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Some Fixed Point Results for Contraction in Dislocated Metric Space

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ABSTRACT: In this paper, we prove some fixed point results in dislocated metric space under contraction condition using linear and rational expression in single as well as in couple of mappings.

Keywords: Fixed point, Complete metric space, Dislocated metric space, Convergent sequence, Contraction.

I. INTRODUCTION

Fixed point was introduced to solve the problems of existence and uniqueness. Many researchers gave many theories and results related to fixed point theory in many spaces. This is one of the most powerful tool for the problems growing in differential and integral equations. Fixed point theory is related to existence of fixed point and uniqueness. A very important result in this field is Banach Contraction Principle [1] which was given by S. Banach in 1992. Kannan [2] rectified lagoon of Banach contraction principle and proved a fixed point theorem for operators that need not be continuous. Further, Chatterjea [3] proved a result for discontinuous mapping which is a kind of dual of Kannan mapping. A lucid survey shows that there exists a vast literature available on fixed point theory. They are applicable in iteration methods, partial differential equations, integral differential equations, variational inequalities etc. There are lots of authors who extended the Banach Contraction Principle in different directions. Hitzler and Seda [7] have presented the notion of dislocated metric space in the year 2000. In dislocated metric space distance of a point from itself not necessarily be equal to zero. This idea had been first studied in domain theory [6] where the dislocated metric space was recognized as metric domains. Dislocated metric spaces play a very critical role in the fields of topology as well as in different parts of science including logic programming and electronic engineering. D.S. Jaggi [5] proved fixed point theorem using rational expression.

II. PRELIMINARIES

Here we establish some necessary details and some outcomes in dislocated metric space.

Definition 2.1 [9] Let $K \neq \emptyset$, and let a function $d: K \times K \to R^+$ satisfying the following axioms, then d is dislocated metric space.

(d_1) $d(m.n)= d(n,m) = 0$ implies $m = n$;
(d_2) $d(m,n) = d(n,m)$ for all $m,n \in K$
(d_3) $d(m,n) \leq d(m,l) + d(l,n)$ for all $l,m,n \in K$

Example 2.2 [14] Let $K = \mathbb{R}^+$an $d: K \times K \to \mathbb{R}^+$ is given by

$$d(a_1, a_2) = \max\{a_1, a_2\}$$

Definition 2.3 [16] Let a sequence $\{a_j\}$ in dislocated metric space(A,d) if

1. $\{a_j\}$ is known as convergent to $a \in A$ as,
$$\lim_{j\to\infty} a_j = 0, if \lim_{j\to\infty} T_p(a_j, a) = 0$$
2. $\{a_j\}$is known as Cauchy sequence in A, if $\lim_{j,i\to\infty} T_p(a_j, a_i) = 0$.
3. Every Cauchy sequence(A,d) is a convergent sequence then that sequence (A,d) is called complete.

Lemma 2.4 Let sequence $\{\mathcal{X}_j\}$ in a dislocated metric space (H,d) s.t

$$d(\mathcal{X}_j, \mathcal{X}_{j+1}) \leq hd(\mathcal{X}_{j-1}, \mathcal{X}_j)$$

where, $h \in [0,1)$ and $n = 1,2,3 \ldots$..
Then $\{\mathcal{X}_j\}$ is a Cauchy sequence in (H,d)

Proof: Let $j > i \geq 1$, it follows that

$$d(\mathcal{X}_j, \mathcal{X}_i) \leq d(\mathcal{X}_j, \mathcal{X}_{j+1}) + d(\mathcal{X}_{j+1}, \mathcal{X}_{j+2}) \ldots . d(\mathcal{X}_{i-1}, \mathcal{X}_i)$$
$$\leq (h^j + h^{j+1} \ldots \ldots . h^{i-1}) d(\mathcal{X}_0, \mathcal{X}_1)$$

