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On the Efficient Shape Retrieval of Symmetric Curves using Fourier Descriptors

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ABSTRACT: Summarizing complex boundary shapes can be challenging. Approximation tools that already exist may require tweaking and refinement or novel approximation techniques need to be designed and tested for robustness. The problems arising in contexts like biological, satellite and medical imagery pose several challenging problems where object recognition becomes the focus of study. One of the means of recognizing objects is to describe their shapes. Boundary description is one of the means of summarizing shapes. Adding texture and color analysis to boundary is expected to take us closer to object recognition and understanding. Boundary of a shape can be modelled by spatial methods or through spectral approaches. Bilateral and radial symmetries are the popular areas of study. Since symmetry holds high level structural information, a study of symmetries can aid high level processing goals like segmentation and template matching to mention a few. The effectiveness of a near exact or exact reproduction of shape can be used for studying a variety of biological situations including leaf shape analysis and study of human parts. We focus on some shapes having bilateral symmetry, synthetic in nature, and try to retrieve a fair approximation using Fourier descriptors. We hope to extend this approach to some real-world problems where symmetries are observed. Three well known shape signatures viz. the centroid distance, area and cumulative angular functions are considered in this study. The performance of the cumulative angular function has been observed to be superior. This was tested on some shapes having bilateral symmetry and test results justify our findings.

Keywords: Boundary representation, Fourier descriptors, Object recognition, shape signatures, Shape, symmetries

Abbreviations: FD, Fourier descriptor, BS, bilateral symmetry, SS, Shape signature, CDF, Centroid distance function, AF, Area function, CAF, Cumulative angular function, DFT, Discrete Fourier transform.

I. INTRODUCTION

Humans are bestowed with an instinct to recognize objects and patterns that are associated in a scene even when the context is not very clear. Processing, analysis and understanding a scene almost happens seamlessly. It is however quite possible that human recognition fails particularly if what is to be expected in an image is unknown. One area where human vision may fail is in the study of tumors where one may not have the knowledge of human anatomy or microbiology. While the cognitive power of the human is very difficult to replicate through the machine, a certain class of problems in pattern recognition and scene analysis have been successfully tried even when the object of interest has many features. This specialized area of study has given scope to researchers to design various domain specific algorithms for complex problems. In the realm of computer vision, describing shape has opened new vistas of research. It is obvious that one cannot ideally define what *shape* means and what one must look for in shape. In the external point of view, shape could be described through object skeleton or region boundary or some topological features, while from the interior, it could be described through color and texture.

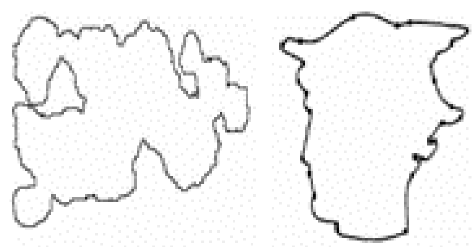

Fig. 1. Complex planar closed curve.

Object recognition is a problem that arises in many fields of study like military target recognition, biomedical imaging, satellite imagery and cartography, to mention a few. Recognition of objects by analyzing object shapes has been an active area of research. There are many applications where image analysis can be reduced to the analysis of shapes. To describe shape through object boundary is a preliminary but critical tepin the overall description of shape. Many real-world objects are three dimensional in nature but their projections onto two dimensional planes still make them recognizable. An aerial photograph of an aircraft parked on an airstrip yields a shape which is easily recognizable.

In simpler cases, the object boundaries are two dimensional closed curves, some which can be handled using simpler mathematical techniques, while other requiring advanced methods. The goal of object recognition can be achieved to a great extent by describing object shape which comprises extracting the boundary. Nature has thrown before us a variety of challenges. There exist objects which have symmetry, though no many of them. Vagaries of nature reduce originally symmetric objects into deformed shapes over a period in time. Describing shapes shown below pose several challenges. No easy methods can be conceived to summarize such shapes. Lack of axes of symmetry perhaps adds to the complexity. The study is motivated through the situations arising in the medical field where analysis of shape is crucial in decision making. New mathematical models and machine learning techniques go hand in hand in solving some of the impregnable problems that hitherto were not attempted. Mathematical research focusses on finding techniques that find analytical expression to describe the boundary.

