

International Journal on Emerging Technologies 10(2b): 137-142(2019)

## On Acceptance Sampling Plans under Truncated Life Tests

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ABSTRACT: This paper discusses acceptance sampling plans based on truncated life test. The first objective of this paper is to illustrate various acceptance sampling plans such as single acceptance sampling plan, double acceptance sampling plan, group acceptance sampling plan and two-stage group acceptance sampling plan. The second objective is on the design of parameters for such sampling plans followed by a discussion on the quality measure. The procedures are also illustrated with the help of exponential distribution, and further with some of the work done on various lifetime distributions in the existing literature.

Keywords: Single acceptance sampling plan, Double acceptance sampling plan, Group acceptance sampling plan, Two-stage group acceptance sampling plan, Life time data.

## I. INTRODUCTION

In statistical analysis, usually the attention lies in studying the various features of individuals or items relating to a particular group, this group of individuals or items are often known as population. For example, in any manufacturing company if we want to know the quality of manufactured goods during the day, then the day's total production refers to population. Thus in statistical point of view, population is the aggregate of objects and in sampling theory the population is group of individuals or items from which samples are drawn. Further sample is the subset of population and the number of items in a sample is known as size of sample. In simple words, a part selected from the population is called a sample, and the process of selecting a sample is called sampling, a technique which allows us to draw conclusions about the population only those included in the sample. The main objective of sampling is to obtain the failure results and possible estimates of the parameter. Sampling may be of various types such as random sampling, stratified sampling and systematic sampling. This paper is based on simple random sampling which is the subset of population in which samples are randomly selected, means each member of population has an equal chance of being selected. It does not effect on quality of samples as samples are drawn randomly without determining any technique for selection, and this sampling is suitable for such population which is highly homogeneous.

The objective of this paper is to discuss sampling plan. various acceptance sampling plans and design of acceptance sampling plans based on random sampling in the presence of truncated life test. The rest of this paper is organized as follows: Section II deals with details on acceptance sampling plan, history and the assumptions for the plans under truncated life tests. Section III and IV are respectively on single and double acceptance sampling plans. Section V is dedicated to group acceptance sampling plan. Section VI is on the design of acceptance sampling plans. Section VII is on the discussion of quality measure, and further two-stage group acceptance sampling plans are discussed in Section VIII.

#### **II. ACCEPTANCE SAMPLING PLAN**

Sampling plans are one of the old techniques which came into existence during the period of world-war 2, [1]. In fact, large number of ammunitions was purchased at that time. In order to check the quality of these ammunitions such kind of plans were introduced because if all the ammunitions were tested then no such product would be left for shipment. Basically it was applied for testing of ammunitions by US military to check the standard of ammunitions under MIL-STD 105 (military standard) which provides tables and methods for sampling plans based on some mathematical theories, [2]. When sampling plans are used to make a decision on the proposed lot whether to accept or reject. the plans are called acceptance sampling plans (ASPs). These plans are useful for life testing experiments to check the quality of product. In fact from a proposed lot of items, the inspection of some items use to be done with sampling plans which makes the producer to decide whether the proposed lot can be accepted or rejected based on the quality of the drawn sample, see Fig. 1.



Fig. 1. Illustration for acceptance sampling plan.

More precisely, to make a decision about a proposed lot, one of the usual way is to take a sample from the proposed lot and check the quality of the products. Further if the drawn sample satisfies the quality as the claim made in the proposed lot then a decision of accepting the whole lot will be made otherwise the 10(2b): 137-142(2019)

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whole lot will be rejected. It can be noticed that it may happen that the drawn sample turn out to be bad by chance but the proposed lot was good, also the drawn sample may turn out to be good when the proposed lot was bad. So acceptance sampling always involve these two risks known as producer's risk and consumer's risk. We next discuss some assumptions before proceeding for the ASPs under truncated life test.