Since $h < 1$. Assume that $(\mathcal{X}_0, \mathcal{X}_1) > 0$.
By taking $\lim_{j,i\to+\infty}$in above inequality, we get

$$\lim_{j,i\to+\infty} d(\mathcal{X}_j, \mathcal{X}_i) = 0$$

Therefore, $\{\mathcal{X}_j\}$ is a Cauchy sequence in K
Also $d\ (\mathcal{X}_0, \mathcal{X}_1) = 0$
Then $d(\mathcal{X}_j, \mathcal{X}_i) = 0\ \forall\, m > n$
$\therefore$ $\{\mathcal{X}_j\}$ is a Cauchy sequence in H

Definition 2.5 [7] Let (X,d) be a dislocated metric space and a function $f: X \to X$ is called a contraction if there exists $0 \leq \lambda < 1$ such that $d(f(x), f(y)) \leq \lambda d(x,y)$ for all $x, y \in X$.

III. MAIN RESULTS

In this section, we prove some fixed point theorems in single as well as in couple of mappings for contraction in dislocated metric spaces. We will use the abbreviation CDMS for complete dislocated metric space.

Theorem 3.1 Let (S,d) is a CDMS and $T: A \to A$ be a continuous mapping and satisfying the condition:

$$d(Tl,Tm) \leq \alpha_1 d(l,m) + \alpha_2[d(l,Tl) + d(m,Tm)]$$
$$+\alpha_3[d(l,Tm) + d(m,Tl)] + \alpha_4\left[\frac{d(l,m)d(l,Tm)}{d(l,m) + d(m,Tm)}\right]$$
$$+ \quad \alpha_5\left[\frac{d(l,Tm)d(m,Tm)}{d(l,m)+d(m,Tm)}\right] \tag{1}$$

Where $\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 \geq 0$ with $\alpha_1 + 2\alpha_2 + 4\alpha_3 + \alpha_4 + \alpha_5 < 1$ for all $l,m \in S$, then T has a unique fixed point.

Proof: Let x_0 is arbitrary. Consider a sequence $\{x_n\}$ and Picard's iteration $x_{n+1} = Tx_n$. If for any n,$x_{n+1} = x_n$, then x_n is a fixed point. Therefore, there is no need to go further. Assert $x_{n+1} \neq x_n$ for any n. Now using Eqn. (1) we have

$$d(x_{n+1}, x_{n+2}) = d(Tx_n, Tx_{n+1})$$

$$\leq \alpha_1 d(x_n, x_{n+1}) + \alpha_2[d(x_n, Tx_n) + d(x_{n+1}, Tx_{n+1})]$$
$$+\alpha_3[d(x_n, Tx_{n+1}) + d(x_{n+1}, Tx_n)]$$
$$+\alpha_4\left[\frac{d(x_n, x_{n+1}).d(x_n, Tx_{n+1})}{d(x_n, x_{n+1}) + d(x_{n+1}, Tx_{n+1})}\right]$$
$$+\alpha_5\left[\frac{d(x_n, Tx_{n+1}).d(x_{n+1}, Tx_{n+1})}{d(x_n, x_{n+1}) + d(x_{n+1}, Tx_{n+1})}\right]$$

$$\leq \alpha_1 d(x_n, x_{n+1}) + \alpha_2[d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2})]$$
$$+\alpha_3[d(x_n, x_{n+2}) + d(x_{n+1}, x_{n+1})]$$
$$+\alpha_4\left[\frac{d(x_n, x_{n+1})d(x_n, x_{n+2})}{d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2})}\right]$$
$$+\alpha_5\left[\frac{d(x_n, x_{n+2})d(x_{n+1}, x_{n+2})}{d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2})}\right]$$
$$\leq \alpha_1 d(x_n, x_{n+1}) + \alpha_2[d(x_n, x_{n+1}) + \alpha_2 d(x_{n+1}, x_{n+2})]$$
$$+2\alpha_3[d(x_n, x_{n+1}) + \alpha_2 d(x_{n+1}, x_{n+2})]$$
$$+\alpha_4 d(x_n, x_{n+1}) + \alpha_5\, d(x_{n+1}, x_{n+2})$$

$$d(x_{n+1}, x_{n+2}) \leq \frac{\alpha_1 + \alpha_2 + 2\alpha_3 + \alpha_4}{1 - (\alpha_2 + 2\alpha_3 + \alpha_5)} d(x_n, x_{n+1})$$

