)(E)$ respectively.

Definition 2.4 [17] A soft fuzzy set S in $\tilde{X}$ is a set of ordered pairs:

$$S = \{(\widetilde{x_e}, \mu_s(\widetilde{x_e})): \widetilde{x_e} \in \tilde{X}, e \in E\},$$

where $\mu_s: \tilde{X} \to [0,1](E)$ is called the soft membership function and $\mu_s(\widetilde{x_e})$ is grade of soft membership of $(\widetilde{x_e})$ in S.

Definition 2.5 [18] Let $\tilde{*}: [0,1](E) \times [0,1](E) \to [0,1]\ (E)$. $\tilde{*}$ is called continuous soft t-norm if $\tilde{*}$ satisfying the following conditions:

(i). $\tilde{*}$ is commutative and associativite;

(ii). $\tilde{*}$ is continuous;

(iii). $\tilde{a} \tilde{*} \tilde{1} = \tilde{a}$, $\forall\, \tilde{a} \tilde{\in} [0,1](E)$.

(iv). $\tilde{a} \tilde{*} \tilde{b} \tilde{\leq} \tilde{c} \tilde{*} \tilde{d}$, whenever

$\tilde{a} \leq \tilde{c}$ and $\tilde{b} \tilde{\leq} \tilde{d}$ and $\tilde{a}, \tilde{b}, \tilde{c}, \tilde{d}\ \tilde{\in}\ [0,1]\ (E)$.

Definition 2.6 [3] A mapping $d: FSC(\widetilde{F_G}) \times FSC(\widetilde{F_G}) \to \mathbb{R}(A)^*$ is said to be a fuzzy soft metric on FE if d satisfies the following conditions.

$(FSM_1) d(\widetilde{F_{g^1}}, \widetilde{F_{g^2}}) \geq 0\ \forall \widetilde{F_{g^1}}, \widetilde{F_{g^2}} \in FSM(\widetilde{F_G})$.

$(FSM_2) d(\widetilde{F_{g^1}}, \widetilde{F_{g^2}}) = 0$ if and only $if\ \widetilde{F_{g^1}} = \widetilde{F_{g^2}}$

$(FSM_3) d(\widetilde{F_{g^1}}, \widetilde{F_{g^2}}) = d(\widetilde{F_{g^1}}, \widetilde{F_{g^2}})\ \forall \widetilde{F_{g^1}}, \widetilde{F_{g^2}} \in FSM(\widetilde{F_G})$

$(FSM_4) d(\widetilde{F_{g^1}}, \widetilde{F_{g^2}}) =$

$d(\widetilde{F_{g^1}}, \widetilde{F_{g^2}}) + d(\widetilde{F_{g^2}}, \widetilde{F_{g^3}}) \forall \widetilde{F_{g^1}}, \widetilde{F_{g^2}}, \widetilde{F_{g^3}} \in FSM(\widetilde{F_G})$

Definition 2.7: [3] Let $(\widetilde{F_G}, d, \tilde{*})$ be a fuzzy soft metric space and $\widetilde{H_G}$, be a fuzzy soft subspace of $\widetilde{F_G}$ then distance between a fuzzy soft point $\widetilde{F_g}$ and $\widetilde{H_G}$ is defined by

$$d(\widetilde{F_g}, \widetilde{H_G}) = \sup\left\{\frac{d(\widetilde{F_g}, \widetilde{H_G})}{\text{for}} \text{every fuzzy soft point } \widetilde{F_g}\ in\ \widetilde{H_G}\right\}$$

Definition 2.8: [4] A fuzzy soft metric space $(\widetilde{F_G}, d, \tilde{*})$ is said to be complete, if every Cauchy sequence in $\widetilde{F_G}$ converges to some fuzzy soft point of $\widetilde{F_G}$.

Definition 2.9: [3] Let $(\tilde{E}, \tilde{d})$ and $(\tilde{E}', \tilde{\rho})$ be two fuzzy soft metric spaces. Then the mappings $\varphi_\psi = (\varphi, \psi: (\tilde{E}, \tilde{d}) \to (\tilde{E}', \tilde{\rho})$ is called a fuzzy soft mapping, if $\varphi: \tilde{E} \to \tilde{E}'$ and $\psi: E \to \tilde{E}'$ are two mappings.

