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Fixed Point Theorems for Partial Cone b-Metric Space

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ABSTRACT: In present work, we prove some fixed point theorems which satisfy the new contractive condition on the Partial Cone b-Metric Space with normal cone. These theorems are an extension of work proved by Lu Shi and Shaoyuan Xu in the paper –A common unique fixed point theorem for two weakly compatible self-mapping on cone b-metric space published in Fixed Point Theory and Application, v.1, N.20, 2013, pp.1-11.

Keywords: Contractive Mappings, Fixed Point, b-Metric space, Partial Metric, Cone Metric.

I. INTRODUCTION

Theory of fixed point plays an important role in various disciplines. This theory has been received more attention because of its broad applications in area of applied as well as in pure mathematics. Theory of the fixed point is a major tool to find the solution of existence and uniqueness. A mathematician Banach [1] proved a theorem named Banach Contraction Principle in 1922, which ensures the suitable conditions for uniqueness and existence of fixed points. The generalization of usual metric space named as the b-metric space which was presented by Bakhtin [2] and Czerwik [3] in 1989.

Steve G Matthew [4] proposed the partial metric space in 1994 and proved Banach contraction theorem under the conditions of partial metric space.

Huang and Zhang [5] proposed a cone metric space in 2007. This is generalisation of metric space. In this type of metric space, distance between the two elements m and n is defined over a vector in the Banach space E whereas the distance of two elements is a non negative real number. Recently, in the cone metric space existence of fixed point is considered [6-9]. The generalization of cone and partial metric space was proposed by Ayse Sonmez [10] in 2011 as partial cone metric space. Hussain and Shah [11] established the of cone b-metric space idea in 2011, which is the combined form of the b-metric and cone metric space. Some results were given by Huang and Xu [12] on basis of cone b-metric spaces for contractive mappings.

In 2014, Satish Shukla [13] proposed the partial b-metric space and by using Kannan type mapping, Banach Contraction theorem was proved under partial b-metric space conditions.

Very recently in 2016, Fernandez *et al.*, [14] proposed the idea of partial b-cone metric space which is a generalization of cone, b-metric and partial metric space. In the present work, we prove some new fixed point theorems for partial cone b-metric space under new contractive condition for single as well as for pair of weakly compatible mapping.

Definition 1.1 [22] Let F be the subset of real Banach space E. Let θ is represented as the zero element of E and interior F by the int. F. Then subset F is known as cone iff

(i) $F \neq \emptyset$, closed and $F \neq \{\theta\}$;
(ii) $m, n \in \mathbb{R}, m, n \geq 0,\ i, k \in F \Rightarrow mi + nk \in F;$
(iii) $F \cap (-F) = \{\theta\}$

On this basis, a relation $\preccurlyeq$ w.r.t. F is defined by $i \preccurlyeq k$ which is partial order relation iff $k - i \in F$. To indicate $i \preccurlyeq k$ we write $i \prec k$ but when $i \neq k$, then $i \preccurlyeq k$ indicates the $k - i \in \operatorname{int} F$. Write norm as $\|.\|$ on E then F is known as the normal cone if there is a number $L > 0$ s.t. $\forall\, i, k \in E, \theta \preccurlyeq i \preccurlyeq k$ implies that $\|i\| \leq L\|k\|$. The least positive number L satisfying the above is known as the normal constant of F where $L \geq 1$.

We know that cone F in Banach space E with $\operatorname{int} F \neq \emptyset$ and $\preccurlyeq$ is a partial order relation w. r. t. F.

Definition 1.2 [15] Let Y$\neq \emptyset$ and a function $d'_c: Y \times Y \to E$ is known as the cone metric when it satisfied the below conditions:

(i) $\theta < d'_c(m_1', n_1')\ \forall\, m_1', n_1' \in Y$
(ii) $d'_c(m_1', n_1') = \theta\ iff\ m_1' = n_1'$
(iii) $d'_c(m_1', n_1') = d'_c(n_1', m_1')$
(iv) $d'_c(m_1', o_1') \leq [d'_c(m_1', n_1') + d'_c(n_1', o_1')]$; $\forall\ m_1', n_1', o_1' \in Y$

Then the pair (Y,d'_c) is known as the cone metric space.

Definition 1.3 [22] Let Y$\neq \emptyset$ and t is a non-zero +ve real number. A function $d'_{cb}: Y \times Y \to E$ is known as the cone b-metric when it satisfies the following axioms:

(i) $\theta < d'_{cb}(r_1', r_2')\ \forall\ r_1', r_2' \in Y$
(ii) $d'_{cb}(r_1', r_2') = \theta\ iff\ r_1' = r_2'$
(iii) $d'_{cb}(r_1', r_2') = d'_{cb}(r_2', r_1')$

(iv) $d'_{cb}(r_1', r_3') \le t[d'_{cb}(r_1', r_2') + d'_{cb}(r_2', r_3')]$; $\forall\ r_1', r_2', r_3' \in Y$

Then, the pair (Y,d'_{cb}) is known as the cone b-metric space.

Definition 1.4 [15] Let Y≠ ∅ and a function $d'_p: Y \times Y \to R^+$is partial-metric when it satisfies the below conditions:

(i) $r_1'' = r_2'' \Leftrightarrow d'_p(r_1'', r_1'') = d'_p(r_1'', r_2'') = d'_p(r_2'', r_2'')$

(ii) $d'_p(r_1'', r_1'') \le d'_p(r_1'', r_2'')$

(iii) $d'_p(r_1'', r_2'') = d'_p(r_2'', r_1'')$

(iv) $d'_p(r_1'', r_2'') \le d'_p(r_1'', r_3'') + d'_p(r_3'', r_2'') - d'_p(r_3'', r_3'')$

Then the pair (Y,d'_p) is known as the partial metric space.

