International Journal on Emerging Technologies 11(1): 409-413(2020)

ISSN No. (Print): 0975-8364
ISSN No. (Online): 2249-3255

Line Weight based PMU Placement for Observability in Test Case Power Systems

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(Received 09 October 2019, Revised 05 December 2019, Accepted 13 December 2019)
(Published by Research Trend, Website: www.researchtrend.net)

ABSTRACT: This paper proposes an optimal Phasor Measurement Units (PMU) deployment method considering branch weight and Zero Injection (ZI) constraints in Binary Integer Programming (BIP) formulation to attain complete observability. The installation cost of PMU for different buses varies with number branches connected. BIP method is formulated in such a way that PMUs are installed at buses connected with minimum weight branches in prior, to minimize cost of installation without loss of observability. To optimize placement locations in network, a ZI constraint model is proposed. ZI constraint modeling decreases the locations in the network for PMU Placement thereby reducing cost of installation. The observability performance is checked with Branch Redundancy Index (BRI) proposed. IEEE -14, -24, -30 and 57 bus systems are tested with MATLAB software and compared with methods to show its effectiveness.

Keywords: Branch weight; PMU; Redundancy; Synchrophasors; Zero Inkection.

Abbreviations: PMU-Phasor Measurement Unit; GPS- Global Position Systems; OPP- Optimal PMU placement; MICA- Modified Imperialist Competitive Algorithm; MBCOA-Modified Binary Cuckoo Optimization Algorithm; TBLO-Teaching-Learning-Based-Optimization; CNBOI-Complete Network Bus Observability Index; SE-State Estimation; BOI-Observability Index.

I. INTRODUCTION

In power grid network, State Estimation (SE) process of finding voltage and current at particular buses plays a key role in protection and control of power system. Due to inaccurate estimation of states of system network in the cases where generation failure or sudden load increases that may change to blackouts in the power system. In this case there is in need to find accurate states of the system. With introducing synchro phasors in to power system leads to development of SE process with accurate measurements. PMUs installed at different locations in the network synchronized with the time stamp provided by Global Position Systems (GPS). PMUs provide accurate phasor measurements associated with voltage and current. PMU placement at every bus of system is not feasible due to economic cost of it. PMUs installed at different locations differ in cost due to number of branches connected to it. PMU with a smaller number of channels decreases cost of installation. Optimal placement of PMUs in power system network is developed in 1990s [1, 2]. Different SE methods are developed by them with optimal PMU and conventional measurements. Optimal PMU placement (OPP) methods to determination of power system observability with topology-based algorithms is formulated in [3, 4]. Many workers have proposed linear programming with different techniques. Also concentrated to optimize the channels of PMUs while installing at the bus [5-8]. A channel oriented OPP is proposed such a way that channels are treated optimization variables. PMUs are installed to observe voltage stability status [9]. The Modified Imperialist Competitive Algorithm (MICA) is use for contingency analysis, as well as normal operation in OPP case [10]. However, a fuzzy based binary linear integer program is proposed to obtain the total observability of network in both cases [11]. To increase the measurement redundancy of the network, which, complete observability with binary particle swarm optimization technique [12]. A novel evolution algorithm is Modified Binary Cuckoo Optimization Algorithm (MBCOA) and Teaching-Learning-Based-Optimization (TBLO) are also used to find complete observability with OPP [13, 14]. Optimal placement of PMUs with limited number of channels is discussed in [15-16]. In this it has been assumed that PMUs with sufficient number of channels such that PMU placed at any bus would make neighboring buses observable. Different manufacturers of PMUs introduced them with different number of channels such as TESLA 4003 and QPMU have 5 channels, ABBRES521 have 6 channels, SEL487E have 15 channels etc. To estimate observability performance of complete network, a Complete Network Bus Observability Index (CNBOI) is suggested.

This paper proposes optimal PMU installation considering minimum branch weight including ZI constraints to attain complete observability. As different manufactures produce different PMUs with different number of channels, a minimum branch weight is considered to optimize PMU locations which can be used for all type of PMUs.

