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Solving Fuzzy Fractional Klein-Gordon-Fock Equation by the VIM, ADM and NIM in Fluid Mechanics

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ABSTRACT: In this paper we are extending one dimensional fractional partial differential Klein–Gordon–Fock equation to Trapezoidal fuzzy fractional partial differential equation under Riemann-Liouville and caputo fractional derivatives, namely Variational iteration method, Adomain Decomposition method, and New iterative method and this method has applied to fuzzy fractional Klein-Gordon equation with initial conditions as in fuzzy.

Keywords: Fuzzy Fractional equation, Adomain Decomposition Method, Variational Iteration Method, New iterative Method.

Abbreviations: ADM, adomain decomposition method; VIM, variational iterational method; NIM, new iterative method; IVP, initial value problem; FFDEs, fuzzy fractional differential equations.

I. INTRODUCTION

In now a day, there have been many implementations in obtaining exact solutions in the subject of fuzzy fractional partial differential equation. Amiri define the fuzzy generalized Pantograph Equation under Hukuhara differentiability [1]. The concept of fuzzy derivative was initially defined by Chang and Zadeh [2]. The concept of fuzzy matrix is first introduced by Thomason (1977) [3]. VIM for obtaining the analytical solution of nonlinear fuzzy IVP relating to fuzzy Duffing's equation without changing of first order system [4]. Applications of VIM, and finding the exact solution of fractional order by using fuzzy IVP is compared [5]. Jafari and Tazadodi had explain Huan VIM for fractional Riccati differential equation with following nonlinear equations [6],

$$ {}^{C}_{.}D^{\alpha}_{0^+}u(t) = A(t) + B(t)u + C(t)u + D(t)u^2, $$
$$ t > 0, m-1, < \alpha \le m, \quad (1) $$

with fuzzy initial condition,

$$ u^j(0) = \tilde{C}_j, j = 1, \cdots, m-1, m \in \mathbb{N}, $$

where A(t), B(t) and C(t) are given functions, $\tilde{C}_j, j = 1, \cdots, m-1, m \in \mathbb{N}$, are arbitrary fuzzy number and α is an order of the fractional derivative [6]. N-th order fuzzy differential equation for VIM is done by Abbasbandy *et al*., [7]. Using Laplace transforms method with fuzzy fractional differential equations; Kumar and Kumar (2017) had found the exact solutions [8]. Fuzzy fractional differential equations (FFDEs) under Riemann Liouville H-differentiability by fuzzy Laplace transforms are done Salahshour *et al*., [9]. Introducing of the particle and antiparticle wave functions to represent the Klein-Gordon wave function and its time derivative and separating into two coupled time dependent Schrodinger equation is done in [10]. A comparison of the ADM, VIM, and NIM in one dimension equations is done by Ghadle and Khan [11]. Time-fractional diffusion equation, time-fractional telegraphic equation and the time fraction wave equation in three dimensions with three systematic schemes, namely the ADM, VIM and the NIM is done [12].

The Klein Gordon Fock equation was named after the physicist Oskar Klein, Walter Gordon, and Vladimir Fock, proposed in 1926. In this year the equation appeared in many papers to describe relativistic massive particles without spin [14]. Klein Gordon Fock equation is used in Euler equation for the motion of the probability fluid of a particle and antiparticle is provide insight the dynamic. Wong (2010) [10] specialize the simplified case and the motion of the probability fluids of particle and antiparticle which obey relativistic fluid dynamics equations, and the quantum stress tensor. The Klein Gordon Fock equation is put in the form of Schrodinger equation and it is express in two coupled differential equations for first order in time [15]. The equation is also useful in describing some vibrating system in classical mechanics [16] has relate the classical and quantum setting to this equation to explain the concept of mass. Fuzzy Volterra-Fredholm integral equation of second kind is solved by [18] with Adomian Decomposition method, VIM and Homotopy analysis method. Volterra-Fredholm integral equations are solved by Modified Adomian Decomposition method with uniqueness and existence [19].

In second section, we have explain VIM by using examples like, linear inhomogeneous time fractional wave equation in one dimensions with suitable fuzzy initial conditions and analysis of the VIM are used from [13].

II. PRELIMINARIES

Definition. A fuzzy set A is called trapezoidal fuzzy number with tolerance interval [a, b] left width α and right width β if its membership function has the following form

$$\begin{cases} 1-\dfrac{a-t}{\alpha}, \text{if } a-\alpha \le t \le a \\ 1 \quad , \qquad \text{if } a \le t \le b \\ 1-\dfrac{t-b}{\beta}, \text{if } b \le t \le b+\beta \\ 0 \quad , \qquad \text{otherwise} \end{cases}$$

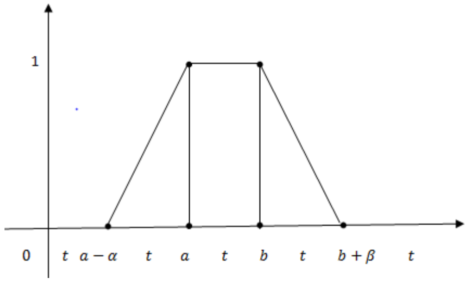

Fig. 1. Trapezoidal fuzzy number.