Definition 1.5 [21] Let $Y \neq \emptyset$ and t be a +ve integer. A function $d'_{pb}: Y \times Y \to R^+$is known as partial b-metric when it satisfies the following axioms: $\forall\ f_1'', f_2'', f_3'' \in Y$

(i) $f_1'' = f_2'' \Leftrightarrow d'_{pb}(f_1'', f_1'') = d'_{pb}(f_1'', f_2'') = d'_{pb}(f_2'', f_2'')$

(ii) $d'_{pb}(f_1'', f_1'') \le d'_{pb}(f_1'', f_2'')$

(iii) $d'_{pb}(f_1'', f_2'') = d'_{pb}(f_2'', f_1'')$

(iv) $d'_{pb}(f_1'', f_3'') + d'_{pb}(f_2'', f_2'') \le t[d'_{pb}(f_1'', f_2'') + d'_{pb}(f_2'', f_3'')]$;

Then the pair (Y,d'_{pb}) is partial b-metric space.

Definition 1.6 [15] Let Y≠ ∅ and a function $d'_{pc}: Y \times Y \to E$ known as the partial cone metric when it satisfies the below axioms:

(i) $e_1' = f_1' \Leftrightarrow d'_{pc}(e_1', e_1') = d'_{pc}(e_1', f_1') = d'_{pc}(f_1', f_1')\ \ \forall\ e_1', f_1' \in Y$

(ii) $\theta \le d'_{pc}(e_1', e_1') \le d'_{pc}(e_1', f_1')$

(iii) $d'_{pc}(e_1', f_1') = d'_{pc}(f_1', e_1')$

(iv) $d'_{pc}(e_1', g_1') + d'_{pc}(f_1', f_1') \le \begin{bmatrix} d'_{pc}(e_1', f_1') + \\ d'_{pc}(f_1', g_1') \end{bmatrix}$

; $\forall\ e_1', g_1', f_1' \in Y$

Then (Y,d'_{pc}) is known as partial cone metric space.

Theorem 1.1 [10] Let (X,d'_{pc}) is partial cone metric space and F is the normal cone with constant L. Leta sequence $\{w_z\}$ for X. Then $\{w_z\}$ converge to x iff $d'_{pc}(w_z, w) \to d'_{pc}(w, w)$ as $z \to \infty$

Sonmez [10] also mentioned that if (X,d'_{pc}) is the partial cone metric space and normal cone F with normal constant L and $d'_{pc} w_z, w) \to d'_{pc}(w, w)$ as $z \to \infty$, then $d'_{pc} w_z, w_z) \to d'_{pc}(w, w)$.

Definition 1.7 [14] Let $Y \neq \emptyset$ and $t \neq 0$. Define a function $d'_{pcb}: Y \times Y \to E$ is partial cone b-metric when it satisfies the below conditions:

$$\theta \le d'_{pcb}(f_1^*, f_1^*) \le d'_{pcb}(f_1^*, f_2^*)\ ;\ \forall\ f_1^*, f_2^* \in Y$$

(i) $f_1^* = f_2^* \Leftrightarrow d'_{pb}(f_1^*, f_1^*) = d'_{pb}(f_1^*, f_2^*) = d'_{pb}(f_2^*, f_2^*)$

(ii) $d'_{pcb}(f_1^*, f_2^*) = d'_{pcb}(f_2^*, f_1^*)$

(iii) $d'_{pcb}(f_1^*, f_3^*) \le t \begin{bmatrix} d'_{pcb}(f_1^*, f_2^*) + \\ d'_{pcb}(f_2^*, f_3^*) \end{bmatrix} - d'_{pcb}(f_2^*, f_2^*)$; $\forall\ f_1^*, f_2^*, f_3^* \in Y$

Then (Y,d'_{pcb}) be the partial cone b-metric space and it is the generalisation of b-metric, cone metric and partial metric space.

Definition 1.8 [14] Let(Y,d'_{pcb}) is partial cone b-metric space and a sequence $\{y_n\}$is known as convergent in Y and $y \in Y$ if for every $c \in F$, there is a +ve integer N, $d'_{pcb}(y_n, y) \ll c + d'_{pcb}(y, y)$ then sequence $\{y_n\}$ converges to y and this point is known as limit point of sequence $\{y_n\}$ and it is represented as $\lim_{n\to\infty}\{y_n\} = y$.

Definition 1.9 [14] Let a partial cone b-metric space (Y,d'_{pcb}) and a sequence $\{y_n\}$ is known as the Cauchy sequence if there is $a \in F$ such that $f$or every $\in >0$, there is $\|d'_{pcb}(y_n, y_m) - a\| < \in$ when $m, n \to \infty$.

Definition 2.0 [14] Let (Y,d'_{pcb}) be the partial cone b-metric space then it is called as complete if every Cauchy sequence converges in this space.

II. MAIN RESULT

In this portion, the main purpose is to show some fixed point theorems for single as well as for the pair of weakly compatible mapping on partial cone b-metric space.

Example 2.1 Let $E = R,\ F = \{\{g_n^*\} \in E : g_n^* \ge 0, \forall\ n \ge 1\}$. Let a function $d'_{pcb}: Y \times Y \to E$ is defined by $d'_{pcb}(g^*, h^*) = \sum_{n=1}^{m}(|g_n^* - h_n^*|^p)^{1/p}$. Now we conclude that (Y,d'_{pcb}) be the partial cone b-metric space where $p \ge 1$.

$$d'_{pcb}(g^*, h^*) = \sum_{n=1}^{m}(|g_n^* - h_n^*|^p)^{1/p}$$

(i) $|g_n^* - h_n^*| \ge \theta$

$$|g_n^* - h_n^*|^p \ge \theta$$

$$\sum_{n=1}^{m}(|g_n^* - h_n^*|^p)^{\frac{1}{p}} \ge \theta$$

$$d'_{pcb}(g^*, h^*) \ge \theta$$

When we take $g_n^* = h_n^*$

Then, we get $|g_n^* - g_n^*| \ge \theta$

$$|g_n^* - g_n^*|^p \ge \theta$$

$$\sum_{n=1}^{m}(|g_n^* - g_n^*|^p)^{\frac{1}{p}} \ge \theta$$

$$d'_{pcb}(g^*, g^*) \ge \theta$$

So, $\theta \le d'_{pcb}(g^*, g^*) \le d'_{pcb}(g^*, h^*)$

(ii) $g^* = h^* \Leftrightarrow d^*_{pcb}(g^*, g^*) = d^*_{pcb}(g^*, h) = d'_{pcb}(h^*, h^*)$

$$\sum_{n=1}^{m}(|g_n^* - g_n^*|^p)^{1/p} = \sum_{n=1}^{m}(|g_n^* - h_n^*|^p)^{1/p}$$

$$= \sum_{n=1}^{m}(|h_n^* - h_n^*|^p)^{1/p}$$

$$\theta = \sum_{n=1}^{m}(|g_n^* - h_n^*|^p)^{1/p} = \theta$$

$$(|g_n{}^* - h_n{}^*|^p)^{1/p} = \theta$$
$$|g_n{}^* - h_n{}^*| = \theta$$
$$\Leftrightarrow g_n{}^* = h_n{}^*$$
$$\Leftrightarrow g^* = h^*$$

