MHD Oscillatory Flow of Non Newtonian Fluid through Porous Medium in the Presence of Radiation and Chemical Diffusion with Hall Effects

R. Sakthikala¹ and V. Lavanya²
¹Assistant Professor, Department of Mathematics, PSGR Krishnammal College for Women, Coimbatore (Tamilnadu), India. ²Research Scholar, Department of Mathematics, PSGR Krishnammal College for Women, Coimbatore (Tamilnadu), India.

(Corresponding author: R. Sakthikala)
(Received 14 March 2020, Revised 08 April 2020, Accepted 11 April 2020)
(Published by Research Trend, Website: www.researchtrend.net)

ABSTRACT: A study has been carried out to analyze MHD flow of fluid through vertical porous medium placed in a magnetic field with the effect of suction/injection on the unsteady second grade fluid flow through a vertical channel with non-uniform wall temperature. The governing equations under a flow parameters on velocity, temperature and concentration profiles, are solved using regular perturbation method and the results are represented graphically.

Keywords: Hall effects, oscillatory flow, porous medium, magnetic field, fluid slip, suction/injection.

I. INTRODUCTION

Keeping the above mentioned facts, in this paper, we considered the effect of suction/injection on the unsteady oscillatory second grade fluid flow through a vertical channel filled with saturated porous medium in the presence of hall current, radiation, and thermal molecular diffusion.

II. MATHEMATICAL FORMULATION

Consider the MHD flow of mass transfer through unsteady laminar flow of an incompressible viscous electrically conducting second grade fluid through vertical porous channel filled with porous medium having inclined magnetic field with slip at the cold plate. The fluid flow put through the suction at the cold wall and injection at the heated wall. Consider a Cartesian coordinate system (x, y, z) where x lies along the center of the channel, and z is the distance measured in the normal section such that z = d is the channel’s half width.

By the Boussinesq’s approximation the governing equations flow are as follows:

**Equations of continuity**

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \]  

**Momentum of equations**

\[ \frac{\partial u}{\partial t} - \nu \frac{\partial u}{\partial x} = - \frac{1}{\rho} \frac{\partial P}{\partial x} + \frac{1}{\rho} \frac{\partial \tau_{xx}}{\partial x} + \frac{1}{\rho} \frac{\partial \tau_{xy}}{\partial y} + \frac{1}{\rho} \frac{\partial \tau_{xz}}{\partial z} + g \beta T \frac{\partial q}{\partial x} + \frac{q}{\alpha_x} \]  

**Equations of energy**

\[ \frac{\partial T}{\partial t} - \nu \frac{\partial T}{\partial x} = \frac{1}{\rho c_p} \frac{\partial q}{\partial x} + \frac{1}{\rho c_p} \frac{\partial \tau_{xx}}{\partial x} + \frac{1}{\rho c_p} \frac{\partial \tau_{xy}}{\partial y} + \frac{1}{\rho c_p} \frac{\partial \tau_{xz}}{\partial z} + \frac{q}{\alpha_x} \]  

Further it is assumed that \( \alpha_x T \approx 0 \) (1) and \( \alpha_x \beta \ll 1 \), where \( \alpha_x \) and \( \beta \) are the cyclotron frequency and collision time for ions respectively. In the equation (6) the electron pressure gradient, the ion-slip and thermo-electric effects are neglected. Suppose that the electric field \( E = 0 \) under assumptions reduces to

\[ J = \frac{\omega_x B_0}{\eta} \]  

Further it is assumed that \( \omega_x e \tau \approx 0 \) (1) and \( \omega_x \beta \approx 1 \), where \( \omega_x \) and \( \beta \) are the cyclotron frequency and collision time for ions respectively. In the equation (6) the electron pressure gradient, the ion-slip and thermo-electric effects are neglected. Suppose that the electric field \( E = 0 \) under assumptions reduces to

\[ j_x + m j_y = \sigma B_0 v \]  

\[ j_x - m j_y = -e B_0 v \]  

By equating (7) and (8) we get,

\[ j_x = \frac{\sigma B_0}{\eta} (v + mu) \]  