It is not within the scope of approximation theory to design an analytical function to such shapes. Attempting a global fit is mired with disastrous consequences. Using piecewise approximation methods like cubic splines can be a choice, but they come with a cost. The author has been engaged in working with complex shapes, both open and closed, in the spatial domain, using a novel parametrization scheme. An alternative could be the use of spectral methods. Fourier descriptors [FD] have been successfully employed in the approximation of complex shapes. This study is on summarizing symmetric two-dimensional shapes. We study different shape signatures [SS]- Centroid distance function [CDF], Area function [AF] and Cumulative angular function [CAF], which form the basis for using Fourier descriptors, and report on their relative performances on boundary approximation.

The concept of Fourier descriptors dealt with the study of shapes and effective retrieval [1, 2]. Tomakova *et al.*, (2017) have applied Fourier descriptors in the classification of blood smears [3]. The shape analysis deals with using Fourier descriptors for 2-D closed curves which also included symmetric shapes [4, 5]. Cosgriff (1960) is one of the earliest papers that has contributed to the line of research on Fourier descriptors [6]. Fourier descriptors for character recognition [7, 8]. Wang *et al.*, (2003) have applied Fourier descriptors for leaf classification problems [9]. The mathematical aspects of approximations using Fourier descriptors are treated in classic texts [10, 11, 12, 13]. Some researchers contributed to the study of leaf classification using Fourier descriptors and other techniques [19-24]. Bertamini *et al.,* deal with natural way to estimate the axis or plane of symmetry using the complete information in the data and extract components of symmetric shape variation [25]. This was studied a dataset from the lower pharyngeal jaw of the Neotropical cichlid fish [26] examined the preference for symmetry using a deliberately heterogeneous set of images; specifically, faces and abstract shapes The aim was to compare the role of perfect bilateral symmetry in the context of multiple categories, and to relate preference to rating of the symmetry salience of the items. Kayaert and Wagemans, discuss about the benefits of delayed shape matching from the perspective of simplicity and symmetry of structures [26].

Many researchers did study aspects like symmetry for a variety of situations but were short of recommending a best shape signature that captures the shape in an efficient manner. It is this gap in research we would like to fill.

II. METHODOLOGY

The proposed approach tries to approximate closed symmetric shapes using spectral methods. We have chosen to use Fourier descriptors to approximate the target shapes by using different shape descriptors and select the best of them after trying them on some test cases.

A. Boundary description

One can approximate an object boundary using spatial and spectral methods. Spatial methods can be a good choice when the boundary points are largely free from noise. Synthetic curves may have such a feature. Spatial descriptors are based on region boundary measures such as boundary length, curvature, bending energy and chord distribution, to mention a few. Real world problems do not lead to object boundary that is free from noise. Digital images are stored in discrete data structures and are highly sensitive to scale change. Most spatial boundary approximation techniques fail to work under noisy conditions. Pre-processing for noise removal becomes an essential step before encoding the boundary in the spectral domain. They can be designed to be independent of scaling, translation, or rotation because shape is that aspect of the object that is invariant of the above.

B. Shape signatures

A shape signature function is a one-dimensional function defined on the pixel information of the region boundary. Among the several shape-signature functions that have been used, the Complex shape function, Centroid distance function, Area function, Curvature function, Cumulative angle function are some. Using these signatures, which form the basis for defining Fourier descriptors, the boundary of an image is approximated without a noticeable loss of information. We propose to test the relative performance of three shape signatures- the Centroid distance function, Area function, Curvature function and Cumulative angle function in this study.

(i) **Centroid distance function:** In this signature, centroid of a boundary is computed and distance from the centroid to each boundary point is calculated and stored in a vector. The obtained distance vector is a resultant of this signature's implementation.