(iii) $d'_{pcb}(g^*, h^*) = \sum_{n=1}^{m}(|g_n{}^* - h_n{}^*|^p)^{1/p}$
$$= \sum_{n=1}^{m}(|(-h_n{}^* + g_n{}^*)|^p)^{1/p}$$
$$= \sum_{n=1}^{m}(|h_n{}^* - g_n{}^*|^p)^{1/p}$$
$$= d'_{pcb}(h^*, g^*)$$

(iv) Let $g^*, h^*, i^* \in Y$ then,

$$d'_{pcb}(g^*, h^*) = \sum_{n=1}^{m}(|g_n{}^* - h_n{}^*|^p)^{1/p}$$

Since, $(r^{**} + s^{**})^p \leq (2\max(r^{**}, s^{**}))^p \leq 2^p(|r^{**}|^p + |s^{**}|^p)\ \forall\, r^{**}, s^{**} \geq 0$

Let $m^{**} = (g_n{}^* + i_n{}^* - i_n{}^*)$ and $g_n{}^* - h_n{}^* = m^{**} + n^{**}$

Then, $(|g_n{}^* - h_n{}^*|)^p \leq (|m^{**} + n^{**}|)^p$
$$\leq (|m^{**}| + |n^{**}|)^p$$
$$\leq 2^p\left(\sum |m^{**}|^p + \sum |n^{**}|^p\right)$$
$$\leq 2^p\left(\sum |m^{**}|^p + \sum |n^{**}|^p\right)^{\frac{1}{p}}$$
$$\leq (2^p \sum |m^{**}|^p + 2^p \sum |n^{**}|^p)^{\frac{1}{p}}$$
$$\leq (2^p \sum |g_n{}^* + i_n{}^* - i_n{}^*|^p + 2^p \sum |i_n{}^* - i_n{}^* - h_n{}^*|^p)^{\frac{1}{p}}$$
$$\leq \left(2^p \sum |g_n{}^* + i_n{}^* - i_n{}^* + i_n{}^* - i_n{}^* - h_n{}^*|^p\right)^{\frac{1}{p}}$$
$$\leq 2^p\left[\sum_{n=1}^{m}(|g_n{}^* - i_n{}^*|^p)^{\frac{1}{p}} + \sum_{n=1}^{m}(|i_n{}^* - h_n{}^*|^p)^{\frac{1}{p}} - \sum_{n=1}^{m}(|i_n{}^* - i_n{}^*|^p)^{\frac{1}{p}}\right]$$
$$\leq 2^p[d'_{pcb}(g^*, i^*) + d'_{pcb}(i^*, h^*) - d'_{pcb}(i^*, i^*)]$$
$$As\ p \geq 1$$
$$\leq 2^p[d'_{pcb}(g^*, i^*) + d'_{pcb}(i^*, h^*)] - d'_{pcb}(i^*, i^*)$$

Since, all conditions of partial cone b-metric space have been satisfied. So (Y,d'_{pcb}) is a partial cone b-metric space.

Example 2.2 : Let $E = R^2$ and $F = \{(h_1{}^*, h_2{}^*) \in E; h_1{}^*,\ h_2{}^* \in R\}$ and $Y = R$ then a function d'_{pcb}: Y×Y → E by $d'_{pcb}(h_1{}^*, h_2{}^*) = \max\{h_1{}^*, h_2{}^*\}$. Now we conclude that (Y,d'_{pcb}) is partial cone b-metric space.

Proof: Let (Y, d'_{pcb}) is a partial cone b-metric space with coefficient $t \geq 1$. Let $h_1{}^*, h_2{}^*, h_3{}^* \in$ Y be an arbitrary point, then

(i) $d'_{pcb}(h_1{}^*, h_2{}^*) = \max\{h_1{}^*, h_2{}^*\}$

$\max\{h_1{}^*, h_2{}^*\} \geq \theta$

$\max\{h_1{}^*, h_1{}^*\} \geq \theta$

then, $\theta \leq d'_{pcb}(h_1{}^*, h_1{}^*) \leq d'_{pcb}(h_1{}^*, h_2{}^*)$

(ii) $d'_{pcb}(h_1{}^*, h_2{}^*) = \max\{h_1{}^*, h_2{}^*\}$
$$d'_{pcb}(h_1{}^*, h_1{}^*) = d'_{pcb}(h_1{}^*, h_2{}^*) = d'_{pcb}(h_2{}^*, h_2{}^*)$$
$$\Leftrightarrow h_1{}^* = h_2{}^*$$
$$\max\{h_1{}^*, h_1{}^*\} = \max\{h_1{}^*, h_2{}^*\} = \max\{h_2{}^*, h_2{}^*\}$$
$$h_1{}^* = \max\{h_1{}^*, h_2{}^*\} = h_2{}^*$$
$$\Leftrightarrow h_1{}^* = h_2{}^*$$