Let $h = \frac{\alpha_1+\alpha_2+2\alpha_3+\alpha_4}{1-(\alpha_2+2\alpha_3+\alpha_5)}$

$$\alpha_1 + 2\alpha_2 + 4\alpha_3 + \alpha_4 + \alpha_5 < 1$$

Therefore

$$d(x_{n+1}, x_{n+2}) \leq hd(x_n, x_{n+1})$$

Similarly $d(x_n, x_{n+1}) \leq hd(x_{n-1}, x_n) \leq h.\, hd(x_{n-1}, x_n)$

$$\Rightarrow d(x_n, x_{n+1}) \leq h^2 d(x_{n-1}, x_n)$$

Using iteration up to n times,

$$d(x_{n+1}, x_{n+2}) \leq h^n d(x_0, x_1)$$

where, $0 \leq h \leq 1$ $\Rightarrow h^n \to 0$ as $n \to \infty$

By **(iv) Lemma** $\{x_n\}$ is a Cauchy sequence. Therefore $\exists\, \mu \in S$ such that $\{x_n\} \to \mu$ as $n \to \infty$

Now, we will prove μ is fixed point of T. Since $\{x_n\} \to \mu$ as $n \to \infty$. Using the definition of continuity for T, we have

$$\lim_{n\to\infty} Tx_n = T\mu$$

$$\lim_{n\to\infty} x_{n+1} = T\mu$$

Then $T\mu = \mu$,

Hence μ is a fixed point of T.

Now, for uniqueness μ and ρ be the two fixed point of T for $\mu \neq \rho$, using Eqn. (1), we have

$$d(\mu, \rho) \leq \alpha_1 d(\mu, \rho) + \alpha_2[d(\mu, T\rho) + d(l_2{}^*, T\rho)]$$
$$+\alpha_3[d(\mu, T\rho) + d(\mu, T\mu)] + \alpha_4\left[\frac{d(\mu, \rho)d(\mu, T\rho)}{d(\mu, \rho) + d(\rho, T\rho)}\right]$$
$$+\alpha_5\left[\frac{d(\mu, T\rho)d(\rho, T\rho)}{d(\mu, v) + d(\rho, T\rho)}\right]$$
$$d(\mu, \rho) \leq \alpha_1 d(\mu, \rho) + \alpha_2[d(\mu, \rho) + d(\rho, \rho)]$$
$$+\alpha_3[d(\mu, \rho) + d(\rho, \mu)] + \alpha_4\left[\frac{d(\mu, \rho).d(\mu, \rho)}{d(\mu, \rho)}\right]$$
$$+\alpha_5\left[\frac{d(\mu, \rho).d(\rho, \rho)}{d(\mu, \rho)}\right]$$

As μ and ρ are fixed point of T.

Therefore, by above equation we have,

$$d(\mu, \mu) = 0 \text{ and } d(\rho, \rho) = 0$$

So, above equation become

$$d(\mu, \rho) \leq |\alpha_1 + \alpha_3 + \alpha_4||d(\mu, \rho) + \alpha_3 d(\rho, \mu)| \quad (2)$$

Similarly,

$$d(\rho, \mu) \leq |\alpha_1 + \alpha_3 + \alpha_4||d(\rho, \mu) + \alpha_3 d(\mu, \rho)|` \quad (3)$$

Subtract (2) from (3)

$$|d(\mu, \rho) - d(\rho, \mu)| \leq |(\alpha_1 + \alpha_3 + \alpha_4) - \alpha_3||(d(\mu, \rho) - d(\rho, \mu)|$$
$$\leq |\alpha_1 + \alpha_4||d(\mu, \rho) - d(\rho, \mu)| \quad (4)$$

Here, $|\alpha_1 + \alpha_4| < 1$, above inequality hold.

$$\Rightarrow d(\mu, \rho) - d(\rho, \mu) = 0 \quad (5)$$

From Eqns. (2), (3) and (5), we have

$$d(\mu, \rho) = 0 \text{ and } d(\rho, \mu) = 0$$
$$\Rightarrow \mu = \rho$$

Therefore, T has a unique fixed point.