(i) Life time Model: Life time data arises from the units put on life test experiments, and lifetime model are the class of models, or say distributions, used to model the lifetime data. Lifetime distributions can help the experimenter to further analyse data statistically, provide inference or predict the future behaviour of the product. In fact characteristics of a product such as mean lifetime can be studied through the characteristics of the considered distribution. These distributions consist of the hazard rate with associated parameters of the distribution. The hazard rate is the part of statistical branch called survival analysis which predicts the measure of time until a specific occasion happens, such as (death or failure). It refers to the death/failure rate for an item of a given age, say X. It examines the probability that an item will survive to a specific point in time based on its survival to a prior time. The hazard rate just applies to items that can't be fixed, and is denoted by Greek letter  $\lambda$  or h. The hazard rate or failure rate  $h_X(x)$  of the distribution of a continuous random variable X with cumulative distribution function (CDF)  $F_X(x) = P(X \le x)$  and probability density function (PDF)  $f_X(x) = F_{X'}(x)$ , is defined as

$$h_X(x) = -\frac{d}{dx}\log(1 - F_X(x)) = \frac{f_X(x)}{1 - F_X(x)}$$

Notice that the failure rate can't be negative. In reliability analysis, the most well known statistical tests try to establish whether the times between failures are efficiently increasing, constant or decreasing. It can be clearly understood from the concept of Bathtub curve which is often used to describe the behaviour of failed items, [3].

Our first assumption is to suppose that lifetimes of the units put on life test experiment follow a distribution having PDF  $f_X(x;\theta)$  and CDF  $F_X(x;\theta)$ , where  $\theta$  represents the unknown or a vector of unknown parameters of the distribution.

(ii) Quality of the Units: Suppose that the actual lifetime of the units is m units of time, and the specified lifetime as claim made by the producer is  $m_0$  units of the time. The lifetime can be considered as an average mean lifetime or median lifetime of the units. Now the ratio of the actual lifetime and specified lifetime, say  $r = m/m_0$  can be seen as a quality level.

(iii) **Truncated Life Test:** In life test experiment failure times are observed for the units put on the experiment. However failing all the units may take a long time which may increase the cost of the experiment and also there may have restriction on the time. So often the life tests are run for pre-specified units of time, and the experiments are terminated once the pre-specified time reaches. Such life tests are called truncated life test. Often to check the quality of sample, life test experiments are conducted for pre-specified  $t_0$  units of time, such that  $t_0 = am_0$  where  $m_0$  represent the

specified lifetime of the units as claimed by producer and a is any positive constant, [4].

Now based on the above assumptions the idea of ASP is to consider a plan so a decision can be made whether to accept or reject the proposed lot based on the quality of the drawn sample. However during the experiment how many units need to be drawn as a sample and how the quality of the units in sense of lifetime need to checked play important roles. We next discuss the answers to these questions through various ASPs in subsequent sections.

## **III. SINGLE ACCEPTANCE SAMPLING PLAN**

In single acceptance sampling plan (SASP), a sample containing n number of units is drawn from the proposed lot, and all the units are put on the life test experiments for the  $t_0$  units of time. During the experiment, if more than c number of units fail then a decision of rejecting the lot will be made otherwise it will be accepted. Here sample size n and acceptance number c are called the design parameters of the SASP. As in this plan a decision depends on the single sample drawn from the proposed lot and single acceptance number so the name is SASP.

Now if we consider p (consider success) as probability that a unit fails before the prescribed time  $t_0$ . Then recall that the CDF also represents the same, and therefore pbecomes  $p = F_X(t_0; \theta)$ . Further each unit will either fail before the time  $t_0$  or will not fail, so there are two possible outcomes which resemble the Bernoulli trial. Notice that the total number of successes in n number of Bernoulli trials follow Binomial distribution with parameter n and p, see [5]. Therefore the probability of accepting a lot becomes

$$P_{a}(p) = \sum_{i=0}^{c} {n \choose i} p^{i} (1-p)^{n-i}$$
(1)

#### IV. DOUBLE ACCEPTANCE SAMPLING PLAN

Double acceptance sampling plans (DASPs) can be considered as the generalization of the SASPs. In DASP a decision depends on two samples and acceptance numbers, also see [6]. The main advantage of the DASPs is that it provides another chance to the producer to check the quality by taking another sample. So if the first sample turns out to be bad by chance still the second sample may provide more information to make a final decision. The following is the procedure under double acceptance sampling, also see Fig. 2:

1. Draw a random sample of size, say  $n_1$  from the proposed lot, and put all the  $n_1$  number of units simultaneously on the life test experiment for the  $t_0$  units of time. Then

(a) Accept the lot if  $c_1$  or less number of units fail during the life test experiment

(b) Reject the lot if  $c_2 + 1$  or more number of units fail during the experiment.