III. NUMERICAL RESULT

In this section, we present the illustrative examples, by using definitions i.e. fuzzy and Triangular fuzzy number, and key lemma, essential for the build the solutions, the properties of Modified Fuzzy Riemann-Liouville derivative are explained [13] and using (1).

Example 1. Consider the following one dimensional linear inhomogeneous fuzzy fractional Klein Gordon Fock equation.

$${}^{C}_{.}D^{\alpha}_{0^+}u(t) - u_{xx} + u = 6x^3 t + (x^3-6x)t^3, \tag{2}$$

Subject to the fuzzy initial condition

$$u(0) = \tilde{0} = [r-1, 1-r], \tag{3}$$

Now, the VIM correction function for (2) form as

$$\underline{u}_{n+1}(x,t,r) = \underline{u}_n(x,t,r) + \frac{1}{\Gamma(\alpha+1)}\int_0^t \lambda(\xi)\left[\frac{d^\alpha \underline{u}_n}{d\xi^\alpha}(x,\xi,r) - \frac{d^2}{dx^2}\underline{u}_n + \underline{u}_n(x,\xi,r) - 6x^3 t - (x^3-6x)t^3\right](d\xi^\alpha). \tag{4a}$$

$$\overline{u}_{n+1}(x,t,r) = \overline{u}_n(x,t,r) + \frac{1}{\Gamma(\alpha+1)}\int_0^t \lambda(\xi)\left[\frac{d^\alpha \overline{u}_n}{d\xi^\alpha}(x,\xi,r) - \frac{d^2}{dx^2}\overline{u}_n + \overline{u}_n(x,\xi,r) - 6x^3 t - (x^3-6x)t^3\right](d\xi^\alpha). \tag{4b}$$

Where $\frac{\partial^\alpha[u_n(\xi,x)]^r}{\partial\xi^\alpha} = {}^{C}_{.}D^{\alpha}_{0^+}[u_n(\xi)]^r$. This yields the stationary conditions $\lambda(\xi) = 0$ and $\overline{\lambda}(\xi) = 0{,}1 + \overline{\lambda}(\xi) = 0$ which gives $\underline{\lambda}(\xi) = \overline{\lambda}(\xi) = -1$, using the Lagrangen multiplier of VIM.

$$\underline{u}_{n+1}(x,t,r) = \underline{u}_n(x,t,r) - \frac{1}{\Gamma(\alpha+1)}\int_0^t \left[\frac{d^\alpha \underline{u}_n}{d\xi^\alpha}(x,\xi,r) - \frac{d^2}{dx^2}\underline{u}_n + \underline{u}_n(x,\xi,r) - 6x^3 t - (x^3-6x)t^3\right](d\xi^\alpha). \tag{5a}$$

$$\overline{u}_{n+1}(x,t,r) = \overline{u}_n(x,t,r) - \frac{1}{\Gamma(\alpha+1)}\int_0^t \left[\frac{d^\alpha \overline{u}_n}{d\xi^\alpha}(x,\xi,r) - \frac{d^2}{dx^2}\overline{u}_n + \overline{u}_n(x,\xi,r) - 6x^3 t - (x^3-6x)t^3\right](d\xi^\alpha). \tag{5b}$$

Beginning with

$$\underline{u}_0(x,t,r) = (r-1)\frac{t^\alpha}{\Gamma(\alpha+1)}, \tag{6a}$$

$$\overline{u}_0(x,t,r) = (1-r)\frac{t^\alpha}{\Gamma(\alpha+1)}, \tag{6b}$$

$$\underline{u}_1(x,t,r) = -(r-1)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + 6x^3\frac{t^{\alpha+1}}{\Gamma(\alpha+2)} + (x^3-6x)\frac{t^{\alpha+3}}{\Gamma(\alpha+4)}, \tag{7a}$$

$$\overline{u}_1(x,t,r) = -(1-r)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + 6x^3\frac{t^{\alpha+1}}{\Gamma(\alpha+2)} + (x^3-6x)\frac{t^{\alpha+3}}{\Gamma(\alpha+4)}, \tag{7b}$$

$$\underline{u}_2(x,t,r) = (r-1)\frac{t^{3\alpha}}{\Gamma(3\alpha+1)} + 6x^3\frac{t^{\alpha+1}}{\Gamma(\alpha+2)} + (x^3-6x)\frac{6t^{\alpha+3}}{\Gamma(\alpha+4)} + 36x\frac{t^{2\alpha+1}}{\Gamma(2\alpha+2)} + 6x^3\frac{t^{2\alpha+3}}{\Gamma(2\alpha+4)} - 6x^3\frac{t^{2\alpha+1}}{\Gamma(2\alpha+2)} - (x^3-6x)\frac{t^{2\alpha+3}}{\Gamma(2\alpha+4)}, \tag{8a}$$