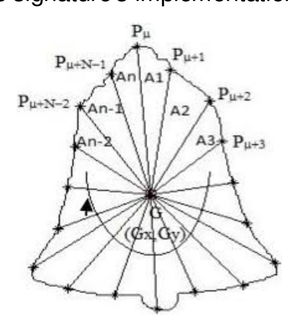

Fig. 2. Centroid distance function.

The centroid is (g_u, g_v) where

$$g_u = \frac{1}{6A}\sum(x_i + x_{i+1})(x_i y_{i+1} - y_i x_{i+1}) \quad (1)$$

$$g_v = \frac{1}{6A}\sum(y_i + y_{i+1})(x_i y_{i+1} - y_i x_{i+1}) \quad (2)$$

In which

$$A = \frac{1}{2}\sum(x_i y_{i+1} - y_i x_{i+1}) \quad (3)$$

determines the area.

If $X_1, X_2, X_3, \ldots, X_n$ be triangles partitioning the polygon of interest having (g_u, g_v) as centroid, the centroid distance function is defined as

$$r(t) = (x(t) - g_u)^2 + (y(t) - g_v)^2 \quad (4)$$

This is one of the shape signatures under consideration in this study.

(ii) **Area function:** Imagine a plane lamina having n corner points whose centroid is $G \equiv (g_u, g_v)$. Assume that the points are parametrically expressed as $(x(k), y(k))$ where k is a shape parameter like the arc length.

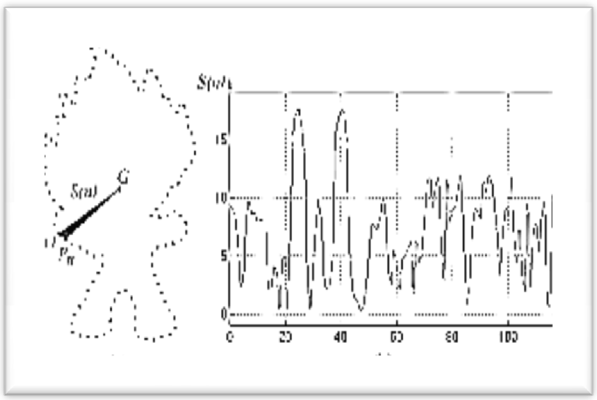

Fig. 3. The idea of triangle area function.

One can know compute the elementary triangle areas between consecutive corner points and the centroid by the rule

$$\Delta_k \ \frac{1}{2}\begin{vmatrix} x_k & x_{k+1} & 1 \\ y_k & y_{k+1} & 1 \\ g_u & g_v & 1 \end{vmatrix} \quad (5)$$

(iii) **Cumulative angular function:** Tangent angles indicate the change of angular directions of the boundary shape. Human perception of sudden change in angle direction is quite natural and hence this shape signature is one that can naturally be adopted. We define the tangent angle at any point on the boundary by the rule

$$\theta(k) \equiv arctan\left(\frac{y(k)-y(k-\tau)}{x(k)-x(k-\tau)}\right) mod(2\pi) \quad (6)$$

To compensate for any possible discontinuities that arise as the boundary is encountered. This basic rule can be digitally implemented by customizing it as

$$\emptyset(k) \equiv arctan\big(\theta(k) - \theta(0)\big) mod(2\pi) \quad (7)$$

$\emptyset(k)$is the net amount of angular bend between the starting point *z*(0) and the current point *z(k)* on the shape boundary. This signature was further improved by [5].

The tangent angles for every adjacent boundary point pairs is computed and stored as a one dimensional vector. The obtained vector of tangent angles is the resultant of cumulative angular function.

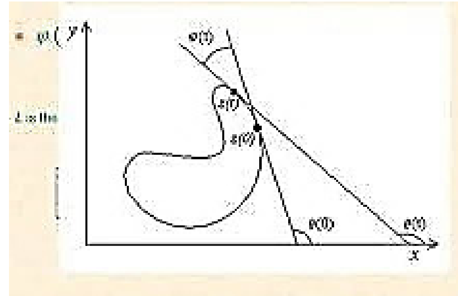

Fig. 4. Cumulative angle function.