II. PROPOSED FORMULATION FOR PLACEMENT OF PMU

A. Formulation of Problem

The objective function is formulated for placement of PMUs considering minimum branch weights as the

priority to obtain complete observability of system network.
The objective function formulated subjected to observability constraints for placement of PMU is as follows:

$$Min \sum_{p=1}^{N} W_p y_p \qquad (1)$$

Subjected to

$$y(p) = \begin{cases} B-1 = y_1 + y_2 + y_5 \geq 1 \\ B-2 = y_1 + y_2 + y_3 + y_4 + y_5 \geq 1 \\ B-3 = y_2 + y_3 + y_4 \geq 1 \\ B-4 = y_2 + y_3 + y_4 + y_5 + y_7 + y_9 \geq 1 \\ B-5 = y_1 + y_2 + y_4 + y_5 + y_6 \geq 1 \\ B-6 = y_5 + y_6 + y_{11} + y_{12} + y_{13} \geq 1 \\ B-9 = y_4 + y_7 + y_9 + y_{10} + y_{14} \geq 1 \\ B-10 = y_9 + y_{10} + y_{11} \geq 1 \\ B-11 = y_6 + y_{10} + y_{11} \geq 1 \\ B-12 = y_6 + y_{13} + y_{12} \geq 1 \\ B-13 = y_6 + y_{12} + y_{13} + y_{14} \geq 1 \\ B-14 = y_9 + y_{13} + y_{14} \geq 1 \end{cases} \qquad (2)$$

where number of buses (N), cost coefficient (WP) of PMU installed at the Pth bus in the network, based on the consideration of the diagonal property of matrix the PMU cost equal to 1 p.u.

$$y_p = \begin{cases} 1 & \text{if PMU is allocated at bus } p \\ 0 & \text{otherwise} \end{cases}$$

III. BRANCH WEIGHT CONSTRAINED OPTIMAL PMU PLACEMENT

A. ZI modeling

In large power systems, certain buses are not associated to any generators, compensators or load, in such cases, current flow in such buses is approximately equal to zero which are considered as ZI buses. These buses are considered in optimization for OPP. Modeling of ZI constraints in BIP frame work has remained challenge. Here we suggest a technique to model ZI constraints with in a linear frame of work.
The Fig. 3, shows ZI of the bus at 4th node.
The voltage phasors from bus 1 to (m – 1) are voltage phasor buses and the current $I_{j,1}$ calculated as follows.

$$I_{j,1} = Y_{j,1}[e_j - e_1] \qquad (3)$$

where $Y_{j,1}$ is the admittance line connected between bus 1, j
The observed m bus voltage computed as follows.

$$e_m = V_1 - Z_{1,m} \sum_{j=2}^{m-1} I_{j,1} \qquad (4)$$

From Eqn. 4, $Z_{1,m}$ is the line impedance between buses 1, j.
Based on the all ZIs of network the minimum number of PMUs required for complete observability, here, every ZI node consider as additional constraint.
From the above equation consider bus 2 is ZI bus

$$I_{24} = I_{32} + I_{12}$$

As we know line currents, voltage at bus 4 is calculated as

$$V_4 = V_2 - (I_{12} + I_{32})Z_{24}$$

As per KVL, the voltage at 4th node calculated and confirmed the node does not require PMU.

B. Branch weight constraint modeling

Installation cost of PMUs at different buses with different number of branches is more, so to reduce cost of installation, total branch weight of bus with minimum number of branches is selected for installing PMU. Branch weight assumed for each branch is 0.1 p.u. and for example, branch weight of 14- bus system is shown in Fig. 1.

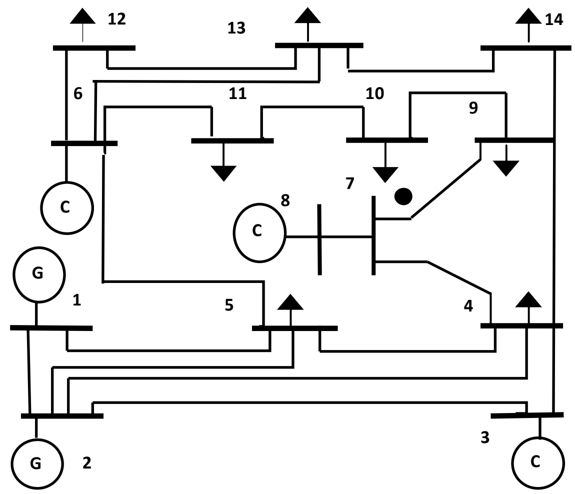

Fig. 1. Single Line diagram of 14- bus system.