Definition 2.10: [1] Let $A \subseteq E$. A pair (F, A) is called a soft set over (X, E), if F is a mapping $F: A \to P(X)$.

III. MAIN RESULTS

Fixed point theorems for multivalued contractive mappings in fuzzy soft metric space

Let $(\widetilde{F_G}, d, \tilde{*})$ be a complete fuzzy soft metric space and let $T: \widetilde{F_G} \to G(\widetilde{F_G})$ be a multivalued mapping, we define a mapping $h: \widetilde{F_G} \to 2^{\widetilde{F_G}} \backslash \{0\}$ as $h(\widetilde{F_G}) = (\widetilde{F_G}, T(\widetilde{F_G}))$ for $b \in (0,1]$ we define a set $J_b^{\widetilde{F_G}} \in \widetilde{F_G}$ as $J_b^{\widetilde{F_G}} = \{\widetilde{F^{g'}} \in T(\widetilde{F_G}): b\left[d\left(\widetilde{F_G}, \widetilde{F^{g'}}\right) + d\left(T(\widetilde{F_G}), T\left(\widetilde{F^{g'}}\right)\right) \leq d\left(\widetilde{F^{g'}}, T\left(\widetilde{F^{g'}}\right)\right)\right\}$.

Theorem 3.1: Let $(\widetilde{F_G}, d, \tilde{*})$ be a complete fuzzy soft metric space and let $T: \widetilde{F_G} \to G(\widetilde{F_G})$ be a multivalued mapping. If there exists a constant $k \in (0,1)$ such that for any $\widetilde{F_G} \in \widetilde{F_G}$ there exists $\widetilde{F^{g'}} \in J_b^{\widetilde{F_G}}$ satisfying

$$d\left(\widetilde{F^{g'}}, T\left(\widetilde{F^{g'}}\right)\right) \leq k\left[d\left(\widetilde{F_G}, \widetilde{F^{g'}}\right) + d\left(T(\widetilde{F_G}), T(\widetilde{F^{g'}})\right) + d\left(T\left(\widetilde{F^{g'}}\right), T(\widetilde{F_G})\right)\right], \qquad (1)$$

where,

$J_b^{\widetilde{F_G}} = \{\widetilde{F^{g'}} \in T(\widetilde{F_G}): b\left[d\left(\widetilde{F_G}, \widetilde{F^{g'}}\right) + d\left(T\left(\widetilde{F^{g'}}\right), T(\widetilde{F_G})\right]\right. \leq d\left(\widetilde{F^{g'}}, T\left(\widetilde{F^{g'}}\right)\right)\}$. Then This is a fixed point in $\widetilde{F_G}$ provided $k < b$ and $h$is sequentially lower semi continuous.

Proof: For $\widetilde{F_G} \in \widetilde{F_G}$, $T(\widetilde{F_G}) \in G(\widetilde{F_G})$. For any constant $b \in (0,1) J_b^{\widetilde{F_G}}$ is nonempty.

For $\widetilde{F_1^g} \in \widetilde{F_G}, \exists\, \widetilde{F_1^g} \in J_b^{\widetilde{F_0^g}}$ such that

$$d(\widetilde{F_1^g}, T(\widetilde{F_1^g})) \leq k[d\left(\widetilde{F_0^g}, \widetilde{F_1^g}\right) + d\left(\widetilde{F_0^g}, T\left(\widetilde{F_1^g}\right)\right) + d\left(T\left(\widetilde{F_0^g}\right), \widetilde{F_1^g}\right)]$$

For $\widetilde{F_1^g} \in \widetilde{F_G}, \exists\, \widetilde{F_2^g} \in J_b^{\widetilde{F_1^g}}$, satisfying

$$d(\widetilde{F_2^g}, T(\widetilde{F_2^g})) \leq k[d\left(\widetilde{F_1^g}, \widetilde{F_2^g}\right) + d(\widetilde{F_1^g}, T\left(\widetilde{F_2^g}\right) + d(T\left(\widetilde{F_1^g}\right), \widetilde{F_2^g})$$