C. Fourier descriptors

One can determine FD by applying the DFT on the shape signature function. The DFT of this function generates Fourier coefficients a_u determined by

$$a_u = \sum_{k=0}^{K-1} z(k).\exp\left(-i\, 2\pi k u/K\right) \quad (8)$$

where $u = 0, 1, 2, \ldots, (K-1)$

The complete set of coefficients a_k constitute the Fourier descriptors of the boundary. The inverse discrete Fourier transform (IDFT) of these coefficients restores $z(k)$.

$$z(k) = \frac{1}{K}\sum_{u=0}^{K-1} a_u.\exp(2\pi\mathrm{i}uk/\mathrm{K}) \quad (9)$$

This result is an analysis technique credited to [6, 8]. Fourier descriptors are used as a symbolic representation of shape for subsequent representation. With the aid of a few descriptors one can visualize the gross shape and by considering more descriptors, finer details of the shape reveal themselves. The number of descriptors to be considered will depend on the amount of detail one is looking for.

For applications in medical imaging, one may need to compute Fourier descriptors of high order to capture fine details that is so essential for diagnosis. It is therefore necessary not to invest too much on computing lower order descriptors. We make a comparative study on the number of Fourier descriptors needed for a fair recognition of the original shape by confining to three shape signatures- the centroid distance signature, the area function signature and the cumulative angle signature on a data base of symmetric shapes.

III. RESULTS AND DISCUSSION

The tremendous scope to research on shape analysis in a variety of scientific fields provided the necessary motivation to scan in to the literature to identify application areas. The contribution of [19, 20, 21, 22, 23] is significant in the context of leaf classification where both spatial and spectral methods were used. Klingenberg *et al.*, (2002) offer a natural way to estimate the axis or plane of symmetry using the complete information in the data and extract components of symmetric shape variation for the many studies in which asymmetry is not the main interest. They demonstrate their approach with a dataset from the lower pharyngeal jaw of the Neotropical cichlid fish to good effect [24]. Bertamini *et al.,* (2019) used a simple rating task with images that were manipulated to introduce perfect bilateral symmetry around the vertical axis [25]. Kayaert and Wagemans (2009) evaluated the influence of complexity and symmetry on shape recognition, by measuring the recognition of unfamiliar shapescreated using Fourier boundary descriptors through a delayed matching task [26].

Taking a cue from the above, we decided upon a task- to suggest a reliable shape signature that suits shapes having bilateral symmetry. The relative performance of each of the shape descriptors, viz., CDF, AF and CAF un that order, are first presented with respect to two shapes, one a non-symmetric shape (a crane), and another, a symmetric shape (a butterfly). The purpose is to arrive at a decision as to which is a better choice as a shape signature. We then proceed to study these shape signatures on a database of nine symmetric shapes, and their rotations in steps of 90^0. The one-dimensional vector is converted into frequency domain by applying the fast Fourier transform algorithm, which results in a complex vector. Inverse Fourier transform is applied on this complex vector and the values of the vector which are above the threshold value are chosen to reconstruct the boundary.

We set the threshold : $\frac{1}{2}(Max(\text{inverse FFT vector}) + \min(\text{inverse FFT vector})$ (10)

We consider all points which are less than or equal to the threshold in the retrieval process. This reconstruction does not affect the originality of the shape. It only eliminates the corner points which are invisible to human eye