$$\overline{u}_2(x,t,r) = (1-r)\frac{t^{3\alpha}}{\Gamma(3\alpha+1)} + 6x^3\frac{t^{\alpha+1}}{\Gamma(\alpha+2)} + (x^3-6x)\frac{6t^{\alpha+3}}{\Gamma(\alpha+4)} + 36x\frac{t^{2\alpha+1}}{\Gamma(2\alpha+2)} + 6x^3\frac{t^{2\alpha+3}}{\Gamma(2\alpha+4)} - 6x^3\frac{t^{2\alpha+1}}{\Gamma(2\alpha+2)} - (x^3-6x)\frac{t^{2\alpha+3}}{\Gamma(2\alpha+4)}, \tag{8b}$$

$$\underline{u}_3(x,t,r) = -(r-1)\frac{t^{4\alpha}}{\Gamma(4\alpha+1)} + 6x^3\frac{t^{\alpha+1}}{\Gamma(\alpha+2)} + (x^3-6x)\frac{6t^{\alpha+3}}{\Gamma(\alpha+4)} + 36x\frac{t^{2\alpha+1}}{\Gamma(2\alpha+2)} - 6x^3\frac{t^{2\alpha+1}}{\Gamma(2\alpha+2)} - (x^3-6x)\frac{6t^{2\alpha+3}}{\Gamma(2\alpha+4)} + 6x\frac{t^{2\alpha+1}}{\Gamma(2\alpha+2)} + 36x\frac{t^{2\alpha+3}}{\Gamma(2\alpha+4)} + 36x\frac{t^{3\alpha+3}}{\Gamma(3\alpha+4)} - 36x\frac{t^{3\alpha+3}}{\Gamma(3\alpha+4)} - 6x^3\frac{t^{2\alpha+1}}{\Gamma(2\alpha+2)} - 36x\frac{t^{3\alpha+1}}{\Gamma(3\alpha+2)} - 6x^3\frac{t^{3\alpha+3}}{\Gamma(3\alpha+4)} + 6x^3\frac{t^{3\alpha+1}}{\Gamma(3\alpha+2)} + (x^3-6x)\frac{6t^{3\alpha+3}}{\Gamma(3\alpha+4)}, \tag{9a}$$

$$\overline{u}_3(x,t,r) = -(1-r)\frac{t^{4\alpha}}{\Gamma(4\alpha+1)} + 6x^3\frac{t^{\alpha+1}}{\Gamma(\alpha+2)} + (x^3-6x)\frac{6t^{\alpha+3}}{\Gamma(\alpha+4)} + 36x\frac{t^{2\alpha+1}}{\Gamma(2\alpha+2)} - 6x^3\frac{t^{2\alpha+1}}{\Gamma(2\alpha+2)} - (x^3-6x)\frac{6t^{2\alpha+3}}{\Gamma(2\alpha+4)} + 6x\frac{t^{2\alpha+1}}{\Gamma(2\alpha+2)} + 36x\frac{t^{2\alpha+3}}{\Gamma(2\alpha+4)} + 36x\frac{t^{3\alpha+3}}{\Gamma(3\alpha+4)} - 36x\frac{t^{3\alpha+3}}{\Gamma(3\alpha+4)} - 6x^3\frac{t^{2\alpha+1}}{\Gamma(2\alpha+2)}$$

$$-36x\frac{t^{3\alpha+1}}{\Gamma(3\alpha+2)}-6x^3\frac{t^{3\alpha+3}}{\Gamma(3\alpha+4)}$$
$$+6x^3\frac{t^{3\alpha+1}}{\Gamma(3\alpha+2)}$$
$$+(x^3-6x)\frac{6t^{3\alpha+3}}{\Gamma(3\alpha+4)}, \qquad (9b)$$

In the view of the ADM the first few components of above problem are derived as follows.