Here if we observe from the Fig. 1. The bus-8 has minimum number of branches that is one. Bus 1, 3, 9, 10, 11, 12, 14 has minimum two branches connected.

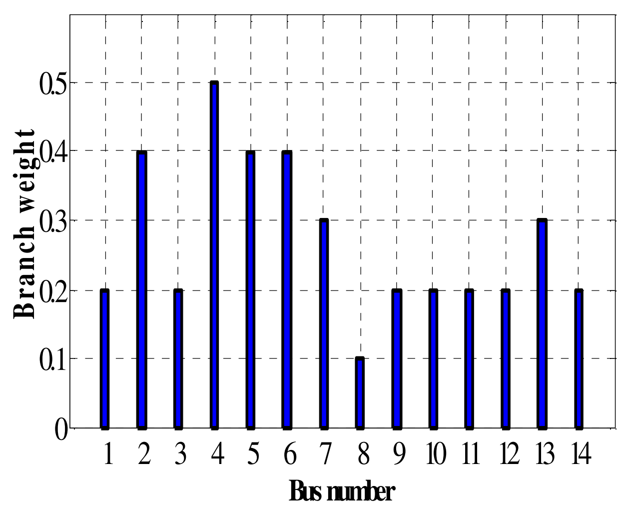

Fig. 2. Branch weight of 14 bus system.

In our problem, one PMU is constrained to be located at Bus-8 and another minimum PMU number to achieve complete observability shown in Fig. 2. This is formulated by substituting $y_8 = 1$ in Eqn. 2 which is shown as follows

$$Min \sum_{p=1}^{N} W_p y_p \qquad (5)$$

Subjected to

$$y(p)=\begin{cases} B-1=y_1+y_2+y_5\geq 1 \\ B-2=y_1+y_2+y_3+y_4+y_5\geq 1 \\ B-3=y_2+y_3+y_4\geq 1 \\ B-4=y_2+y_3+y_4+y_5+y_7+y_9\geq 1 \\ B-5=y_1+y_2+y_4+y_5+y_6\geq 1 \\ B-6=y_5+y_6+y_{11}+y_{12}+y_{13}\geq 1 \\ B-9=y_4+y_7+y_9+y_{10}+y_{14}\geq 1 \\ B-10=y_9+y_{10}+y_{11}\geq 1 \\ B-11=y_6+y_{10}+y_{11}\geq 1 \\ B-12=y_6+y_{13}+y_{12}\geq 1 \\ B-13=y_6+y_{12}+y_{13}+y_{14}\geq 1 \\ B-14=y_9+y_{13}+y_{14}\geq 1 \end{cases} \tag{6}$$

Single line diagram of 30-bus system is shown in Fig. 3. In this 30 line bus system contingency has been decreased and improves the performance.

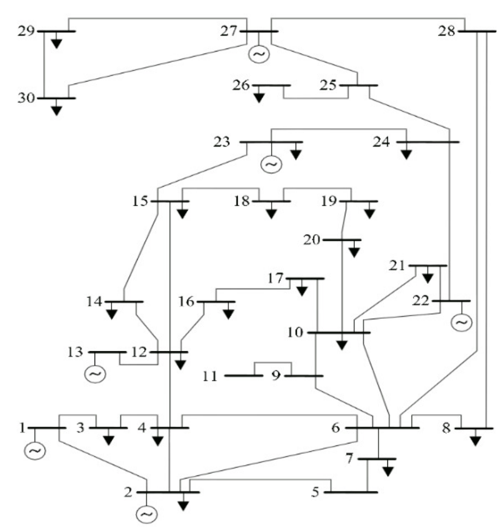

Fig. 3. Single Line diagram of 30- bus system.

Branch weight of 30-bus system is shown in Fig. 4.