(iii) $d'_{pcb}(h_1{}^*, h_2{}^*) = \max\{h_1{}^*, h_2{}^*\}$

$= \max\{h_2{}^*, h_1{}^*\}$

$= d'_{pcb}(h_2{}^*, h_1{}^*)$

(iv) $d'_{pcb}(h_1{}^*, h_3{}^*) = \max\{h_1{}^*, h_3{}^*\}$
$$= \max\{h_1{}^*, h_3{}^*\} + \max\{h_2{}^*, h_2{}^*\} - \max\{h_2{}^*, h_2{}^*\}$$
$$\leq \max\{h_1{}^*, h_2{}^*\} + \max\{h_2{}^*, h_3{}^*\} - \max\{h_2{}^*, h_2{}^*\}$$
$$\leq t[\max\{h_1{}^*, h_2{}^*\} + \max\{h_2{}^*, h_3{}^*\} - \max\{h_2{}^*, h_2{}^*\}]$$
$$[\text{since } t \geq 1]$$
$$\leq t[\max\{h_1{}^*, h_2{}^*\} + \max\{h_2{}^*, h_3{}^*\}] - \max\{h_2{}^*, h_2{}^*\}$$
$$\leq t[d'_{pcb}(h_1{}^*, h_2{}^*) + d'_{pcb}(h_2{}^*, h_3{}^*)] - d'_{pcb}(h_2{}^*, h_2{}^*)$$

Therefore, all four conditions are satisfied and so (Y,d'_{pcb}) is a partial cone b-metric space.

Theorem 2.1 Let (W,d'_{pcb}) is partial cone b-metric space with constant$t \geq 1$ and F is the normal cone and W is a self-mapping $W: Y \to Y$ and $\lambda \in \left[\frac{1}{t+1}, 1\right)$ such that

$$d'_{pcb}(Wp^*, Wq^*) \leq \lambda \max\left\{\begin{array}{c} d'_{pcb}(p^*, Wp^*), d'_{pcb}(q^*, Wq^*), \\ \frac{d'_{pcb}(p^*, Wq^*) + d'_{pcb}(q^*, Wp^*)}{2t} \end{array}\right\}$$

$\forall\, p^*, q^* \in Y$

Then W has a common unique fixed point.

Proof: We start with initial point k_0 in Y s. t.,

$k_0 \neq W(k_0)$

Then we construct a sequence $\{k_s\}$ in Y, we get

$k_1 = Wk_0, k_2 = Wk_1, k_3 = Wk_2, \ldots\ldots.., k_{s+1} = Wk_s$

∴ $d'_{pcb}(k_s, k_{s+1}) = d'_{pcb}(Wk_{s-1}, Wk_s)$

$$\leq \lambda \max\left\{\begin{array}{c} d'_{pcb}(k_{s-1}, Wk_{s-1}), d'_{pcb}(k_s, Wk_s), \\ \frac{d'_{pcb}(k_{s-1}, Wk_s) + d'_{pcb}(k_s, Wk_{s-1})}{2t} \end{array}\right\}$$
$$\leq \lambda \max\left\{\begin{array}{c} d'_{pcb}(k_{s-1}, k_s), d'_{pcb}(k_s, k_{s+1}), \\ \frac{d'_{pcb}(k_{s-1}, k_{s+1}) + d'_{pcb}(k_s, k_s)}{2t} \end{array}\right\}$$
$$\leq \lambda \max\left\{\begin{array}{c} d'_{pcb}(k_{s-1}, k_s), d'_{pcb}(k_s, k_{s+1}), \\ \frac{td'_{pcb}(k_{s-1}, k_s) + td'_{pcb}(k_s, k_{s+1}) - d'_{pcb}(k_s, k_s) + d'_{pcb}(k_s, k_s)}{2t} \end{array}\right\}$$

$$\leq \lambda \max\left\{\begin{array}{c} d'_{pcb}(k_{s-1},k_s), d'_{pcb}(k_s,k_{s+1}), \\ \dfrac{d'_{pcb}(k_{s-1},k_s)+d'_{pcb}(k_s,k_{s+1})}{2} \end{array}\right\}$$

$$d'_{pcb}(k_s,k_{s+1}) \leq \lambda \max\{d'_{pcb}(k_{s-1},k_s), d'_{pcb}(k_s,k_{s+1})\}$$

if $\max\{d'_{pcb}(k_{s-1},k_s), d'_{pcb}(k_s,k_{s+1})\} = d'_{pcb}(k_s,k_{s+1})$

then $d'_{pcb}(k_s,k_{s+1}) \leq \lambda d'_{pcb}(k_s,k_{s+1})$ it is a contradiction,

then, $d'_{pcb}(k_s,k_{s+1}) \leq \lambda d'_{pcb}(k_{s-1},k_s)$

$$\leq \lambda^2 d'_{pcb}(k_{s-2},k_{s-1})$$

Similarly, we get

$$d'_{pcb}(k_s,k_{s+1}) \leq \lambda^s d'_{pcb}(k_0,k_1)$$

Now we show that $d'_{pcb}(k_s,k_s) = \theta$

$$d'_{pcb}(k_s,k_s) = d'_{pcb}(Wk_{s-1},Wk_{s-1})$$

$$\leq \lambda \max\left\{\begin{array}{c} d'_{pcb}(k_{s-1},Wk_{s-1}), d'_{pcb}(k_{s-1},Wk_{s-1}), \\ \dfrac{d'_{pcb}(k_{s-1},Wk_{s-1})+d'_{pcb}(k_{s-1},Wk_{s-1})}{2t} \end{array}\right\}$$

$$\leq \lambda \max\left\{\begin{array}{c} d'_{pcb}(k_{s-1},k_s), d'_{pcb}(k_{s-1},k_s), \\ \dfrac{d'_{pcb}(k_{s-1},k_s)+d'_{pcb}(k_{s-1},k_s)}{2t} \end{array}\right\}$$

$$\leq \lambda \max\{d'_{pcb}(k_{s-1},k_s), d'_{pcb}(k_{s-1},k_s)\}$$

So, $d'_{pcb}(k_s,k_s) \leq \lambda^s d'_{pcb}(k_0,k_1)$

As we take limit $s \to \infty$, we get

$$d'_{pcb}(k_s,k_s) = \theta$$

Now, let r and s be two positive integers, $r < s$

$$d'_{pcb}(k_r,k_s) \leq t[d'_{pcb}(k_r,k_{r+1}) + d'_{pcb}(k_{r+1},k_s)] - d'_{pcb}(k_{r+1},k_{r+1})$$