Theorem 3.2 Let (S, d) be a CDMS and $T: S \to S$ be a continuous mapping and satisfying the condition:

$$d(Tl, Tm) \leq \alpha_1 d(l, m) + \alpha_2[d(l, Tl) + d(m, Tm)]$$
$$\left[\frac{d(l, m) + d(m, Tm)}{d(l, Tm)}\right] + \alpha_3[d(l, Tm) + d(m, Tl)]$$
$$\left[\frac{d(l,m)+d(m,Tm)+d(l,Tm)}{d(l,Tm)}\right] \quad (6)$$

Where $\alpha_1, \alpha_2, \alpha_3 \geq 0$ with $\alpha_1 + 2\alpha_2 + 8\alpha_3 < 1$ for all $l, m \in S$ .Then T has a unique fixed point.

Proof: Let x_0 is arbitrary. Consider a sequence $\{x_n\}$ and Picard's iteration $x_{n+1} = Tx_n$. If for any n, $x_{n+1} = x_n$, then x_n is a fixed point. Therefore, there is no need to go further. Assert $x_{n+1} \neq x_n$ for any n.

Now using Eqn. (1) we have

$$d(l, m) = d(Tx_n, Tx_{n+1}) \leq \alpha_1 d(x_n, x_{n+1})$$
$$+\alpha_2[d(x_n, Tx_n) + d(x_{n+1}, Tx_{n+1})]$$
$$\left[\frac{d(x_n, x_{n+1}) + d(x_{n+1}, Tx_{n+1})}{d(x_n, Tx_{n+1})}\right]$$
$$+\alpha_3[d(x_n, Tx_{n+1}) + d(x_{n+1}, Tx_n)]$$
$$\left[\frac{d(x_n, x_{n+1}) + d(x_{n+1}, Tx_{n+1}) + d(x_n, Tx_{n+1})}{d(x_n, Tx_{n+1})}\right]$$

$$\leq \alpha_1 d(x_n, x_{n+1}) + \alpha_2[d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2})]$$
$$\left[\frac{d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2})}{d(x_n, x_{n+2})}\right]$$
$$+\ \alpha_3[d(x_n, x_{n+2}) + d(x_{n+1}, x_{n+1})]$$
$$\left[\frac{d(x_n, x_{n+2}) + d(x_{n+1}, x_{n+1}) + d(x_n, x_{n+2})}{d(x_n, x_{n+2})}\right]$$
$$\leq \alpha_1 d(x_n, x_{n+1}) + \alpha_2[d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2})]$$
$$\left[\frac{d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2})}{d(x_n, x_{n+2})}\right]$$
$$+\ \alpha_3[d(x_n, x_{n+2}) + d(x_{n+1}, x_{n+2}) + d(x_n, x_{n+1})]$$
$$2\left[\frac{d(x_n, x_{n+1})}{d(x_n, x_{n+2})}\right]$$
$$\leq \alpha_1 d(x_n, x_{n+1}) + \alpha_2[d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2})]$$
$$+\ 4\,\alpha_3[d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2})]$$
$$d(x_{n+1}, x_{n+2}) \leq \frac{\alpha_1 + \alpha_2 + 4\alpha_3}{1 - \alpha_2 - 4\alpha_3} d(x_n, x_{n+1})$$