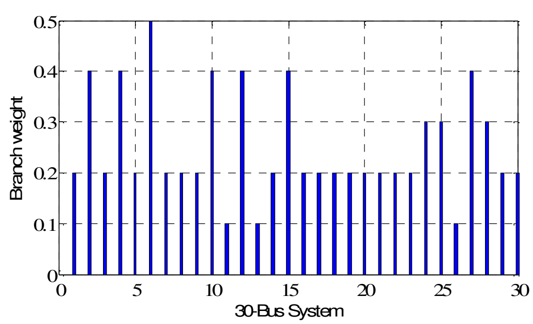

Fig. 4. Branch weight of 30 bus system.

C. Branch weight including ZI constraint Modeling

In this the 14-bus case is meant for placement of PMU. In which 7th bus is consider as ZI bus of the network, as well as 7, 4, 8 and 9th nodes are consider to find optimized constraints .The following rules are applied it $A\subset B$ is $A\cup B=B$.

$$Min\ \sum_{p=1}^{N} W_p y_p \tag{7}$$

Subjected to observability constraints p

$$y(p)=\begin{cases} B-1=y_1+y_2+y_5\geq 1 \\ B-3=y_2+y_3+y_4\geq 1 \\ B-4=y_2+y_3+y_4+y_5+y_7+y_9\geq 1 \\ B-5=y_1+y_2+y_4+y_5+y_6\geq 1 \\ B-6=y_5+y_6+y_{11}+y_{12}+y_{13}\geq 1 \\ B-9=y_4+y_7+y_9+y_{10}+y_{14}\geq 1 \\ B-10=y_9+y_{10}+y_{11}\geq 1 \\ B-11=y_6+y_{10}+y_{11}\geq 1 \\ B-13=y_6+y_{12}+y_{13}+y_{14}\geq 1 \\ B-14=y_9+y_{13}+y_{14}\geq 1 \end{cases} \tag{8}$$

D. Single Line Contingency with PMU placement

The minimum branch weight constrained optimization problem with single line outage can be formulated as follows:

$$Min\ \sum_{p=1}^{n} W_p y_p \tag{9}$$

Subjected to $y_8=2$,

$$y(p)=\begin{cases} bus-1=y_1+y_2+y_5\geq 2 \\ bus-3=y_2+y_3+y_4\geq 2 \\ bus-4=y_2+y_3+y_4+y_5+y_7+y_9\geq 2 \\ bus-5=y_1+y_2+y_4+y_5+y_6\geq 2 \\ bus-6=y_5+y_6+y_{11}+y_{12}+y_{13}\geq 2 \\ bus-9=y_4+y_7+y_9+y_{10}+y_{14}\geq 2 \\ bus-10=y_9+y_{10}+y_{11}\geq 2 \\ bus-11=y_6+y_{10}+y_{11}\geq 2 \\ bus-13=y_6+y_{12}+y_{13}+y_{14}\geq 2 \\ bus-14=y_9+y_{13}+y_{14}\geq 2 \end{cases} \tag{10}$$

Substituting $y_8=2$, in $y(x)\geq 2$ results in the subjected constraints (10) for the problem

where W_p is defined as cost coefficient of PMU installed at the bus p in the bus network.

E. Observability Performance

The Observability Index (BOI) in total observable system is measured as follows:

$$\beta_p\leq \Re_p+1 \tag{11}$$

The BOI is based on number of incident branches and one. The number of PMUs used to measure in this system is expressed as BOI. However, the complete observability index can derive from sum of indices at the bus in the system.

$$CSBOI=\sum_{p=1}^{N}\beta_p \tag{12}$$

F. Optimal PMU Placement utilizing BIP Formulation

The procedure for PMU placement is shown in Fig. 5 the basic model of optimal placement of PMUs is shown in Eqns. (1-2), branch weight constraint modeling is shown in Eqns. (3-4). Branch weight and ZI constraint modeling is shown in Eqns. (4-8). OPP considering single line outage constraints is shown in Eqns. (9-10). Performance of observability is checked considering CSBOI shown in Eqns. (11-12). The BIP is modeled with these constraints from the Eqns. (1-12) and programmed as shown in Fig. 5.