Substitute equations (9) and (10) in (3) and (2), we obtain

\[ \frac{\partial u}{\partial t} - \nu \frac{\partial u}{\partial x} = - \frac{1}{\rho} \frac{\partial P}{\partial x} + \frac{1}{\rho} \frac{\partial \tau_{xx}}{\partial x} + \frac{1}{\rho} \frac{\partial \tau_{xy}}{\partial y} + \frac{1}{\rho} \frac{\partial \tau_{xz}}{\partial z} + g \beta T \frac{\partial q}{\partial x} + \frac{q}{\alpha_x} \]  

\[ \frac{\partial T}{\partial t} - \nu \frac{\partial T}{\partial x} = \frac{1}{\rho c_p} \frac{\partial q}{\partial x} + \frac{1}{\rho c_p} \frac{\partial \tau_{xx}}{\partial x} + \frac{1}{\rho c_p} \frac{\partial \tau_{xy}}{\partial y} + \frac{1}{\rho c_p} \frac{\partial \tau_{xz}}{\partial z} + \frac{q}{\alpha_x} \]  

On combining Eqs. (11) and (12), in terms of \( q = u + iv \) and \( \xi = x + iy \), we obtain

\[ \frac{\partial q}{\partial t} - \nu \frac{\partial q}{\partial x} = - \frac{1}{\rho} \frac{\partial P}{\partial x} + \frac{1}{\rho} \frac{\partial \tau_{xx}}{\partial x} + \frac{1}{\rho} \frac{\partial \tau_{xy}}{\partial y} + \alpha \frac{\partial ^2 q}{\partial x^2} + \beta \frac{\partial ^2 q}{\partial y^2} + \frac{q}{\alpha_x} \]  

The boundary conditions are

\[ q = \frac{v}{\alpha_x}, \quad T = T_0, \quad C = C_0 \text{ at } z = 0 \]  

(14)
\[
q_0 = -\frac{e^{m_5y}}{e^{m_6y} - e^{m_5y}} - \left(\frac{e^{m_6y} - e^{m_5y}}{1 - b_1(e^{m_4y} - 1)} + b_2(e^{m_3y} - 1) + b_3(e^{m_3y} - 1) + b_4(e^{m_2y} - 1) + b_5(e^{m_1y} - 1) + b_6(e^{m_1y} - b_6e^{m_2y} + b_6e^{m_2y} - b_6e^{m_4y} - e^{m_6y}) - b_1e^{m_2y} + b_2e^{m_3y} - b_3e^{m_3y} - b_4e^{m_2y} - b_5e^{m_1y} - b_6e^{m_2y} + b_7e^{m_1y} - b_8e^{m_2y} + b_9e^{m_1y} - b_{10}e^{m_2y} - b_{11}e^{m_3y} + b_{12}e^{m_3y} - b_{13}e^{m_2y} - b_{14}e^{m_2y} - b_{15}e^{m_3y}
\right)
\]

\[
\theta_0 = \frac{e^{m_4y} - e^{m_3y}}{e^{m_4y} - e^{m_3y}}
\]

\[
\varphi_0 = \left(h_1(1 - A_3h_2 + A_4h_3 + A_5h_4 - A_6h_5)\right)
\]

**IV. RESULTS AND DISCUSSION**

The computational data have been presented in graphical form were evaluated analytically in the following Fig.s (1) – (4) represents the velocity profiles for u & v, Fig. (5) represents the temperature profiles for \( \theta \). The governing flow by dimensionless parameters M Hartmann’s number, K permeability parameter, m hall parameter, \( \alpha \) viscoelastic parameter, Gr thermal Grashof number, Gm mass Grash of number, Sc Schmidt number, \( \omega \) the frequency of oscillation, Kc chemical reaction parameter, \( \gamma \) slip velocity are discussed. In Fig. 1 (a & b), the velocity u and v, increases with increasing hall parameter due to the effect of inclined magnetic field in terms of hall parameter which have no significance effect in velocity profile. Fig. (a & b), represents the velocity u and v, decreasing by Hartmann number or due to the increasing magnetic field then the resultant velocity also reduces with increase in the intensity of the magnetic field. The effect of inclined magnetic field on an electrically conducting fluid (Lorentz force) similar to drag force will increase the drag force and slow down the motion of the fluid. Fig. 3 (a & b), the velocity u and v, reduces with the v increasing by radiation parameter N because of the left half of the channel, the effect of N on the velocity is insignificant while in the right half of the channel velocity decreases with increase of N. Fig. 4 (a & b), the velocity u and v, increasing the permeability parameter K, such that increasing the permeability higher the fluid speed, in the absence of \( \gamma \), there is no flow and separation does not take place at boundary. In Fig. 5(a, b, c, d), the temperature magnitude is decreasing with suction parameter, frequency of oscillation \( \delta \). By the fluid temperature increases with increasing the channel, injection increases the plate heated and suction increases the plate while cooled. Thus, the temperature magnitude is increasing with the Prandtl parameter Pr and suction parameter s. In Fig. 6 (a, b, c, d), the concentration magnitude is decreasing with suction parameter, chemical reaction parameter, frequency of oscillation, therefore it increases with Schmidt number Sc. When Schmidt number increases the diffusivity will be small, it decreases the diffusivity which will be high.
Fig. 2 (b) The velocity $v$ profiles against $M$ with $m=0.5;=1; K=1; Gr=3; Gm=4; s=1; pr=0.71; Sc=0.22; Kc=1; r=0.25; w=\pi/6; d=0.5; So=1; i=1.0; N=3.0$.