A. Study on a non-symmetric shape

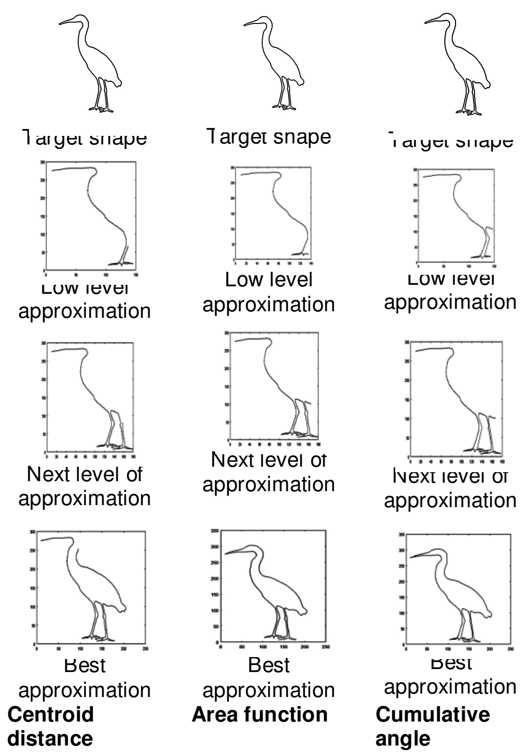

Fig. 5. Approximation of crane boundary (1577 boundary points).

Table 1: Summary of results of the shape in Fig. 5.

Shape descriptor	Centroid distance	Area	Cumulative angle
FDs for best approximation	1402	1250	821

Of the three shape signatures, the cumulative angular function clearly out-performs the centroid distance and area shape descriptors.

B. Study on a symmetric shape

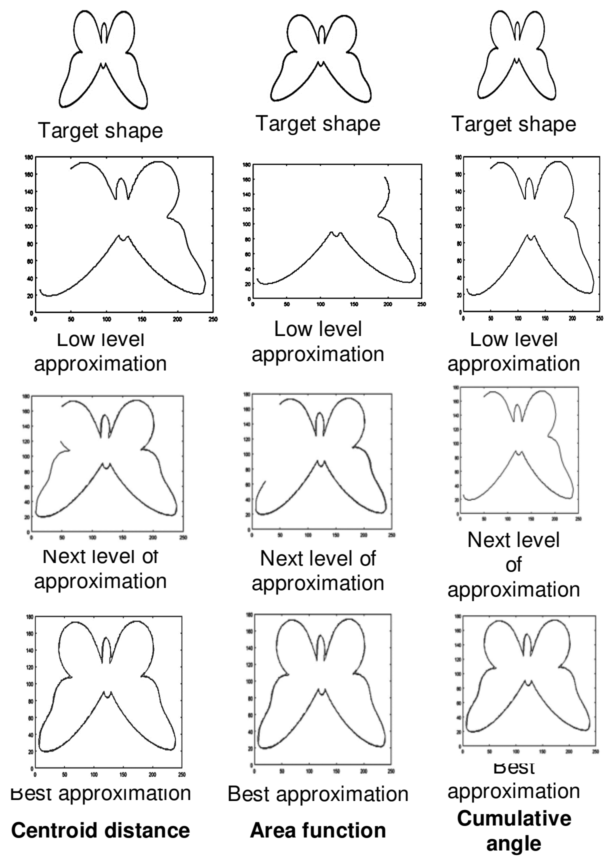

Fig. 6. Approximation of the butterfly boundary (2300 boundary points).

Table 2: Summary of results of the shape in Fig. 6.

Shape descriptor	Centroid distance	Area	Cumulative angle
Fourier descriptors for best approximation	2202	2191	1198

As observed in the case of the crane boundary, the cumulative angular function is observed as the best signature for approximation as it requires less information about the boundary. To conclusively arrive at the decision that the cumulative angle shape signature is reliable, it is proposed to study the relative performances of the three shape signatures on a database of nine symmetric shapes (a) in their normal positions and (b) their rotations through 90°, 180° and 270°. The performance of each shape descriptor is analyzed. The results is presented in the following figures/tables. The order of the shapes in various rotations is preserved.

Fig. 7 is a database of nine target shapes used to further this study. The corresponding approximations are presented in Fig. 8. The number of Fourier descriptors consumed for a fair approximation of each target is displayed in Table 3 and shape statistics are graphically represented in Fig. 9, for a quick grasp of the performance of each signature.