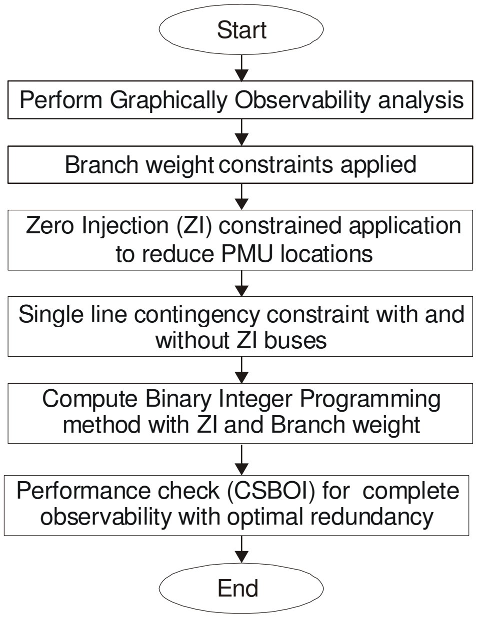

Fig. 5. Multiple-Constrained Modeling of BIP method.

With the application of BIP approach, to the objective functions (7-8) with subjected constraints gives the solution of optimal placement at 2, 8, 10 and 13 making the system completely observable.

IV. RESULTS AND ANALYSIS

The minimum branch weight constrained and ZI modeled PMU placement for various test case power systems are modelled with BIP technique and simulation performance with MATLAB Programming. All these test case systems such as 14, 24, and 30 bus and 57 bus systems are simulated with Intel(R) core (TM), i3 processor at 2.20 GHz, 4 GB of RAM on computer system. The data of minimum branch weight buses and ZI buses is shown in Table 1.

Table 1: Minimum Branch weight buses and ZI Buses.

Test case	Required branch weight buses	ZI- buses
14 bus	8	7
24 bus	7	11, 12, 17, 24
30 bus	11, 13, 26	6, 9, 22, 25, 27, 28
57 bus	15	4, 7, 11, 21, 22, 24, 26, 34, 36, 37, 39, 40, 45, 46, 48

As shown in Table 2 and 3 the minimum branch weight constrained with ZI modelling on the Comparison of tables, it can identify the location of PMU with ZI decreases the required PMUs, therefore, reducing the installation cost of PMU.

Table 2: Minimum Branch Weight constrained PMU Locations.

Test case	Required PMU's	Required branch weight buses
14 bus	5	2 ,7, 8, 11, 13
24 bus	9	1, 7, 9, 10, 11, 15, 17, 20, 21
30 bus	11	1, 7, 10, 11, 12, 13, 19, 24, 26, 27, 28
57	19	1, 4, 9, 10, 15, 20, 23, 27, 29, 30, 32, 36, 38, 39, 41, 46, 49, 53, 56

Table 3: Minimum Branch Weight constrained PMU Locations with ZI Modeling.

Test case	Required PMU's	Branch weight constrained PMU placement
14 bus	4	2, 8, 10, 13
24 bus	8	1, 2, 7, 12, 14, 15, 17, 19
30 bus	10	2, 4, 11, 13, 15,16, 19, 22, 26, 27
57 bus	15	1, 9, 10, 15, 20, 23, 27, 29, 30, 32, 38, 41, 49, 53, 56

Consider the disconnected line problem, it illustrates the usefulness of location of PMUs at minimum branch weight nodes and the lead of Zero injection model for this condition. Minimum branch weight constrained optimal placement for disconnected line problem with and without considering ZI modeling is shown in Table 4 and 5. From Table 4, it is detected that reduction of PMU locations with observability using ZI modeling.

Table 4: Disconnected line using ZI modeling.

Test case	Required PMU's	Branch weight constrained PMU placement
14 bus	8	2, 4, 5, 6, 8, 9, 10, 13
24 bus	14	1, 2, 3, 7, 8, 9, 10, 11, 15, 16, 17, 20, 21, 23
30 bus	17	2, 3, 4, 7, 10, 11, 12, 13, 15, 17, 19, 20, 22, 24, 26, 27, 29
57 bus	33	1, 3, 4, 6, 9, 12, 15, 19, 20, 22, 24, 26, 28, 29, 30, 31, 32, 33, 35, 36, 38, 39, 41, 43, 45, 46, 47, 50, 51, 53, 54, 55, 57

Table 5: Without ZI Modeling for Single Line Outage.