Continuing this process, we can find a sequence $\left\{\widetilde{F_n^g}\right\} \subset \widetilde{F_G}$ such that $\widetilde{F_{n+1}^g} \in J_b^{\widetilde{F_n^g}}$

and

$$d\left(\widetilde{F_{n+1}^g}, T\left(\widetilde{F_{n+1}^g}\right)\right) \leq k\left[d\left(\widetilde{F_n^g}, \widetilde{F_{n+1}^g}\right) + d\left(\widetilde{F_n^g}, T\left(\widetilde{F_{n+1}^g}\right)\right) + d(T\left(\widetilde{F_n^g}\right), \widetilde{F_{n+1}^g})\right], \quad n = 0,1,2,3,4, \ldots\ldots\ldots$$

Now we will show $\left\{\widetilde{F_n^g}\right\}$ Cauchy sequence in $\widetilde{F_G}$. On the one hand,

$$d\left(\widetilde{F_{n+1}^g}, T\left(\widetilde{F_{n+1}^g}\right)\right) \le k\left[d\left(\widetilde{F_n^g}, \widetilde{F_{n+1}^g}\right) + d\left(\widetilde{F_n^g}, T\left(\widetilde{F_{n+1}^g}\right)\right) + d(T\left(\widetilde{F_n^g}\right), \widetilde{F_{n+1}^g})\right],$$

$$n = 0,1,2,3,4, \dots \dots \dots \dots$$

and from the other hand

$$\widetilde{F_{n+1}^g} \in J_b^{\widetilde{F_n^g}} \Rightarrow b\left[d\left(\widetilde{F_n^g}, \widetilde{F_{n+1}^g}\right) + d\left(T\left(\widetilde{F_n^g}\right), T\left(\widetilde{F_{n+1}^g}\right)\right)\right] \le d\left(\widetilde{F_{n+1}^g}, T\left(\widetilde{F_{n+1}^g}\right)\right),$$

$$n = 0,1,2,3,4, \dots \dots \dots \dots$$

By the above inequalities we have

$$d\left(\widetilde{F_{n+1}^g}, \widetilde{F_{n+2}^g}\right) + d\left(T(\widetilde{F_{n+1}^g}), T(\widetilde{F_{n+2}^g})\right) \le \frac{k}{b} d\left(\widetilde{F_n^g}, \widetilde{F_{n+1}^g}\right),$$

$$n = 0,1,2,3,4, \dots \dots \dots \dots$$

$$d\left(\widetilde{F_{n+1}^g}, T\left(\widetilde{F_{n+1}^g}\right)\right) \le \frac{k}{b}\left(\widetilde{F_n^g}, T\left(\widetilde{F_{n+1}^g}\right)\right),$$

$$n = 0,1,2,3,4, \dots \dots \dots \dots$$

Hence, easy to prove,

$$d\left(\widetilde{F_n^g}, \widetilde{F_{n+1}^g}\right) + d\left(T\left(\widetilde{F_n^g}\right), T\left(\widetilde{F_{n+1}^g}\right)\right) \le C^n d\left(\widetilde{F_0^g}, \widetilde{F_1^g}\right),$$

$$n = 0,1,2,3,4, \dots \dots \dots \dots$$

$$d\left(\widetilde{F_n^g}, T\left(\widetilde{F_n^g}\right)\right) \le C^n d\left(\widetilde{F_0^g}, \widetilde{F_1^g}\right),$$

$$n = 0,1,2,3,4, \dots \dots \dots \dots$$

where $C = \frac{k}{b}$

For $m, n \in G, m > n$ we have

$$d\left(\widetilde{F_n^g}, \widetilde{F_m^g}\right) \le d\left(\widetilde{F_n^g}, \widetilde{F_{n+1}^g}\right) + d\left(\widetilde{F_{n+1}^g}, \widetilde{F_{n+2}^g}\right) + \dots \dots \dots \dots + d\left(\widetilde{F_{m-1}^g}, \widetilde{F_m^g}\right)$$