C. A study on nine symmetric shapes in the normal position and their different orientations

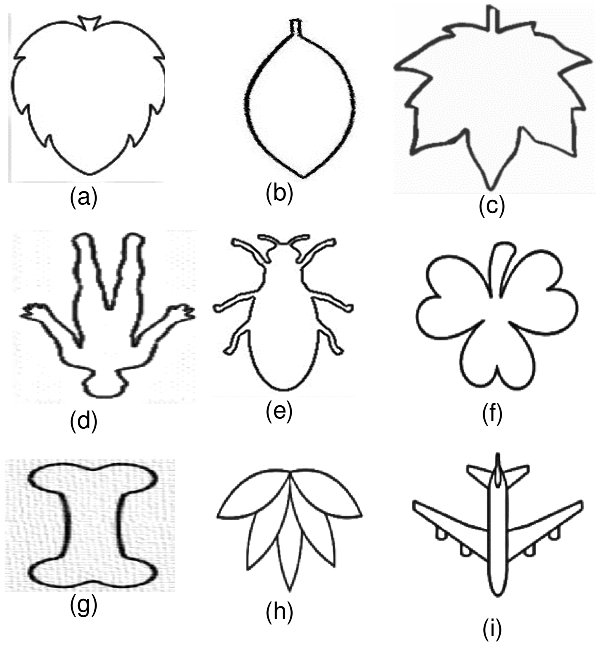

Fig. 7. Database of nine Symmetrical shapes.

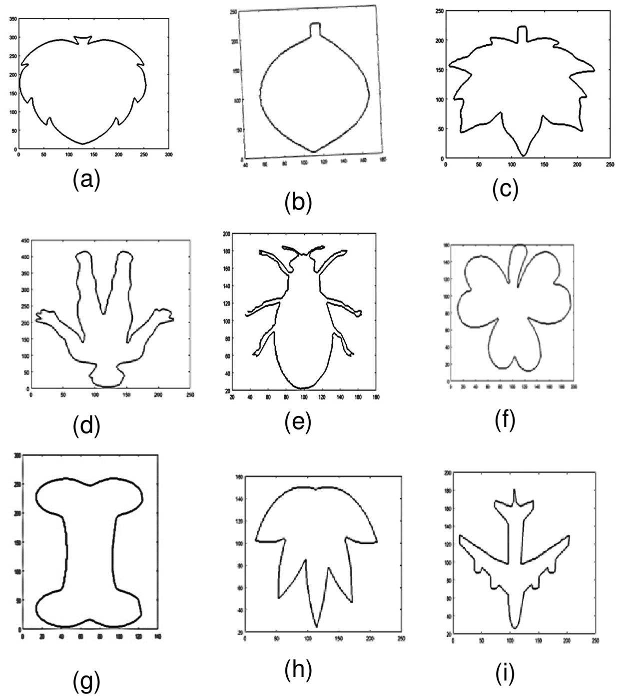

Fig. 8. Approximation of the shapes in the database using the cumulative angle shape signature.

Table 3: Shape statistics for the nine shapes in Fig. 8.

Figure	Number of corner points	No. of Fourier descriptors used for fair representation		
		Centroid distance function	Area function	Cumulative angular function
8(a)	1375	1198	908	716
8(b)	659	645	595	314
8(c)	1199	981	900	717
8(d)	2246	2051	2161	1127
8(e)	1261	1210	1190	531
8(f)	1031	987	837	527
8(g)	907	830	796	501
8(h)	845	802	680	433
8(i)	997	990	902	510