Let $h = \frac{\alpha_1+\alpha_2+4\alpha_3}{1-\alpha_2-4\alpha_3}$

as $h < 1 \Rightarrow$ $\alpha_1 + 2\alpha_2 + 8\alpha_3 < 1$

Therefore, $d(x_{n+1}, x_{n+2}) \leq hd(x_n, x_{n+1})$,

Similarly, $d(x_n, x_{n+1}) \leq hd(x_{n-1}, x_n)$

$$d(x_{n+1}, x_{n+2}) \leq h.\, hd(x_{n-1}, x_n) \leq h^2 d(x_{n-1}, x_n)$$

Using iteration up to n times,

$$d(x_n, x_{n+1}) \leq h^n d(x_0, x_1)$$

Where, $0 \leq h \leq 1$ $\Rightarrow h^n \to 0$ as $n \to \infty$

By (iv) Lemma $\{x_n\}$ is a Cauchy sequence. So $\exists \mu \in S$ such that $\{x_n\} \to \mu$ as $n \to \infty$.

Now we will prove μ is a fixed point of T.

As $\{x_n\} \to \mu$ as $n \to \infty$

By means of continuity of T, we have

$$\lim_{n\to\infty} Tx_n = \mu \Rightarrow \lim_{n\to\infty} x_{n+1} = \mu$$

Then μ is a fixed point of T.

Now, for uniqueness μ and ρ be the two fixed point of T for $\mu \neq \rho$, we have

$$d(\mu, \rho) \leq \alpha_1 d(\mu, \rho) + \alpha_2[d(\mu, T\mu) + d(\rho, T\rho)]$$
$$\left[\frac{d(\mu, \rho) + d(\mu, T\rho)}{d(\mu, T\rho)}\right]$$
$$+\alpha_3[d(\mu, T\rho) + d(\rho, T\mu)]$$

$$\left[\frac{d(\mu,\rho)+d(\rho,T\rho)+d(\mu,T\rho)}{d(\mu,T\rho)}\right]$$

Here, μ and ρ are fixed point of T.

Therefore, by given condition, we have

$$d(\mu,\mu)=0 \text{ and } d(\rho,\rho)=0$$

So, above equation become

$$d(\mu,\rho)\le \alpha_1 d(\mu,\rho)+2\alpha_3 d(\mu,\rho)+2\alpha_3 d(\rho,\mu)$$

And $d(\mu,\rho)\le(\alpha_1+2\alpha_3)d(\mu,\rho)+2\alpha_3 d(\rho,\mu)$ (7)

Similarly,

$d(\rho,\mu)\le(\alpha_1+2\alpha_3)d(\rho,\mu)+2\alpha_3 d(\mu,\rho)$ (8)

Subtract above two equations, we get

$|d(\mu,\rho)-d(\rho,\mu)|\le|\alpha_1+2\alpha_3-2\alpha_3||d(\mu,\rho)-d(\rho,\mu)|$

$|d(\mu,\rho)-d(\rho,\mu)|\le|\alpha_1||d(\mu,\rho)-d(\rho,\mu)|$ (9)

Clearly, $|\alpha_1|<1$

So, above inequality holds.

If $|d(\mu,\rho)-d(\rho,\mu)|=0$ (10)

From Eqns. (6), (7) and (10), we have

$$d(\mu,\rho)=0 \text{ and } d(\rho,\mu)=0$$

$$\Rightarrow \mu=\rho$$

Therefore, T has a unique fixed point.

Example 3.3 Let (A,d) be a CDMS and $T:R^+\to R^+$ defined as

$d(\mu,\rho)=t\mid\mu-\rho\mid$ Such that

$\mu_\alpha=1+\frac{1}{\alpha}$ and $\rho_\alpha=1+\frac{2}{\alpha}$

Therefore, $d(\mu_\alpha,\rho_\alpha)=t\mid\mu_\alpha-\rho_\alpha\mid$

$=t\mid 1+\frac{1}{\alpha}-1-\frac{2}{\alpha}\mid$

$=t\mid-\frac{1}{\alpha}\mid=t\left(\frac{1}{\alpha}\right)$

$$\log_{\alpha\to\infty} d(\mu_\alpha,\rho_\alpha)=\log_{\alpha\to\infty} t\frac{1}{\alpha}=\text{t}\log_{\alpha\to\infty}\frac{1}{\alpha}=0$$

$$=\log_{\alpha\to\infty} d(\mu_\alpha,\rho_\alpha)\to 0$$

as both $\mu_\alpha=1+\frac{1}{\alpha}$ and $\rho_\alpha=1+\frac{2}{\alpha}$ tend to 1 as $\alpha\to\infty$. Hence 1 is the fixed point.