(c) Otherwise, go to next step.

2. Draw a second sample of size, say  $n_2$  from the proposed lot, and again put all the  $n_2$  number of units simultaneously on the life test experiment for the same  $t_0$  units of time. Then

(a) Accept the lot if at most  $c_2$  number of units have failed from the two samples.

(b) Reject the lot otherwise, that is if  $c_2 + 1$  or more number of units have failed from the two samples.



Here  $c_1 < c_2$ , Design Parameter  $(n_1, n_2, c_1, c_2)$ , Single acceptance sampling plan for  $c_1 = c_2$ 

Fig. 2. Procedure for double acceptance sampling plan.

Here  $(n_1, n_2, c_1, c_2)$  are the design parameters. Observe that DASPs reduce to SASP, if one considers  $n = n_1 = n_2$  and  $c = c_1 = c_2$ . Further the probability of accepting a lot for the DASP is

$$P_a(p) = P_a^1(p) + P_a^2(p)$$
 (2)

Here  $P_a^1(p)$  and  $P_a^2(p)$  are the probabilities of accepting a lot respectively from the first sample and the second sample, and are given by

$$P_a^{1}(p) = \sum_{i=0}^{c_1} {n_1 \choose i} p^i (1-p)^{n_1-i}$$

and

$$P_a^2(p) = \sum_{j=c_1+1}^{c_2} \binom{n_1}{j} p^j (1-p)^{n_1-j} [\sum_{i=0}^{c_2-j} \binom{n_2}{i} p^i (1-p)^{n_2-i}]$$

## V. GROUP ACCEPTANCE SAMPLING PLAN

Group Acceptance Sampling Plans (GASP) are very much useful when units put on the experiment are highly reliable and are of low cost, [7, 8]. In this sampling plan a sample of *n* number of units is drawn and further divided into, say *g* number of groups where each group contains *r* number of units such that  $n = g \times r$ . All the group are then simultaneously tested for a prescribed time period  $t_0$ . The lot under inspection accepts if in each group the number of failed units is smaller than particular specified number. A decision using GASP can be made in the following manner, also see Fig. 3. First select a random sample of *n* units such that  $n = g \times r$ . Assign *r* number of units to each *g* number of groups. Now put all the groups simultaneously on life

of groups. Now put all the groups simultaneously on life test experiment for  $t_0$  units of time. During the experiment, if the number of failure units exceeds the acceptance number c in any group then terminate the experiment and reject the lot.



Fig. 3. Procedure for group acceptance sampling plan.

The GASP can also be seen as the generalization of the SASP as correspond to r = 1 it reduces to SASP. The design parameters for the GASPs are (g,c), here r is prefixed and  $n = g \times r$ . The probability of accepting a lot under this sampling plan is

$$P_{a}(p) = \left[\sum_{i=0}^{c} {r \choose i} p^{i} (1-p)^{r-i}\right]^{g}$$
(3)

Observe that corresponding to g=1 we get n=r, and therefore the probability of accepting a lot under GASP is same as under SASP.