Test case	Required PMU's	Branch weight constrained PMU placement
14 bus	9	2, 4, 5, 6, 7, 8, 9, 11, 13
24 bus	10	1, 2, 7, 8, 9, 10, 18, 19, 20, 22
30 bus	21	1, 3, 5, 7, 8, 9, 10, 11, 12, 13, 15, 17, 19, 20, 22, 24, 25, 26, 27, 28, 29
57 bus	57	1, 2, 4, 6, 9, 12, 15, 19, 20, 22, 24, 25, 28, 29, 30, 32, 33, 35, 36, 38, 41, 45, 48, 49, 50, 51, 53, 54, 56

Table 6: Shows the CSBOI under regular and irregular conditions.

Test case	For Connected Line		For Disconnected line	
	ZI present	ZI not present	ZI present	ZI not present
14 bus	14	18	35	39
24 bus	27	38	38	59
30 bus	36	41	65	76
57 bus	57	67	117	127

As shown in Table 6, the minimum branch weight consider with optimal PMU placement can increases redundancy without Zero injection modeling. The complete observability with optimum redundancy obtains with ZI modeling Complete System Bus Observability Index (CSBOI).

For 14- bus system, The BOI at every bus of the system considering minimum branch weight buses is shown in Table 7.

Table 7: BOI considering with Branch weight.

Bus No.	BS1	BS2	BS3	BS4	BS5	BS6	BS7
BRI	1	1	1	1	1	1	1
Bus No.	**BS8**	**BS9**	**BS 10**	**BS 11**	**BS 12**	**BS 13**	**BS 14**
BRI	1	1	1	1	1	1	1

Table 8: BOI considering Branch weight and ZI constraints with Single Line Outage.

Bus No.	BS1	BS2	BS3	BS4	BS5	BS6	BS7
BRI	2	3	2	4	4	3	3
Bus No.	**BS8**	**BS9**	**BS10**	**BS11**	**BS12**	**BS13**	**BS14**
BRI	1	3	2	2	2	2	2

The Redundancy of the system is increased with branch weight constrained OPP. The BOI at every bus of the system considering branch weight buses under disconnected line is shown in Table 8.

Table 9: Comparison between existing and Proposed Methods for Complete Observability.

Methods	14-Bus	24-Bus	30-Bus	57 -Bus
Generalized ILP	4	—	10	17
MILP	4	—	10	17
MICA	4	—	10	—
FBLP	4	—	—	—
BPSO	4	—	10	—
TBLO	4		10	17
Proposed minimum branch weight constrained BIP Method	4	8	10	15

Table 9 illustrations the assessment of the planned minimum branch weight constrained optimization formed with complete observability with additional methods shown in literature survey that obtained optimal PMU locations with complete observability. By using ZI modeling to minimum branch weight constrained PMU placement, it can be able to the number of PMUs reduced both in normal and abnormal conditions.

V. CONCLUSION

This paper presented the Binary Integer programming method, formulated considering constraints of branch weight and zero injection buses for optimal PMU placement. Zero injection constraint modeling decreases the PMU locations in bus networks thereby minimizing the cost to be occurred for installation. Branch weight constraint modeling decreases the cost of installation of PMUs in network. Disconnected line condition with and without ZI modeling is considered for optimal PMU placement. Complete System Bus Observability Index (CSBOI) is proposed to evaluate the complete observability performance of the system. CSBOI with connected and disconnected line outage with and without ZI modeling for branch weight constrained PMU locations is evaluated. Branch weight constrained BIP approach attains complete observability with optimal PMU locations. Placement of PMUs can be carried out using different criteria depending on the objective of the investigator. In this paper, the main focus is to make the entire system observable by optimal placement of PMUs.

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How to cite this article: Lakshminaraya, P. and Venkatesan M. (2020). Line Weight based PMU Placement for Observability in Test Case Power Systems. *International Journal on Emerging Technologies*, *11*(1): 409–413.