Hence, it satisfies all the condition of CDMS.

For Theorem (i) $=\frac{1}{4},\alpha_2=\frac{1}{3},\alpha_3=\frac{1}{9},\alpha_4=\frac{17}{12},\alpha_5=\frac{1}{18}$

For Theorem (ii) $\alpha_1=\frac{1}{8},\alpha_2=\frac{1}{8},\alpha_3=\frac{1}{32}$

Theorem 3.4 Let (X,d) be a CDMS. Let S,T be two self mappings $S,T:X\to X$

(i) $T(X)\subseteq S(X)$

(ii) S and T is continuous

(iii) $d(Sr,Ts)\le\alpha_1 d(r,s)$

$$+\alpha_2[d(r,Sr)+T_d(s,Ts)]\left[\frac{d(r,s)+d(s,Ts)}{d(r,Ts)}\right]$$

$$+\alpha_3[d(r,Ts)+d(s,Sr)]$$

$\left[\frac{\{d(r,s)+d(s,Ts)+d(r,Ts)\}^2}{d(r,Ts)^2}\right]$ (11)

Where $\alpha_1+\alpha_2+\alpha_3\ge 0$ with $\alpha_1+2\alpha_2+16\alpha_3<1$. for all $x,y\in X$. Then S,T has a unique common fixed point.

Proof: Let $r_0\in X$ be arbitrary and sequence $\{r_j\}_{j\varepsilon N}$ Such That

$r_1=S(r_0),r_2=T(r_1)\ldots\ldots r_{2j+1}=Sr_{2j},r_{2j}=T(r_{2j-1})$.

Using (1), we have

$$d(r_{2j+1},r_{2j+2})=d(Sr_{2j},Tr_{2j+1})\le\alpha_1 d(r_{2j},r_{2j+1})$$

$$+\alpha_2\begin{bmatrix}d(r_{2j},Sr_{2j})\\+d(r_{2j+1},Tr_{2j+1})\end{bmatrix}$$

$$\left[\frac{d(r_{2j},r_{2j+1})+d(r_{2j+1},Tr_{2j+1})}{d(r_{2j},Tr_{2j+1},)}\right]$$

$$+\alpha_3\begin{bmatrix}d(r_{2j},Tr_{2j+1})\\+d(r_{2j+1},Sr_{2j})\end{bmatrix}$$

$$\left[\frac{\{d(r_{2j},r_{2j+1})+d(r_{2j+1},Tr_{2j+1})+d(r_{2j},Tr_{2j+1})\}^2}{\{d(r_{2j},Tr_{2j+1})\}^2}\right]$$

$$\le\alpha_1 d(r_{2j},r_{2j+1})$$

$$+\alpha_2\begin{bmatrix}d(r_{2j},r_{2j+1})\\+d(r_{2j+1},r_{2j+2})\end{bmatrix}\left[\frac{d(r_{2j},r_{2j+1})+d(r_{2j+1},r_{2j+2})}{d(r_{2j},r_{2j+2})}\right]$$

$$+\alpha_3\begin{bmatrix}d(r_{2j},r_{2j+1})\\+d(r_{2j+1},r_{2j+1})\end{bmatrix}$$

$$\left[\frac{\{d(r_{2j},r_{2j+1})+d(r_{2j+1},r_{2j+2})+d(r_{2j},r)\}^2}{\{d(r_{2j},r_{2j+2})\}^2}\right]$$

$$\le\frac{\alpha_1+\alpha_2+12\alpha_3}{1-\alpha_2-4\alpha_3}d(r_{2j},r_{2j+1})$$

$$d(r_{2j+1},r_{2j+2})\le kd(r_{2j},r_{2j+1})$$

Where, $k=\frac{\alpha_1+\alpha_2+12\alpha_3}{1-\alpha_2-4\alpha_3}$; $0<k<1$

Continuing this way, we get

$$d(r_{2j+1},r_{2j+2})\le k^{2j}d(r_{2j},r_{2j+1}) \quad ;0<k<1$$

By **(2.4) Lemma** $k^{2j}\to 0$ as $j\to\infty$. Hence $\{r_j\}_{j\varepsilon N}$ is a Cauchy sequence in (X,d)

So, $\exists\, p\in X$ s.t $\{r_j\}\to p$.