## VI. DESIGN OF ACCEPTANCE SAMPLING PLAN

This section deals with the design of ASP to obtain the design parameters for the considered ASPs. To understand it let us consider an example. Suppose a producer proposes a lot of some units and makes a claim on the quality in sense of lifetime of the units. Now the question is whether to accept or reject the proposed lot based on the claims made by the producer. To make a decision statistically, generally a sample from the proposed lot is drawn to check that whether it satisfy the quality claim as made by the producer. Finally based on the quality of the sample a decision about the whole lot can be taken. However the process contains two types of risk: it may happen that the sample drawn by the consumer turn out good but the proposed lot was bad, and due to this a bad lot can be accepted. This is called consumer's risk, a probability of accepting a bad lot, and generally denoted as  $\beta$ . On the other hand: it may happen that the sample drawn by the consumer turn out bad but the proposed lot was good, and due to this a good lot can be rejected. This is called producer's risk, a probability of rejecting a good lot, and generally denoted as  $\alpha$ . So the decision making process commonly involves both the risks. During the process of decision making, a consumer will always desire to have probability of accepting a lot smaller than the probability of accepting a bad lot when  $m = m_0$ . Similarly, a producer will desire to have probability of rejecting a lot smaller than the probability of rejecting a good lot when  $m > m_0$ , [4] and [6]. Therefore for the given consumer's risk and producer's risk, the design of accepting sampling plan corresponding to a sampling plan for which the probability of accepting a lot is, say  $P_{a}(p)$ , turn out to satisfy the following two inequalities simultaneously

$$P_{a}\left(p_{1} \mid r_{1} = m \mid m_{0}\right) \leq \beta,$$

$$P_{a}\left(p_{2} \mid r_{2} = m \mid m_{0}\right) \geq 1 - \alpha,$$
(4)

Here  $p_1$  and  $p_2$  are the probabilities that a test unit put on the life test experiment fails before the termination time  $t_0$  respectively when the quality levels are  $r_1$  and  $r_2$  corresponding to consumer's risk  $\beta$  and producer's risk  $\alpha$ . Therefore for the various discussed ASPs in the previous section, design parameters can be obtained on solving the system of equations given by Eqn. (4).

## (i) Design of SASP

From section III, the probability of accepting a lot under SASP is given by, see (1)

$$P_{a}(p) = \sum_{i=0}^{c} {n \choose i} p^{i} (1-p)^{n-i}$$

Further the design parameters are (n,c), and to obtain the design parameters one need to have the values of  $a, m, m_0, \beta, \alpha$  and  $p = F(t_0; \theta)$ . Observe that the probability that a unit fails before the truncated time  $t_0$ is given by  $p = F(t_0; \theta)$  which depends upon the distribution under consideration to which the lifetime of the units follow. Let us consider the exponential distributions, and obtain the value of p.

#### **Exponential Distribution :**

Observe that for the exponential distribution, CDF is given in [9]. Further the mean lifetime, say m of the exponential distribution is given by  $1/\lambda$ . Therefore if we consider  $p = F_{\chi}(t_0; \lambda)$ , then we get

$$p = 1 - e^{-\lambda t_0}, t_0 > 0 \tag{5}$$

Further if the truncated times is taken as  $t_0 = am_0$ , where  $m_0$  represent the specified mean lifetime. Then on utilizing the relation  $m = 1/\lambda$ , we get

$$p=1-e^{-a\frac{m_0}{m}},$$

which can be easily computed using the given values of a, m and  $m_0$ . Subsequently the design parameters (n,c) can be obtained on solving system of equations given by Eqn. (4) with p given by Eqn. (5).

## (ii) Design of DASP

From section IV, the probability of accepting a lot under DASP is given by,

$$P_a(p) = P_a^1(p) + P_a^2(p)$$

Here  $P_a^1(p)$  and  $P_a^2(p)$  are the probabilities of accepting a lot respectively from the first sample and the second sample, and are given by

$$P_{a}^{1}(p) = \sum_{i=0}^{c_{1}} {n_{1} \choose i} p^{i} (1-p)^{n_{1}-i}$$

and

$$P_{a}^{2}(\rho) = \sum_{j=c_{1}+1}^{c_{2}} {n_{1} \choose j} \rho^{j} (1-\rho)^{n_{1}-j} \left[\sum_{i=0}^{c_{2}-j} {n_{2} \choose i} \rho^{i} (1-\rho)^{n_{2}-i}\right]$$