$$\le (C^n + C^{n+1} + \dots \dots \dots \dots + C^{u-1}) d(\widetilde{F_0^g}, \widetilde{F_1^g})$$

$$\le \frac{C^n}{1-C} d(\widetilde{F_0^g}, \widetilde{F_1^g})$$

Where $C = \frac{k}{b}$, since $k < b \Rightarrow C^n \longrightarrow 0$ as $n \longrightarrow \infty \Rightarrow \left\{\widetilde{F_n^g}\right\}$ is a Cauchy sequence, follows from the completeness of $\widetilde{F_G}, \exists\, \widetilde{F_G} \in \widetilde{F_G}$ such that $lim_{n\to\infty} \widetilde{F_n^g} = \widetilde{F_G}$.

Now we will show that $\widetilde{F_G}$ is a fixed point of T, i.e. $\widetilde{F_G} \in T(\widetilde{F_G})$. Since $\left\{\widetilde{F_n^g}\right\}$ is a Cauchy sequence converging to $\widetilde{F_G}$ and $\left\{h(\widetilde{F_n^g})\right\} = \{d\left(\widetilde{F_n^g}, T(\widetilde{F_n^g}\right)\}$ is decreasing, therefore, converges to 0. h Being lower semi-continuous, therefore, we have

$0 \le h(\widetilde{F_G}) \le \lim_{\overline{v\to\infty}} h(\widetilde{F_n^g}) = 0 \Rightarrow h(\widetilde{F_G}) = 0$. Also by closeness of $T(\widetilde{F_G}) \Rightarrow \widetilde{F_G} \in T(\widetilde{F_G})$

Hence the result.

Corollary 3.2: Let $(\widetilde{F_G}, d, *)$ be a complete fuzzy soft metric space, $T: \widetilde{F_G} \mapsto k(\widetilde{F_G})$ be a contractive mapping. If there exists a constant $k \in (0,1)$ such that for any $\widetilde{F_G} \in \widetilde{F_G}, \widetilde{F^{g'}} \in J_b^{\widetilde{F_G}}$,

$$d\left(\widetilde{F^{g'}}, T\left(\widetilde{F^{g'}}\right)\right) \le k[d\left(\widetilde{F_G}, \widetilde{F^{g'}}\right),$$

Then T has a fixed point in $\widetilde{F_G}$ provided f is lower semi-continuous.

Example 3.3: Let $\mathcal{U}$ be a non-empty finite subset of parameters and $(\widetilde{F_G}, d, \tilde{*})$ be a fuzzy soft metric space. The mapping $T: \widetilde{F_G} \to G(\widetilde{F_G}) \times \tilde{\mathbb{R}} \to \tilde{I}$

$$T(\tilde{u}, \tilde{v}, t) = \min M_e(\tilde{u}(e), \tilde{v}(e), \tilde{t}(e))$$

Is a fuzzy metric on $G(\widetilde{F_G})$.

Example 3.4: Let $X = \{3,9, \dots \dots \dots\ 3^n\} \cup M(x_1, x_2, t) = |x_1 - x_2|$, for $x_1, x_2 \in X$; then X is a complete fuzzy soft metric space. We define a mapping $T: \widetilde{F_G} \to G(\widetilde{F_G})$ as

$$T(x_1) = \begin{cases} \{3^n, 1\}, & x = 3^n, n = 0,1,2,3, \dots. \\ \{0,3\}, & x = 0. \end{cases}$$

Hence T is continuous.

IV. CONCLUSION

In this paper, we have presented a new fixed point theorem for multi-valued contractive mappings using Banach contraction principle in the setting of complete fuzzy soft metric space, which improve and extends the results due to Sayed and Alahmari [24].

V. FUTURE SCOPE

Fixed point for contractive mapping in Fuzzy soft metric space is an interesting concept. There is scope to examine the applicability of this space in different branches to study.

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CONFLICT OF INTEREST

There is no conflict of interest.

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