Further the subsequence $\{Sr_{2j}\}\to p$ and $\{Tr_{2j}\}\to p$

Since $S,T:X\to X$ are continuous, we have

$$Sp=p \text{ and } Tp=p$$

Then, p is a fixed point of S and T.

$$\Rightarrow Sp=p=Tp$$

Now, for uniqueness μ and ρ be the two-fixed point of S and T, then we get

$d(\mu,\rho)=d(S\mu,T\rho)\le\alpha_1 d(\mu,\rho)$

$$+\alpha_2[d(\mu,S\mu)+d(\rho,T\rho)]\left[\frac{d(\mu,\rho)+d(\rho,T\rho)}{d(\mu,T\rho)}\right]$$

$$+\alpha_3\begin{bmatrix}d(\mu,T\rho)\\+d(\rho,S\mu)\end{bmatrix}\left[\frac{\{d(\mu,\rho)+d(\rho,T\rho)+d(\mu,T\rho)\}^2}{[d(\mu,T\rho)]^2}\right]$$

$$\le\alpha_1 d(\mu,\rho)+\alpha_2[d(\mu,\mu)+d(\rho,\rho)]\left[\frac{d(\mu,\rho)+d(\rho,\rho)}{d(\mu,\rho)}\right]$$

$$+\alpha_3[d(\mu,\rho)+d(\rho,\mu)]\left[\frac{\{d(\mu,\rho)+d(\rho,\rho)+d(\mu,\rho)\}^2}{[d(\mu,\rho)]^2}\right]$$

Hence $d(\mu,\mu)=0$ and $d(\rho,\rho)=0$

$$\le\alpha_1 d(\mu,\rho)+\alpha_3[d(\mu,\rho)+d(\rho,\mu)]$$

$$\left[\frac{\{d(\mu,\rho)+d(\mu,\rho)\}^2}{[d(\mu,\rho)]^2}\right]$$

$$\le\alpha_1 d(\mu,\rho)+[4\alpha_3 d(\mu,\rho)+\alpha_3 d(\rho,\mu)]$$

$$\le(\alpha_1+4\alpha_3)d(\mu,\rho)+4\alpha_3 d(\rho,\mu)$$

$d(\mu,\rho)\le(\alpha_1+4\alpha_3)d(\mu,\rho)+4\alpha_3 d(\rho,\mu)$ (12)

Similarly,

$d(\rho,\mu)\le(\alpha_1+4\alpha_3)d(\rho,\mu)+4\alpha_3 d(\mu,\rho)$ (13)

Subtract above two equations, we have

$|d(\mu,\rho)-d(\rho,\mu)|\le|\alpha_1+4\alpha_3-4\alpha_3||d(\mu,\rho)-d(\rho,\mu)|$

$|d(\mu,\rho)-d(\rho,\mu)|\le|\alpha_1||d(\mu,\rho)-d(\rho,\mu)|$ (14)

Clearly, $|\alpha_1|<1$

So, above inequality holds.

If $d(\mu,\rho)-d(\rho,\mu)=0$ (15)

From Eqns. (11), (12) and (15) we have

$$d(\mu,\rho)=0 \text{ and } d(\rho,\mu)=0\Rightarrow\mu=\rho$$

Thus fixed point of S and T is Unique.

IV. CONCLUSION

In the iterative fixed-point procedure, the results vary like some of results are completely significant while the others are formulated. In this paper, we proved fixed point theorems for single and double mapping under contraction condition in complete dislocated metric space.

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