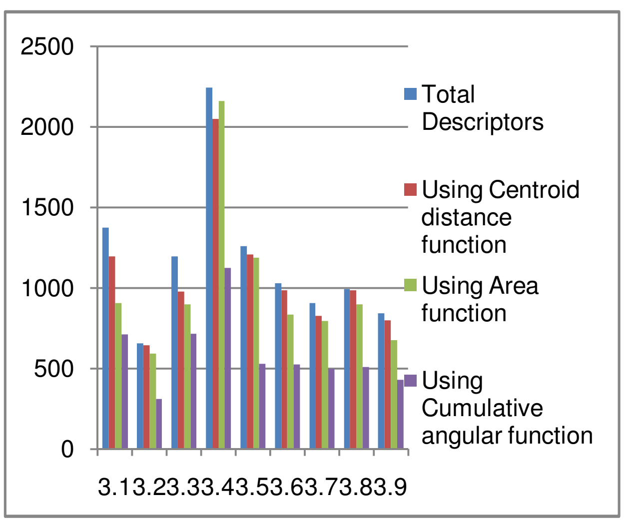

Fig. 9. Summary statistics showing the approximated and actual descriptors for all the figures in the database using the three signatures.

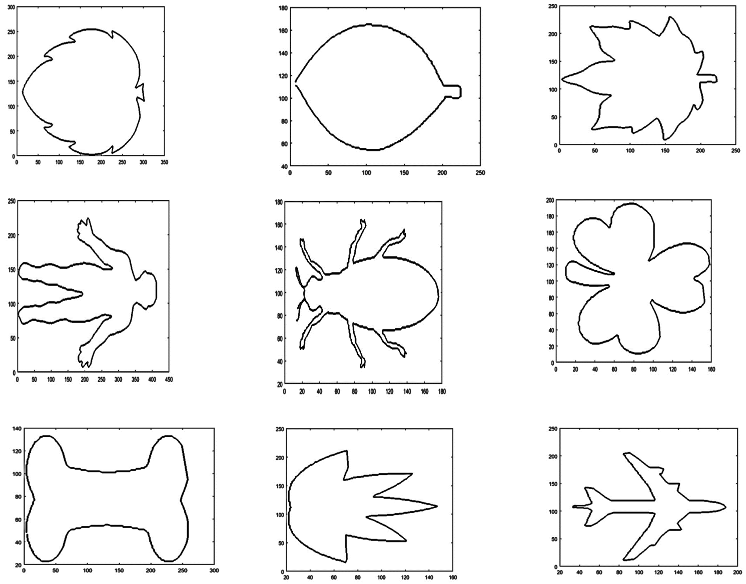

Fig. 10. Approximations obtained by a 90^0 rotation of the target shapes in database [Fig. 7] using the cumulative angle shape signature

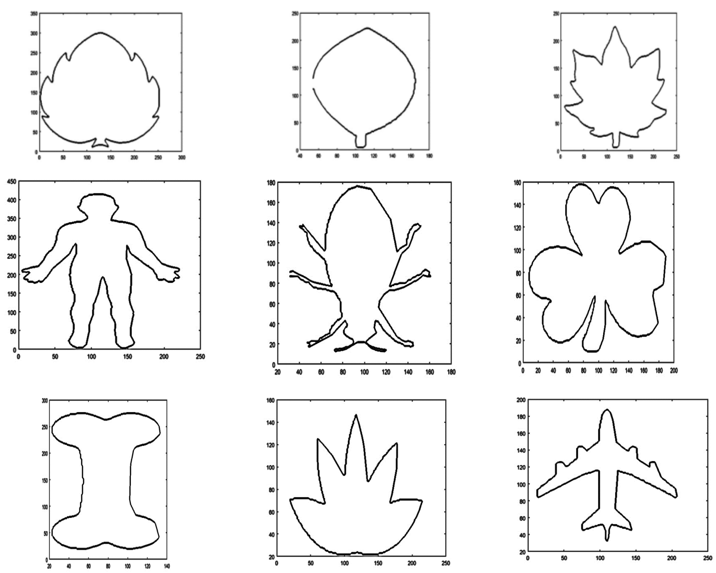

Fig. 11. Approximations obtained by a 180° rotation of the target shapes in database [Fig. 7] using the cumulative angle shape signature.