Further the design parameters are  $(n_1, n_2, c_1, c_2)$ , and to obtain the design parameters one need to have the values of  $a, m, m_0, \beta, \alpha$ , and  $p = F(t_0; \theta)$ . Now we have

already obtained the value of p for exponential distribution. Therefore the design parameters  $(n_1, n_2, c_1, c_2)$  can be obtained on solving the system of equations given by Eqn. (4) with p given by Eqn. (5). However it can be observed that many different combinations of  $(n_1, n_2, c_1, c_2)$  may satisfy the system of equations given by (4) with probability of accepting a lot given by Eqn. (5). So the best and desired combination will be that for which average sample number (ASN) becomes minimum to take a decision. For DASP the ASN is given by

$$ASN(p) = n_1 P_d(p) + (n_1 + n_2) \left( 1 - P_d(p) \right)$$

where  $P_d(p)$  is the probability that a decision has been made based on the first sample, and is given by

$$P_d(p) = 1 - \sum_{j=c_1+1}^{c_2} {n_1 \choose j} p^j (1-p)^{n_1-j}$$
(6)

Now the desired design of parameters for DASP can be obtained on solving the following optimization problem

Minimize 
$$ASN(p_1) = n_1P_d(p_1) + (n_1 + n_2)(1 - P_d(p_1))n$$
  
Subject to  $P_a(p_1 | r_1 = m / m_0) \le \beta$ ,  
 $P_a(p_2 | r_2 = m / m_0) \ge 1 - \alpha$ ,  $1 \le n_2 \le n_1$ ,  $n_1, n_2$  : integers.

#### (iii) Design of GASP

From section V, the probability of accepting a lot under GASP is given by Eqn. (3)

$$P_{a}(p) = \left[\sum_{i=0}^{c} {r \choose i} p^{i} (1-p)^{r-i}\right]^{g}$$

Further corresponding to g = 1 we get n = r, and therefore the probability of accepting a lot under GASP is same as under SASP. Now the design parameters are (g, c) with prefixed r, and these can be obtained for exponential distribution on solving the system of equations given by (4) with p given by Eqn. (5). However likewise in design of DASP the interest will be in those design parameters for which the ASN becomes minimum. Therefore such design parameters can be obtained on solving the following optimization problem Minimize  $ASN = n = q \times rn$ 

Subject to 
$$P_a(p_1 | r_1 = m/m_0) \le \beta$$
,  
 $P_a(p_2 | r_2 = m/m_0) \ge 1 - \alpha$ ,

## VII. DISCUSSION ON QUALITY MEASURE

So far we have discussed SASP, DASP and GASP. In previous section we also discussed the example when lifetimes of the units follow exponential distribution and mean lifetime of the units is considered as a quality measure. However not every time the mean lifetime can be considered. For an instance if the distribution considered for the lifetime modelling does not exhibit the mean or not in the closed form. In such situations and also alternate to mean life time, median lifetime as a quality measure can also be considered. In their work, Aslam et. al., [10] considered generalized exponential distribution with two-parameters,  $GE(\alpha, 1/\lambda)$  when the lifetime experiment is based on truncated life test. Authors considered the median lifetime for the quality measure rather than the mean lifetime. Notice that for the

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$$E(X) = \left[ \psi(\alpha + 1) - \psi(1) \right] / \lambda \text{ and}$$
$$V(X) = \left[ \psi'(1) - \psi'(\alpha + 1) \right] / \lambda^2$$

Here  $\psi(.)$  and  $\psi'(.)$  are the digamma and ploygamma functions respectively given as

$$\psi(u) = \frac{d}{du} \Gamma(u)$$
 and  $\psi'(u) = \frac{d}{du} \psi(u)$ 

where  $\Gamma(u) = \int_0^\infty x^{u-1} e^{-x} dx$ . Now in the case of

generalized exponential distribution the mean doesn't exhibit the closed form so constructing acceptance sampling plans may become difficult. However the median is given by

$$m = -\ln\left(1 - 0.5^{1/\alpha}\right) / \lambda$$

Further by making use of the CDF of  $GE(\alpha, \lambda)$  and m given in the previous equation, the value of  $p = F(t_0, \alpha, \lambda)$  is given by p = . Now by making use of this p the design parameters for the SASP, DASP and GASP can be obtained as discussed in the previous sections. On generalized exponential distribution Aslam et al., [13] also suggested group acceptance sampling plans. Authors made the assumption that the shape parameter of the generalized exponential distribution is known, and obtained design parameters such as the number of groups and the acceptance number by considering both the producer's and consumer's risks at the specified quality levels in terms of medians, under the assumption that the termination time and the number of items in each group are pre-fixed. Rao [12] also constructed group acceptance sampling plan for generalized exponential distribution.