$$\underline{u}_0(x,t,r)=-(r-1)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}+6x^3\frac{t^{\alpha+1}}{\Gamma(\alpha+2)}$$
$$+(x^3-6x)\frac{t^{\alpha+3}}{\Gamma(\alpha+4)}, \qquad (10a)$$

$$\overline{u}_0(x,t,r)$$
$$=-(1-r)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}+6x^3\frac{t^{\alpha+1}}{\Gamma(\alpha+2)}$$
$$+(x^3-6x)\frac{t^{\alpha+3}}{\Gamma(\alpha+4)}, \qquad (10b)$$

$$\underline{u}_1(x,t,r)=-(r-1)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}+36x\frac{t^{2\alpha+1}}{\Gamma(2\alpha+2)}$$
$$+36x\frac{t^{2\alpha+3}}{\Gamma(2\alpha+4)}-6x^3\frac{t^{2\alpha+1}}{\Gamma(2\alpha+2)}$$
$$-(x^3-6x)\frac{6t^{2\alpha+3}}{\Gamma(2\alpha+4)}, \qquad (11a)$$

$$\overline{u}_1(x,t,r)=-(r-1)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}+36x\frac{t^{2\alpha+1}}{\Gamma(2\alpha+2)}$$
$$+36x\frac{t^{2\alpha+3}}{\Gamma(2\alpha+4)}-6x^3\frac{t^{2\alpha+1}}{\Gamma(2\alpha+2)}$$
$$-(x^3-6x)\frac{6t^{2\alpha+3}}{\Gamma(2\alpha+4)}, \qquad (11b)$$

$$\underline{u}_2(x,t,r)=(r-1)\frac{t^{3\alpha}}{\Gamma(3\alpha+1)}-36x\frac{t^{3\alpha+1}}{\Gamma(3\alpha+2)}$$
$$-36x\frac{t^{3\alpha+3}}{\Gamma(3\alpha+4)}-36x\frac{t^{3\alpha+1}}{\Gamma(3\alpha+2)}$$
$$-36x\frac{t^{3\alpha+3}}{\Gamma(3\alpha+4)}+6x^3\frac{t^{3\alpha+1}}{\Gamma(3\alpha+2)}$$
$$+(x^3-6x)\frac{6t^{3\alpha+3}}{\Gamma(3\alpha+4)}, \qquad (12a)$$

$$\overline{u}_2(x,t,r)=(1-r)\frac{t^{3\alpha}}{\Gamma(3\alpha+1)}-36x\frac{t^{3\alpha+1}}{\Gamma(3\alpha+2)}$$
$$-36x\frac{t^{3\alpha+3}}{\Gamma(3\alpha+4)}-36x\frac{t^{3\alpha+1}}{\Gamma(3\alpha+2)}$$
$$-36x\frac{t^{3\alpha+3}}{\Gamma(3\alpha+4)}+6x^3\frac{t^{3\alpha+1}}{\Gamma(3\alpha+2)}$$
$$+(x^3-6x)\frac{6t^{3\alpha+3}}{\Gamma(3\alpha+4)}, \qquad (12b)$$

$$\underline{u}_3(x,t,r)=-(r-1)\frac{t^{4\alpha}}{\Gamma(4\alpha+1)}+36x\frac{t^{4\alpha+1}}{\Gamma(4\alpha+2)}$$
$$+36x\frac{t^{4\alpha+3}}{\Gamma(4\alpha+4)}+36x\frac{t^{4\alpha+1}}{\Gamma(4\alpha+2)}$$
$$+36x\frac{t^{4\alpha+3}}{\Gamma(4\alpha+4)}+36x\frac{t^{4\alpha+1}}{\Gamma(4\alpha+2)}$$
$$-6x^3\frac{t^{4\alpha+1}}{\Gamma(4\alpha+2)}$$
$$-(x^3-6x)\frac{6t^{4\alpha+3}}{\Gamma(4\alpha+4)}, \qquad (13a)$$

$$\overline{u}_3(x,t,r)=-(1-r)\frac{t^{4\alpha}}{\Gamma(4\alpha+1)}+36x\frac{t^{4\alpha+1}}{\Gamma(4\alpha+2)}$$
$$+36x\frac{t^{4\alpha+3}}{\Gamma(4\alpha+4)}+36x\frac{t^{4\alpha+1}}{\Gamma(4\alpha+2)}$$
$$+36x\frac{t^{4\alpha+3}}{\Gamma(4\alpha+4)}+36x\frac{t^{4\alpha+1}}{\Gamma(4\alpha+2)}$$
$$-6x^3\frac{t^{4\alpha+1}}{\Gamma(4\alpha+2)}$$
$$-(x^3-6x)\frac{6t^{4\alpha+3}}{\Gamma(4\alpha+4)}, \qquad (13b)$$

and so on.