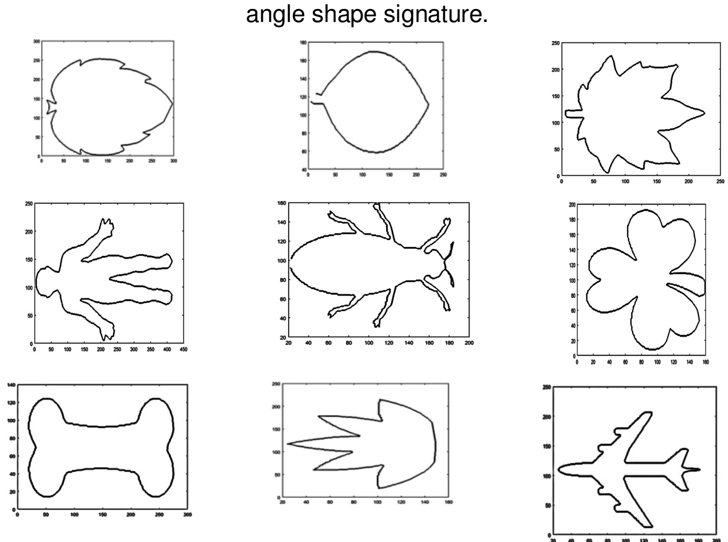

Fig. 12. Approximations obtained by a 270^0 rotation of the target shapes in database [Fig-7] using the cumulative angle shape signature

Table 4: Summary statistics on the performance of cumulative angle shape signature for the four orientations of shapes in the database [Fig. 7].

Fig.	Boundary points	FD for shape in normal position	FD for 90^0 rotation	FD for 180^0 rotation	FD for 270^0 rotation
8(a)	1375	716	735	805	475
8(b)	659	314	406	383	293
8(c)	1199	717	811	599	720
8(d)	2246	1127	889	1152	1134
8(e)	1261	531	702	480	774
8(f)	1031	527	519	391	536
8(g)	907	501	461	430	465
8(h)	845	433	449	451	473
8(i)	997	510	399	446	548

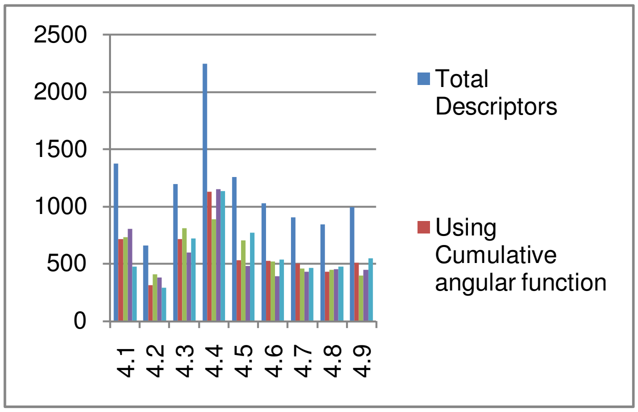

Fig. 13. A graphic showing the performance of cumulative angle shape descriptor for the nine target shapes, their approximations in the normal position and in various rotations.

IV. CONCLUSIONS

A study on approximating closed shapes having symmetries is attempted in the spectral domain using shape signatures. Among the several signatures available in the literature, the centroid distance function, the area function and cumulative angle function were considered. The performance of all three was tested on some synthetic shapes. The study reveals that the cumulative angle functions out-performs the other two by a big margin, consuming significantly fewer Fourier descriptors. An attempt has been made to see if this signature is robust to rotations. By and large, this signature can be used for summarizing closed symmetric shapes. This is the main contribution of the study.

V. FUTURE SCOPE

Future studies are planned to apply spectral methods involving complex shapes that arise in the realm of the medical field, particularly related to tumor lesions. A performance evaluation both spatial and spectral methods to approximate closed boundaries that have many features will be tried. It is also proposed to give an analytic expression to the boundary using novel parametrization techniques.

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Conflict of Interest. No conflict of interest arises as this work was originally conceived by the authors.

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