One may also refer to the work of Aslam and Ali [11] for GASP developed for various lifetime distributions. The value of p given in the Authors work can be further used to compute the design parameters for the DASP. We mention that the work done by Aslam *et al.*, [10] do not obtain the design parameters for the sampling plan as suggested in this work.

# VIII. TWO-STAGE GROUP ACCEPTANCE SAMPLING PLAN

We have seen that the SASP can be viewed as particular case of DASP and GASP. In similar way DASP and GASP can also be considered as the particular cases of the two-stage group acceptance sampling plan (TGASP). Next we discuss TGASP as considered in the work of Singh *et al.*, [4]. Authors suggested TGASP in which the sampling and decision are taken in the following way.

**First stage:** Draw a random sample of  $n_1$  size from the proposed lot, and allocate the units to  $g_1$  number of groups such that each group contain r number of units so that  $n_1 = g_1 \times r$ . Now put all the groups on the experiment simultaneously for  $t_0$  units of time. During the experiment if the number of failures exceed an acceptance number  $c_2$  then the lot will be rejected and if the number of failures are less than or equal to  $c_1$  then the lot will be accepted. Further if number of

failures are between the  $c_1$  and  $c_2$  then move to second stage.

**Second stage:** Draw a second random sample of size  $\mathit{n}_2$  , and again allocate the units to  $\mathit{g}_2$  number of

groups that each group contains the same r number of units so that  $n_2 = g_2 \times r$ . It is to be noticed that here the second sample should be of small size as compared to the sample drawn in first stage, that if  $n_2 < n_1$ . Now again run the experiment for all groups simultaneously for the same  $t_0$  time. If the total number of failures from both the samples are less than  $c_2$  then the lot will be accepted otherwise it will be rejected.

The probability of accepting a lot under the TGASP is given by

$$P_a(p) = P_a^{(1)}(p) + P_a^{(2)}(p)$$

Here,  $P_a^{(1)}(p)$  and  $P_a^{(2)}(p)$  are the probabilities of accepting a lot from the first stage and second stage respectively, and are given by

$$P_a^{(1)}(p) = \sum_{i=0}^{c_1} {n_1 \choose i} p^i (1-p)^{r-i}$$

and

$$P_{a}^{(2)}(\rho) = \sum_{j=c_{1}+1}^{c_{2}} \binom{n_{1}}{j} \rho^{j} (1-\rho)^{n_{1}-j} \left[ \sum_{i=0}^{c_{2}-j} \binom{n_{2}}{i} \rho^{i} (1-\rho)^{n_{2}-i} \right].$$

Observe that the design parameters for TGASP are  $(g_1, g_2, c_1, c_2)$ . Notice that for  $c_1 = c_2 = c$ , the proposed sampling plan reduces to a (single-stage) GASP with parameters (g, c). Accordingly, only the first stage will be taken into consideration with  $q = q_1$ . Also correspond to r = 1, the proposed sampling plan reduces to DASP with design parameters  $(n_1, n_2, c_1, c_2)$ . Furthermore for  $c_1 = c_2 = c$ , it reduces to SASP with and equivalently, (n,c)denotes the  $n = n_1$ corresponding design parameters. This type of plans can be further obtained for various lifetime distributions as discussed in the recent work of Aslam and Ali [11].

## IX. CONCLUSIONS

In this paper, sampling plan, and various acceptance sampling plans such as single acceptance sampling, double acceptance sampling, group acceptance sampling, two-stage group acceptance sampling plans have discussed. Further a discussion on the design of acceptance sampling plan and quality measure are presented. The given discussion may be useful for the early carrier researchers to have basic knowledge to start research work in the field of acceptance sampling plans.

## ACKNOWLEDGEMENTS

Author thanks the Editor and anonymous reviewers for their constructive suggestions. Author is also grateful to Dr. Sukhdev Singh at Department of Mathematics, Chandigarh University, Punjab, India for all the guidance through at this work.

#### CONFLICT OF INTEREST

Author has no conflict of interest.

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