$$\underline{u}(x,t,r)=(r-1)\frac{t^{\alpha}}{\Gamma(\alpha+1)}+6x^3\frac{t^{\alpha+1}}{\Gamma(\alpha+2)}$$
$$+(x^3-6x)\frac{6t^{\alpha+3}}{\Gamma(\alpha+4)}$$
$$-(r-1)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}+36x\frac{t^{2\alpha+1}}{\Gamma(2\alpha+2)}$$
$$+36x\frac{t^{2\alpha+3}}{\Gamma(2\alpha+4)}-6x^3\frac{t^{2\alpha+1}}{\Gamma(2\alpha+2)}$$
$$-(x^3-6x)\frac{6t^{2\alpha+3}}{\Gamma(2\alpha+4)}$$
$$+(r-1)\frac{t^{3\alpha}}{\Gamma(3\alpha+1)}-36x\frac{t^{3\alpha+1}}{\Gamma(3\alpha+2)}$$
$$-36x\frac{t^{3\alpha+3}}{\Gamma(3\alpha+4)}-36x\frac{t^{3\alpha+1}}{\Gamma(3\alpha+2)}$$
$$-36x\frac{t^{3\alpha+3}}{\Gamma(3\alpha+4)}+6x^3\frac{t^{3\alpha+1}}{\Gamma(3\alpha+2)}$$
$$+(x^3-6x)\frac{6t^{3\alpha+3}}{\Gamma(3\alpha+4)}$$
$$-(r-1)\frac{t^{4\alpha}}{\Gamma(4\alpha+1)}+36x\frac{t^{4\alpha+1}}{\Gamma(4\alpha+2)}$$
$$+36x\frac{t^{4\alpha+3}}{\Gamma(4\alpha+4)}+36x\frac{t^{4\alpha+1}}{\Gamma(4\alpha+2)}$$
$$+36x\frac{t^{4\alpha+3}}{\Gamma(4\alpha+4)}+36x\frac{t^{4\alpha+1}}{\Gamma(4\alpha+2)}$$
$$+36x\frac{t^{4\alpha+3}}{\Gamma(4\alpha+4)}-6x^3\frac{t^{4\alpha+1}}{\Gamma(4\alpha+2)}$$
$$-(x^3-6x)\frac{6t^{4\alpha+3}}{\Gamma(4\alpha+4)} \qquad (14)$$

$$\overline{u}(x,t,r)=(1-r)\frac{t^{\alpha}}{\Gamma(\alpha+1)}+6x^3\frac{t^{\alpha+1}}{\Gamma(\alpha+2)}$$
$$+(x^3-6x)\frac{6t^{\alpha+3}}{\Gamma(\alpha+4)}$$
$$-(1-r)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}+36x\frac{t^{2\alpha+1}}{\Gamma(2\alpha+2)}$$
$$+36x\frac{t^{2\alpha+3}}{\Gamma(2\alpha+4)}-6x^3\frac{t^{2\alpha+1}}{\Gamma(2\alpha+2)}$$
$$-(x^3-6x)\frac{6t^{2\alpha+3}}{\Gamma(2\alpha+4)}$$
$$+(1-r)\frac{t^{3\alpha}}{\Gamma(3\alpha+1)}-36x\frac{t^{3\alpha+1}}{\Gamma(3\alpha+2)}$$
$$-36x\frac{t^{3\alpha+3}}{\Gamma(3\alpha+4)}-36x\frac{t^{3\alpha+1}}{\Gamma(3\alpha+2)}$$
$$-36x\frac{t^{3\alpha+3}}{\Gamma(3\alpha+4)}+6x^3\frac{t^{3\alpha+1}}{\Gamma(3\alpha+2)}$$
$$+(x^3-6x)\frac{6t^{3\alpha+3}}{\Gamma(3\alpha+4)}$$
$$-(1-r)\frac{t^{4\alpha}}{\Gamma(4\alpha+1)}$$

$$+36x\frac{t^{4\alpha+1}}{\Gamma(4\alpha+2)}+36x\frac{t^{4\alpha+3}}{\Gamma(4\alpha+4)}+36x\frac{t^{4\alpha+1}}{\Gamma(4\alpha+2)}+36x\frac{t^{4\alpha+3}}{\Gamma(4\alpha+4)}+36x\frac{t^{4\alpha+1}}{\Gamma(4\alpha+2)}+36x\frac{t^{4\alpha+3}}{\Gamma(4\alpha+4)}-6x^3\frac{t^{4\alpha+1}}{\Gamma(4\alpha+2)}-(x^3-6x)\frac{6t^{4\alpha+3}}{\Gamma(4\alpha+4)} \quad (15)$$

Now in the view of NIM the first few components of the problem is,

$$u(x,y,z,t)=\sum_{k=0}^{m-1}h_k(x,y,z)\frac{t^k}{k!}+I_t^\alpha C+I_t^\alpha B+I_t^\alpha A = f+N(u)$$