$$d'_{pcb}(k_r,k_s) \leq t d'_{pcb}(k_r,k_{r+1}) + t^2 d'_{pcb}(k_{r+1},k_{r+2}) + \cdots\ldots + t^{s-r} d'_{pcb}(k_{s-1},k_s)$$

$$d'_{pcb}(k_r,k_s) \leq \left[\begin{array}{c} t\lambda^r d'_{pcb}(k_0,k_1) + \\ t^2\lambda^{r+1} d'_{pcb}(k_0,k_1) + \\ \ldots\ldots + \\ t^{s-r}\lambda^{s-1} d'_{pcb}(k_0,k_1) \end{array}\right]$$

$$d'_{pcb}(k_r,k_s) \leq t\lambda^r \left[\begin{array}{c} 1 + t\lambda + t^2\lambda^2 \\ + \cdots\ldots t^{s-r-1}\lambda^{s-r-1} \end{array}\right] d'_{pcb}(k_0,k_1)$$

$$d'_{pcb}(k_r,k_s) \leq \frac{t\lambda^r}{1-\lambda t} d'_{pcb}(k_0,k_1)$$

$$\therefore d'_{pcb}(k_r,k_s) \leq \frac{t\lambda^r}{1-\lambda t} d'_{pcb}(k_0,k_1)$$

Since F is a normal cone.

Therefore, $\|d'_{pcb}(k_r,k_s\| \leq \frac{t\lambda^r}{1-\lambda t} d'_{pcb}(k_0,k_1) \to \theta$ as $r \to \infty$

As $d'_{pcb}(k_r,k_s) \to \theta$ as $r,s \to \infty$

Hence, sequence $\{k_s\}$ is Cauchy sequence in Y.

By completeness, there exist a point α^* which belongs to Y.

Since, $\{k_s\}$ converge to α^* as $s \to \infty$

$$\lim_{s\to\infty} d'_{pcb}(k_s,\alpha^*) = d'_{pcb}(\alpha^*,\alpha^*) = \lim_{s\to\infty} d'_{pcb}(k_s,k_s) = \theta$$

Now, we need to proof that α^* be the fixed point of W.

$$\therefore d'_{pcb}(\alpha^*,W\alpha^*) \leq t d'_{pcb}(\alpha^*,Wk_s) + t d'_{pcb}(Wk_s,W\alpha^*) - d'_{pcb}(Wk_s,Wk_s)$$

$$d'_{pcb}(\alpha^*,W\alpha^*) \leq t d'_{pcb}(\alpha^*,Wk_s) + \lambda \max\left\{\begin{array}{c} d'_{pcb}(k_s,Wk_s), d'_{pcb}(\alpha^*,W\alpha^*), \\ \dfrac{d'_{pcb}(k_s,W\alpha^*)+d'_{pcb}(\alpha^*,Wk_s)}{2t} \end{array}\right\}$$

$$d'_{pcb}(\alpha^*,W\alpha^*) \leq t d'_{pcb}(\alpha^*,k_{s+1}) + \lambda \max\left\{\begin{array}{c} d'_{pcb}(k_s,k_{s+1}), d'_{pcb}(\alpha^*,W\alpha^*), \\ \dfrac{d'_{pcb}(\alpha^*,W\alpha^*)+d'_{pcb}(\alpha^*,\alpha^*)}{2t} \end{array}\right\}$$

$$d'_{pcb}(\alpha^*,W\alpha^*) \leq t d'_{pcb}(\alpha^*,k_{s+1}) + \lambda d'_{pcb}(\alpha^*,W\alpha^*)$$

$$d'_{pcb}(\alpha^*,W\alpha^*) \leq \lambda d'_{pcb}(\alpha^*,W\alpha^*)$$

$$(1-\lambda) d'_{pcb}(\alpha^*,W\alpha^*) \leq \theta$$

$d'_{pcb}(\alpha^*,W\alpha^*) = \theta$ as $s \to \infty$

So, $W\alpha^* = \alpha^*$

Hence, α^* is fixed point of W.

Now, we show uniqueness

Let β^* is another common fixed point of W.

So, $\beta^* = W\beta^*$

Then, $d'_{pcb}(\alpha^*,\beta^*) = d'_{pcb}(W\alpha^*,W\beta^*)$

$$d'_{pcb}(\alpha^*,\beta^*) \leq \lambda \max\left\{\begin{array}{c} d'_{pcb}(\alpha^*,W\alpha^*), d'_{pcb}(\beta^*,W\beta^*), \\ \dfrac{d'_{pcb}(\alpha^*,W\beta^*)+d'_{pcb}(\beta^*,W\alpha^*)}{2t} \end{array}\right\}$$

$$d'_{pcb}(\alpha^*,\beta^*) \leq \lambda \max\left\{\begin{array}{c} d'_{pcb}(\alpha^*,\alpha^*), d'_{pcb}(\beta^*,\beta^*), \\ \dfrac{d'_{pcb}(\alpha^*,\beta^*)+d'_{pcb}(\beta^*,\alpha^*)}{2t} \end{array}\right\}$$

$$d'_{pcb}(\alpha^*,\beta^*) \leq \lambda \max\left\{\begin{array}{c} d'_{pcb}(\alpha^*,\alpha^*), d'_{pcb}(\beta^*,\beta^*), \\ \dfrac{2\, d'_{pcb}(\alpha^*,\beta^*)}{2t} \end{array}\right\}$$

$$d'_{pcb}(\alpha^*,\beta^*) \leq \lambda \frac{d'_{pcb}(\alpha^*,\beta^*)}{t}$$

$$\left(1-\frac{\lambda}{t}\right) d'_{pcb}(\alpha^*,\beta^*) \leq \theta$$

$$d'_{pcb}(\alpha^*,\beta^*) = \theta$$

implies $\alpha^* = \beta^*$

Hence, α^* is the common unique fixed point of W.