Fig. 3 (a) The velocity $u$ profiles against $a$ with $M=0.5; \alpha=1; K=1; Gr=3; Gm=4; s=1; Pr=0.71; Sc=0.22; Kc=1; r=0.25; w=\pi/6; d=0.5; So=1; i=1.0; N=3.0$.

Fig. 3 (b) The velocity $v$ profiles against $a$ with $M=0.5; \alpha=1; K=1; Gr=3; Gm=4; s=1; Pr=0.71; Sc=0.22; Kc=1; r=0.25; w=\pi/6; d=0.5; So=1; i=1.0; N=3.0$.

Fig. 4 (a) The velocity $u$ profiles against $k$ with $M=0.5; \alpha=1; K=1; Gr=3; Gm=4; s=1; Pr=0.71; Sc=0.22; Kc=1; r=0.25; w=\pi/6; d=0.5; So=1; i=1.0; N=3.0$.

Fig. 4 (b) The velocity $v$ profiles against $k$ with $M=0.5; \alpha=1; K=1; Gr=3; Gm=4; s=1; Pr=0.71; Sc=0.22; Kc=1; r=0.25; w=\pi/6; d=0.5; So=1; i=1.0; N=3.0$.

Fig. 5 (a) The temperature profiles against $\delta$ with $s=3; \delta=2; \omega=\pi/6; Pr=0.71; i=1.0; t=0.2; N=1.0$.
Fig. 5(b) The Temperature profiles against $s=3, \delta=2; \omega=\pi/6; \Pr=0.71; t=0.2; N=1.0$.

Fig. 5(c) The Temperature profiles against $\omega$ $s=3, \delta=2; \omega=\pi/6; \Pr=0.71; t=0.2; N=1.0$.

Fig. 5(d) The Temperature profiles against $\Pr$ $s=3, \delta=2; \omega=\pi/6; \Pr=0.71; t=0.2; N=1.0$.

Fig. 6(a) The concentration profiles against $s=3, \delta=2; \omega=\pi/6; t=0.2; N=1.0; Sc=0.22; Kc=2; So=2; Pr=0.71$.

Fig. 6(b). The concentration profiles against $Sc$ $s=3, \delta=2; \omega=\pi/6; t=0.2; N=1.0; Sc=0.22; Kc=2; So=2; Pr=0.71$.

Fig. 6(c). The concentration profiles against $Kc$ $s=3, \delta=2; \omega=\pi/6; t=0.2; N=1.0; Sc=0.22; Kc=2; So=2; Pr=0.71$. 
velocity slip and temperature jump. It can also further be extended in terms of introducing time.

...— The concentration decrease with increasing parameters M for heated plate.
— The temperature decreases with decreasing parameters Pr Prandtl number, when it increases with time.
— The concentration decrease with increasing parameters Sc Schmidt number, when it increases with time.

It can also further be extended in terms of introducing Soret and Dufour effects, slip conditions.

V. CONCLUSION

We have concluded that MHD oscillatory flow of non-Newtonian fluid in the presence of radiation parameter and thermal diffusivity. The dimensionless governing equations are solved by analytical method due to,
— The velocity increases with increasing parameters m, M.
— The velocity increases with increasing parameters M for heated plate.
— The velocity increases with decreasing parameters α for heated plate.
— The velocity decreases with decreasing parameters M for heated plate.
— The temperature decrease with increasing parameters Pr Prandtl number, when it increases with time.
— The concentration decrease with increasing parameters Sc Schmidt number, when it increases with time.

REFERENCES

[22]. Sivaraj, A. & Jasmine Benazir (2015). Unsteady magneto hydrodynamic mixed convective oscillatory flow...