Where $f=\sum_{k=0}^{m-1}h_k(x,y,z)\frac{t^k}{k!}+I_t^\alpha C+I_t^\alpha B$ and $N(u)=I_t^\alpha A$ and $u_0=f, u_{n+1}=N(u_n), n=0,1,2,\cdots$

$$\underline{u}_0(x,t,r)=(r-1)\frac{t^\alpha}{\Gamma(\alpha+1)}+6x^3\frac{t^{\alpha+1}}{\Gamma(\alpha+2)}+(x^3-6x)\frac{6t^{\alpha+3}}{\Gamma(\alpha+4)}, \quad (16a)$$

$$\overline{u}_0(x,t,r)=(1-r)\frac{t^\alpha}{\Gamma(\alpha+1)}+6x^3\frac{t^{\alpha+1}}{\Gamma(\alpha+2)}+(x^3-6x)\frac{6t^{\alpha+3}}{\Gamma(\alpha+4)}, \quad (16b)$$

$$\underline{u}_1(x,t,r)=-(r-1)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}+36x\frac{t^{2\alpha+1}}{\Gamma(2\alpha+2)}+36x\frac{t^{2\alpha+3}}{\Gamma(2\alpha+4)}-6x^3\frac{t^{2\alpha+1}}{\Gamma(2\alpha+2)}-(x^3-6x)\frac{6t^{2\alpha+3}}{\Gamma(2\alpha+4)}, \quad (17a)$$

$$\overline{u}_1(x,t,r)=-(r-1)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}+36x\frac{t^{2\alpha+1}}{\Gamma(2\alpha+2)}+36x\frac{t^{2\alpha+3}}{\Gamma(2\alpha+4)}-6x^3\frac{t^{2\alpha+1}}{\Gamma(2\alpha+2)}-(x^3-6x)\frac{6t^{2\alpha+3}}{\Gamma(2\alpha+4)}, \quad (17b)$$

$$\underline{u}_2(x,t,r)=(r-1)\frac{t^{3\alpha}}{\Gamma(3\alpha+1)}-36x\frac{t^{3\alpha+1}}{\Gamma(3\alpha+2)}-36x\frac{t^{3\alpha+3}}{\Gamma(3\alpha+4)}-36x\frac{t^{3\alpha+1}}{\Gamma(3\alpha+2)}-36x\frac{t^{3\alpha+3}}{\Gamma(3\alpha+4)}+6x^3\frac{t^{3\alpha+1}}{\Gamma(3\alpha+2)}+(x^3-6x)\frac{6t^{3\alpha+3}}{\Gamma(3\alpha+4)}, \quad (18a)$$

$$\overline{u}_2(x,t,r)=(1-r)\frac{t^{3\alpha}}{\Gamma(3\alpha+1)}-36x\frac{t^{3\alpha+1}}{\Gamma(3\alpha+2)}-36x\frac{t^{3\alpha+3}}{\Gamma(3\alpha+4)}-36x\frac{t^{3\alpha+1}}{\Gamma(3\alpha+2)}-36x\frac{t^{3\alpha+3}}{\Gamma(3\alpha+4)}+6x^3\frac{t^{3\alpha+1}}{\Gamma(3\alpha+2)}+(x^3-6x)\frac{6t^{3\alpha+3}}{\Gamma(3\alpha+4)}, \quad (18b)$$

$$\underline{u}_3(x,t,r)=-(r-1)\frac{t^{4\alpha}}{\Gamma(4\alpha+1)}+36x\frac{t^{4\alpha+1}}{\Gamma(4\alpha+2)}+36x\frac{t^{4\alpha+3}}{\Gamma(4\alpha+4)}+36x\frac{t^{4\alpha+1}}{\Gamma(4\alpha+2)}+36x\frac{t^{4\alpha+3}}{\Gamma(4\alpha+4)}+36x\frac{t^{4\alpha+1}}{\Gamma(4\alpha+2)}-6x^3\frac{t^{4\alpha+1}}{\Gamma(4\alpha+2)}-(x^3-6x)\frac{6t^{4\alpha+3}}{\Gamma(4\alpha+4)}, \quad (19a)$$

$$\overline{u}_3(x,t,r)=-(1-r)\frac{t^{4\alpha}}{\Gamma(4\alpha+1)}+36x\frac{t^{4\alpha+1}}{\Gamma(4\alpha+2)}+36x\frac{t^{4\alpha+3}}{\Gamma(4\alpha+4)}+36x\frac{t^{4\alpha+1}}{\Gamma(4\alpha+2)}+36x\frac{t^{4\alpha+3}}{\Gamma(4\alpha+4)}+36x\frac{t^{4\alpha+1}}{\Gamma(4\alpha+2)}-6x^3\frac{t^{4\alpha+1}}{\Gamma(4\alpha+2)}-(x^3-6x)\frac{6t^{4\alpha+3}}{\Gamma(4\alpha+4)}, \quad (19b)$$

and so on.