Example 2.3 Let $E = R^2$ and (W,d'_{pcb}) be the complete partial cone b-metrics space and coefficient $t \geq 1$ and $\lambda \in \left[\frac{1}{t+1},1\right)$. Then function defined by $W: X \to E$

$$W(r^*) = \begin{cases} -\dfrac{5}{3} & if\ r^* \epsilon\ (-\infty,-1) \\ -\dfrac{1}{2} & if\ r^* \epsilon\ [-1,0) \end{cases}$$

and $d'_{pcb}(r^*,s^*) = \max\{r^*,s^*\}$

Proof : By inequality of above theorem

$$d'_{pcb}(Wr^*,Ws^*) \leq \lambda \max\left\{\begin{array}{c} d'_{pcb}(r^*,Wr^*), d'_{pcb}(s^*,Ws^*), \\ \dfrac{d'_{pcb}(r^*,Ws^*)+d'_{pcb}(s^*,Wr^*)}{2t} \end{array}\right\}$$

Then, let $r^* = -\frac{5}{3}$ and $s^* = -\frac{1}{2}$

$$d'_{pcb}\left(W\left(-\frac{5}{3}\right), W\left(-\frac{1}{2}\right)\right) \leq \lambda \max\left\{\begin{array}{c} d'_{pcb}\left(-\frac{5}{3}, W\left(-\frac{5}{3}\right)\right), \\ d'_{pcb}\left(-\frac{1}{2}, W\left(-\frac{1}{2}\right)\right), \\ \dfrac{d'_{pcb}\left(-\frac{5}{3}, W\left(-\frac{1}{2}\right)\right) + d'_{pcb}\left(-\frac{1}{2}, W\left(-\frac{5}{3}\right)\right)}{2t} \end{array}\right\}$$

$$d'_{pcb}\left(-\frac{5}{3},-\frac{1}{2}\right) \le \lambda \max\left\{ d'_{pcb}\left(-\frac{5}{3},-\frac{5}{3}\right), d'_{pcb}\left(-\frac{1}{2},-\frac{1}{2}\right), \frac{d'_{pcb}\left(-\frac{5}{3},-\frac{1}{2}\right)+d'_{pcb}\left(-\frac{1}{2},-\frac{5}{3}\right)}{2t} \right\}$$

$$\max\left(-\frac{5}{3},-\frac{1}{2}\right) \le \lambda \max\left\{ \max\left(-\frac{5}{3},-\frac{5}{3}\right), \max\left(-\frac{1}{2},-\frac{1}{2}\right), \frac{\max\left(-\frac{5}{3},-\frac{1}{2}\right)+\max\left(-\frac{1}{2},-\frac{5}{3}\right)}{2t} \right\}$$

$$-\frac{1}{2} \le \lambda \max\left\{-\frac{5}{3},-\frac{1}{2},\frac{-\frac{1}{2}+-\frac{1}{2}}{2t}\right\}$$

$$-\frac{1}{2} \le \lambda \max\left\{-\frac{5}{3},-\frac{1}{2},\frac{-1}{2t}\right\}$$

$$-\frac{1}{2} \le -\frac{\lambda}{2t}$$

So, the above inequality satisfied by $t \ge 1$ and $\lambda \in \left[\frac{1}{t+1},1\right)$.

Theorem 2.2 Let P and Q be the two self-mappings $P,Q: Y \to Y$ and (Y, d'_{pcb}) is partial cone b-metric space with $t \ge 1$ and normal cone F and $\lambda \in \left[\frac{1}{t+1},1\right)$ s.t.

(i) $Q(Y) \subseteq P(Y)$ and the closed subset of Y is P(Y).

(ii) Pair (P, Q) be the weakly compatible mapping.

(iii) $d'_{pcb}(Qx,Qy) \le a_1\begin{bmatrix} d'_{pcb}(Px,Qy)+ \\ d'_{pcb}(Qx,Py)\end{bmatrix} + a_2 d'_{pcb}(Px,Qx) + a_3 d'_{pcb}(Py,Qy) \quad \forall\, x,y \in Y$

and $2a_1 + a_2 + a_3 \le 1$ for some constants a_1, a_2, and a_3.
Then, P and Q have a unique common fixed point.

Proof: Let y_0 is any point in Y and then we can choose a point y_1 of Y s.t.
$x_0 = Q(y_0) = P(y_1)$ and $x_1 = Q(y_1) = P(y_2)$
In general, there ∃ a sequence $\{x_s\}$ such that,

$$x_s = Q(y_s) = P(y_{s+1}) \; for \; s = 0,1,2 \; ....$$

Now, we show that $\{x_s\}$ is a cauchy sequence.

Now, $d'_{pcb}(x_s, x_{s+1}) = d'_{pcb}(Qy_s, Qy_{s+1})$

$$\le a_1[d'_{pcb}(Py_s,Qy_{s+1}) + d'_{pcb}(Qy_s,Py_{s+1})] + a_2 d'_{pcb}(Py_s,Qy_s) + a_3 d'_{pcb}(Py_{s+1},Qy_{s+1})$$

$$\le a_1[d'_{pcb}(x_{s-1},x_{s+1}) + d'_{pcb}(x_s,x_s)] + a_2 d'_{pcb}(x_{s-1},x_s) + a_3 d'_{pcb}(x_s,x_{s+1})$$

$$\le a_1\begin{bmatrix} td'_{pcb}(x_{s-1},x_s) + td'_{pcb}(x_s,x_{s+1}) \\ -d'_{pcb}(x_s,x_s) + d'_{pcb}(x_s,x_s)\end{bmatrix} + a_2 d(x_{s-1},x_s) + a_3 d'_{pcb}(x_s,x_{s+1})$$

$$\le a_1\begin{bmatrix} td'_{pcb}(x_{s-1},x_s)+ \\ td'_{pcb}(x_s,x_{s+1})\end{bmatrix} + a_2 d'_{pcb}(x_{s-1},x_s) + a_3 d'_{pcb}(x_s,x_{s+1})$$

$$\le (ta_1 + a_2) d'_{pcb}(x_{s-1},x_s) + (ta_1 + a_3) d'_{pcb}(x_s,x_{s+1})$$

$$\{1-(ta_1+a_3)\} d'_{pcb}(x_s,x_{s+1}) \le (ta_1 + a_2)\, d'_{pcb}(x_{s-1},x_s)$$

$$d'_{pcb}(x_s,x_{s+1}) \le \left[\frac{(ta_1 + a_2)}{\{1-(ta_1+a_3)\}}\right] d'_{pcb}(x_{s-1},x_s)$$

$$d'_{pcb}(x_s,x_{s+1}) \le \lambda\, d'_{pcb}(x_{s-1},x_s)$$

where $\lambda = \left[\frac{(ta_1 + a_2)}{\{1-(ta_1+a_3)\}}\right] < 1$

$$d'_{pcb}(x_s,x_{s+1}) \le \lambda\, d'_{pcb}(x_{s-1},x_s) \le \lambda^2 d'_{pcb}(x_{s-2},x_{s-1})$$

Similarly, we get
$d'_{pcb}(x_s,x_{s+1}) \le \lambda^s d'_{pcb}(x_0,x_1)$
Let r and s be two positive integers, $r < s$
then,

$$d'_{pcb}(x_r,x_s) \le t[d'_{pcb}(x_r,x_{r+1}) + d'_{pcb}(x_{r+1},x_s)] - d'_{pcb}(x_{r+1},x_{r+1})$$

$$d'_{pcb}(x_r,x_s) \le td'_{pcb}(x_r,x_{r+1}) + t^2 d'_{pcb}(x_{r+1},x_{r+2}) + \cdots \ldots \ldots + t^{s-r} d'_{pcb}(x_{s-1},x_s) - d'_{pcb}(x_{r+1},x_{r+1}) - d'_{pcb}(x_{r+2},x_{r+2}) - \cdots \ldots \ldots \ldots - d'_{pcb}(x_{s-1},x_{s-1})$$

$$d'_{pcb}(x_r,x_s) \le [t\lambda^r d'_{pcb}(x_0,x_1) + t^2\lambda^{r+1} d'_{pcb}(x_0,x_1) + \cdots \ldots \ldots + t^{s-r}\lambda^{s-1} d'_{pcb}(x_0,x_1)] - \sum_{m=1}^{s-r-1} d'_{pcb}(x_{r+m},x_{r+m})$$

$$d'_{pcb}(x_r,x_s) \le t\lambda^r \begin{bmatrix} 1+t\lambda+ \\ t^2\lambda^2+ \\ \ldots\ldots+ \\ t^{s-r-1}\lambda^{s-r-1}\end{bmatrix} d'_{pcb}(x_0,x_1) - \sum_{m=1}^{s-r-1} d'_{pcb}(x_{r+m},x_{r+m})$$

$$d'_{pcb}(x_r,x_s) \le \frac{t\lambda^r}{1-\lambda t} d'_{pcb}(x_0,x_1) - \sum_{m=1}^{s-r-1} d'_{pcb}(x_{r+m},x_{r+m})$$

Now we show that $d'_{pcb}(x_s,x_s) = \theta$

$$d'_{pcb}(x_s,x_s) = d'_{pcb}(Qy_s,Qy_s)$$

$$d'_{pcb}(x_s,x_s) \le a_1\begin{bmatrix} d'_{pcb}(Py_s,Qy_s)+ \\ d'_{pcb}(Qy_s,Py_s)\end{bmatrix} + a_2 d'_{pcb}(Py_s,Qy_s) + a_3 d'_{pcb}(Py_s,Qy_s)$$

$$d'_{pcb}(x_s,x_s) \le a_1[d'_{pcb}(x_{s-1},x_s) + d'_{pcb}(x_s,x_{s-1})] + a_2 d'_{pcb}(x_{s-1},x_s) + a_3 d'_{pcb}(x_{s-1},x_s)$$

$$d'_{pcb}(x_s,x_s) \le 2a_1[d'_{pcb}(x_s,x_{s-1})] + a_2 d'_{pcb}(x_{s-1},x_s) + a_3 d'_{pcb}(x_{s-1},x_s)$$

$$d'_{pcb}(x_s,x_s) \le (2a_1 + a_2 + a_3) d'_{pcb}(x_{s-1},x_s)$$

$$d'_{pcb}(x_s,x_s) \le \lambda^s d'_{pcb}(x_0,x_1)$$

By taking limit $s \to \infty$
Then, we get
$d'_{pcb}(x_s,x_s) = \theta$
where $\lambda = 2a_1 + a_2 + a_3$
Hence, $\lim_{r\to\infty} \sum_{m=1}^{s-r-1} d'_{pcb}(x_{r+m},x_{r+m}) = \theta$

$$\therefore\; d'_{pcb}(x_r,x_s) \le \frac{t\lambda^r}{1-\lambda t} d'_{pcb}(x_0,x_1)$$

Since F is normal cone.
Therefore, $\|d'_{pcb}(x_r,x_s)\| \le \frac{t\lambda^r}{1-\lambda t} d'_{pcb}(x_0,x_1) \to \theta$ as $r \to \infty$
As $d'_{pcb}(x_r,x_s) \to \theta$ as $r,s \to \infty$

implies $d'_{pcb}(x_r, x_s) \to \theta$ as $r, s \to \infty$
Hence, sequence $\{x_s\}$ is a Cauchy sequence in Y.
By completeness, there exist a point $\bar{u}^{**}$ which belongs to Y.
Since, $\{x_s\}$ converges to $\bar{u}^{**}$ as s→ ∞
Therefore $\lim_{s\to\infty} x_s = \lim_{s\to\infty} Qy_s = \lim_{s\to\infty} Py_{s+1} = \bar{u}^{**}$

$$\lim_{s\to\infty} d'_{pcb}(x_s, \bar{u}^{**}) = d'_{pcb}(\bar{u}^{**}, \bar{u}^{**}) = \lim_{s\to\infty} d'_{pcb}(x_s, x_s) = \theta$$