Example 2. Consider the following one dimensional linear inhomogeneous fuzzy fractional Klein Gordon Fock equation.

$${}^{C}D_{0^+}^{\alpha}u(t)-u_{xx}+u=6x^3t+(x^3-6x)t^3, \quad (20)$$

$t>0, x\in\mathbb{R}, 1<\alpha\leq 2$

Subject to the fuzzy initial condition

$$u(0)=\tilde{1}=[0.5+0.5r, 1.5-0.5r], \quad (21)$$

Now, the VIM correction function for (20) form as

$$\underline{u}_{n+1}(x,t,r)=\underline{u}_n(x,t,r)+\frac{1}{\Gamma(\alpha+1)}\int_0^t\lambda(\xi)\left[\frac{d^\alpha\underline{u}_n}{d\xi^\alpha}(x,\xi,r)-\frac{d^2}{dx^2}\underline{u}_n+\underline{u}_n(x,\xi,r)-6x^3t-(x^3-6x)t^3\right](d\xi^\alpha). \quad (22a)$$

$$\overline{u}_{n+1}(x,t,r)=\overline{u}_n(x,t,r)+\frac{1}{\Gamma(\alpha+1)}\int_0^t\lambda(\xi)\left[\frac{d^\alpha\overline{u}_n}{d\xi^\alpha}(x,\xi,r)-\frac{d^2}{dx^2}\overline{u}_n+\overline{u}_n(x,\xi,r)-6x^3t-(x^3-6x)t^3\right](d\xi^\alpha) \quad (22b)$$

Where $\frac{\partial^\alpha[u_n(\xi,x)]^r}{\partial\xi^\alpha}={}^{C}D_{0^+}^{\alpha}[u_n(\xi)]^r$. This yields the stationary conditions $\lambda(\xi)=0$ and $\overline{\lambda}(\xi)=0,1+\overline{\lambda}(\xi)=0$ which gives $\underline{\lambda}(\xi)=\overline{\lambda}(\xi)=-1$, using the Lagrangen multiplier of VIM.

$$\delta\underline{y}_{n+1}(t,r)=\delta\underline{y}_n(t,r)+\frac{\delta}{\Gamma(\alpha+1)}\int_0^t\lambda(\xi)(\frac{d^\alpha y_n}{d\xi^\alpha}(\xi,r)+\underline{y}_n(\xi,r))(d\xi^\alpha).$$
$$=\left(1+\lambda|_{\xi=x}\right)\delta\underline{y}_n(x,r)-\frac{1}{\Gamma(\alpha+1)}\int_0^t(\lambda_\xi^\alpha+\lambda)\delta\underline{y}_n(\xi,r)(d\xi)^\alpha. \quad (23a)$$

$$\delta\overline{y}_{n+1}(t,r)=\delta\overline{y}_n(t,r)+\frac{\delta}{\Gamma(\alpha+1)}\int_0^t\lambda(\xi)(\frac{d^\alpha\overline{y}_n}{d\xi^\alpha}(\xi,r)+\overline{y}_n(\xi,r))(d\xi^\alpha).$$

$$= \left(1 + \lambda|_{\xi=x}\right)\delta\overline{y}_n(x,r) - \frac{1}{\Gamma(\alpha+1)}\int_0^t (\lambda_\xi^\alpha + \lambda)\delta\overline{y}_n(\xi,r)(d\xi)^\alpha. \quad (23b)$$

note that $\delta\underline{y}_n(0,r) = 0$ and $\delta\overline{y}_n(0,r) = 0$. and $\lambda(\xi)$ must satisfy $1 + \lambda|_{\xi=x}$ and $\lambda_\xi^\alpha - \lambda = 0$. As a result, $\lambda(\xi)$ can be identified explicitly

$$\lambda(\xi) = -E_{\alpha,1}((\xi - t)^\alpha) \quad (24)$$

Where $E_{\alpha,1}((\xi - t)^\alpha)$ is defined by the classical Mittage-Leffler function.

$$E_{\alpha,1}((\xi - t)^\alpha) = E_{v,\mu}(z) = \sum_{n=0}^{\infty} \frac{((\xi - t)^\alpha)^n}{\Gamma(\alpha n + 1)} \quad (25)$$

Thus, the iteration formula for equation (20) can be written as

$$\underline{y}_{n+1}(x,t,r) = \underline{y}_n(x,t,r) - \frac{1}{\Gamma(\alpha+1)}\int_0^t E_{\alpha,1}((\xi - t)^\alpha)\left[\frac{d^\alpha \underline{y}_n}{d\xi^\alpha}(x,\xi,r) - \frac{d^2\underline{y}_n}{dx^2}(x,\xi,r) + \underline{y}_n(x,\xi,r) - 6x^3 t - (x^3 - 6x)t^3\right](d\xi^\alpha). (26a)$$

$$\overline{y}_{n+1}(x,t,r) = \overline{y}_n(x,t,r) - \frac{1}{\Gamma(\alpha+1)}\int_0^t E_{\alpha,1}((\xi - t)^\alpha)\left[\frac{d^\alpha \overline{y}_n}{d\xi^\alpha}(x,\xi,r) - \frac{d^2\overline{y}_n}{dx^2}(x,\xi,r) + \overline{y}_n(x,\xi,r) - 6x^3 t - (x^3 - 6x)t^3\right](d\xi^\alpha). \quad (26b)$$