Since, P(Y) is closed, then there exist $\bar{v}^{**}$ which belongs to Y.
such that, P$\bar{v}^{**} = \bar{u}^{**}$
Now, we show that Q$\bar{v}^{**} = \bar{u}^{**}$

$$d'_{pcb}(x_s, Q\bar{v}^{**}) = d'_{pcb}(Qy_s, Q\bar{v}^{**})$$

$$d'_{pcb}(\bar{u}^{**}, Q\bar{v}^{**}) \le a_1 \begin{bmatrix} d'_{pcb}(Py_s, Q\bar{v}^{**}) + \\ d'_{pcb}(Qy_s, P\bar{v}^{**}) \end{bmatrix} + a_2 d'_{pcb}(Py_s, Qy_s) + a_3 d'_{pcb}(P\bar{v}^{**}, Q\bar{v}^{**})$$

$$d'_{pcb}(\bar{u}^{**}, Q\bar{v}^{**}) \le a_1 d'_{pcb}(\bar{u}^{**}, Q\bar{v}^{**}) + a_3 d'_{pcb}(\bar{u}^{**}, Q\bar{v}^{**})$$

$$(1 - a_1 - a_3) d'_{pcb}(\bar{u}^{**}, Q\bar{v}^{**}) \le \theta$$

$$d'_{pcb}(\bar{u}^{**}, Q\bar{v}^{**}) = \theta$$

So, Q$\bar{v}^{**} = \bar{u}^{**}$
Since, pair (P,Q) is weakly compatible mapping.

$\therefore\ PQ\bar{v}^{**} = QP\bar{v}^{**}$

$\Rightarrow P\bar{u}^{**} = Q\bar{u}^{**}$

Now, show that $Q\bar{u}^{**} = \bar{u}^{**}$.

As $d'_{pcb}(Py_{s+1}, Q\bar{u}^{**}) = d'_{pcb}(Qy_s, Q\bar{u}^{**})$

$$d'_{pcb}(\bar{u}^{**}, Q\bar{u}^{**}) \le a_1 \begin{bmatrix} d'_{pcb}(Py_s, Q\bar{u}^{**}) + \\ d'_{pcb}(Qy_s, P\bar{u}^{**}) \end{bmatrix} + a_2 d'_{pcb}(Py_s, Qy_s) + a_3 d'_{pcb}(P\bar{u}^{**}, Q\bar{u}^{**})$$

$$d'_{pcb}(\bar{u}^{**}, Q\bar{u}^{**}) \le a_1 \begin{bmatrix} d'_{pcb}(\bar{u}^{**}, Q\bar{u}^{**}) + \\ d'_{pcb}(\bar{u}^{**}, Q\bar{u}^{**}) \end{bmatrix} + a_2 d'_{pcb}(\bar{u}^{**}, \bar{u}^{**}) + a_3 d'_{pcb}(Q\bar{u}^{**}, Q\bar{u}^{**})$$

$$d'_{pcb}(\bar{u}^{**}, Q\bar{u}^{**}) \le 2a_1 d'_{pcb}(\bar{u}^{**}, Q\bar{u}^{**})$$

$$(1 - 2a_1) d'_{pcb}(\bar{u}^{**}, Q\bar{u}^{**}) \le \theta$$

$$\Rightarrow d'_{pcb}(\bar{u}^{**}, Q\bar{u}^{**}) = \theta$$

$$Q\bar{u}^{**} = \bar{u}^{**}$$

Similarly, we get, $P\bar{u}^{**} = \bar{u}^{**}$

$\Rightarrow \bar{u}^{**} = P\bar{u}^{**} = Q\bar{u}^{**}$

So, $\bar{u}^{**}$ is common fixed point of P and Q two self mappings.
Now, we show uniqueness:
Let another fixed point $\bar{y}^{**}$ of P and Q.
therefore, $\bar{\beta}^{**} = P\bar{\beta}^{**} = Q\bar{\beta}^{**}$

Then, $d'_{pcb}\left(\bar{u}^{**}, \bar{\beta}^{**}\right) = d'_{pcb}\left(Q\bar{u}^{**}, Q\bar{\beta}^{**}\right)$

$$d'_{pcb}\left(\bar{u}^{**}, \bar{\beta}^{**}\right) \le a_1 \begin{bmatrix} d'_{pcb}\left(P\bar{u}^{**}, Q\bar{\beta}^{**}\right) \\ + d'_{pcb}\left(Q\bar{u}^{**}, P\bar{\beta}^{**}\right) \end{bmatrix} + a_2 d'_{pcb}(P\bar{u}^{**}, Q\bar{u}^{**}) + a_3 d'_{pcb}\left(P\bar{\beta}^{**}, Q\bar{\beta}^{**}\right)$$

$$d'_{pcb}\left(\bar{u}^{**}, \bar{\beta}^{**}\right) \le a_1 \left[d'_{pcb}\left(\bar{u}^{**}, \bar{\beta}^{**}\right) + d'_{pcb}\left(\bar{u}^{**}, \bar{\beta}^{**}\right) \right] + a_2 d'_{pcb}(\bar{u}^{**}, \bar{u}^{**}) + a_3 d'_{pcb}\left(\bar{\beta}^{**}, \bar{\beta}^{**}\right)$$

$$d'_{pcb}\left(\bar{u}^{**}, \bar{\beta}^{**}\right) \le 2a_1 d'_{pcb}\left(\bar{u}^{**}, \bar{\beta}^{**}\right)$$

$$d'_{pcb}\left(\bar{u}^{**}, \bar{\beta}^{**}\right) \le 2a_1 d'_{pcb}\left(\bar{u}^{**}, \bar{\beta}^{**}\right)$$

$$(1 - 2a_1) d'_{pcb}\left(\bar{u}^{**}, \bar{\beta}^{**}\right) \le \theta$$

$$\Rightarrow d'_{pcb}\left(\bar{u}^{**}, \bar{\beta}^{**}\right) = \theta$$

$$\bar{u}^{**} = \bar{\beta}^{**}$$

Hence, for P and Q common unique fixed point is $\bar{u}^{**}$.

III. CONCLUSIONS

In present paper, some fixed point theorems are proved on partial cone b-metric space by using the new contractive condition. This extends the result of Lu Shi and Shaoyuan Xu [21]. Here, we get the unique fixed point for single as well as for the pair of weakly compatible mappings.

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CONFLICT OF INTEREST

Author has no any conflict of interest.

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