On the other hand, if $\underline{y}_{n+1}(t,r)$ and $\overline{y}_{n+1}(t,r)$ is handled as a limited variation in equation (22a,b), similarly, the Lagrange multiplier can be identified by

$1 + \lambda|_{\xi=x}$ and $\lambda_\xi^\alpha = 0$ (27)

As a result, we can derive the generalized multiplier $\lambda(\xi) = -1$, we can get the iteration form

$$\underline{y}_{n+1}(x,t,r) = \underline{y}_n(x,t,r) - \frac{1}{\Gamma(\alpha+1)}\int_0^t \left[\frac{d^\alpha \underline{y}_n}{d\xi^\alpha}(x,\xi,r) - \frac{d^2\underline{y}_n}{dx^2}(x,\xi,r) + \underline{y}_n(x,\xi,r) - 6x^3 t - (x^3 - 6x)t^3\right](d\xi^\alpha). \quad (28a)$$

$$\overline{y}_{n+1}(x,t,r) = \overline{y}_n(x,t,r) - \frac{1}{\Gamma(\alpha+1)}\int_0^t \left[\frac{d^\alpha \overline{y}_n}{d\xi^\alpha}(x,\xi,r) - \frac{d^2\overline{y}_n}{dx^2}(x,\xi,r) + \overline{y}_n(x,\xi,r) - 6x^3 t - (x^3 - 6x)t^3\right](d\xi^\alpha). \quad (28b)$$

Start from

$$\underline{u}_0(x,t,r) = (0.5 + 0.5r),\ \overline{u}_0(x,t,r) = (1.5 - 0.5r) \quad (29)$$

$$\underline{u}_1(x,t,r) = (0.5 + 0.5r)\left(1 - \frac{t^\alpha}{\Gamma(\alpha+1)}\right) + 6x^3\frac{t^{\alpha+1}}{\Gamma(\alpha+2)} + (x^3 - 6x)\frac{t^{\alpha+3}}{\Gamma(\alpha+4)}, \quad (30a)$$

$$\overline{u}_1(x,t,r) = (1.5 - 0.5r)\left(1 - \frac{t^\alpha}{\Gamma(\alpha+1)}\right) + 6x^3\frac{t^{\alpha+1}}{\Gamma(\alpha+2)} + (x^3 - 6x)\frac{t^{\alpha+3}}{\Gamma(\alpha+4)}, \quad (30b)$$

$$\underline{u}_2(x,t,r) = (0.5 + 0.5r)\left(1 - \frac{t^\alpha}{\Gamma(\alpha+1)} + \frac{t^{2\alpha}}{\Gamma(2\alpha+1)}\right) + (x^3 - 6x)\frac{6t^{\alpha+3}}{\Gamma(\alpha+4)} + 36x\frac{t^{2\alpha+1}}{\Gamma(2\alpha+2)} + 36x\frac{t^{2\alpha+3}}{\Gamma(2\alpha+4)} - 6x^3\frac{t^{2\alpha+1}}{\Gamma(2\alpha+2)} - (x^3 - 6x)\frac{6t^{2\alpha+3}}{\Gamma(2\alpha+4)}, \quad (31a)$$

$$\overline{u}_2(x,t,r) = (1.5 - 0.5r)\left(1 - \frac{t^\alpha}{\Gamma(\alpha+1)} + \frac{t^{2\alpha}}{\Gamma(2\alpha+1)}\right) + 6x^3\frac{t^{\alpha+1}}{\Gamma(\alpha+2)} + (x^3 - 6x)\frac{6t^{\alpha+3}}{\Gamma(\alpha+4)} + 36x\frac{t^{2\alpha+1}}{\Gamma(2\alpha+2)} + 36x\frac{t^{2\alpha+3}}{\Gamma(2\alpha+4)} - 6x^3\frac{t^{2\alpha+1}}{\Gamma(2\alpha+2)} - (x^3 - 6x)\frac{6t^{2\alpha+3}}{\Gamma(2\alpha+4)}, \quad (31b)$$

$$\underline{u}_3(x,t,r) = (0.5 + 0.5r)\left(1 - \frac{t^\alpha}{\Gamma(\alpha+1)} + \frac{t^{2\alpha}}{\Gamma(2\alpha+1)} - \frac{t^{3\alpha}}{\Gamma(3\alpha+1)}\right) + 6x^3\frac{t^{\alpha+1}}{\Gamma(\alpha+2)} + (x^3 - 6x)\frac{6t^{\alpha+3}}{\Gamma(\alpha+4)} + 36x\frac{t^{2\alpha+1}}{\Gamma(2\alpha+2)} + 36x\frac{t^{2\alpha+3}}{\Gamma(2\alpha+4)} - 6x^3\frac{t^{2\alpha+1}}{\Gamma(2\alpha+2)} - (x^3 - 6x)\frac{6t^{2\alpha+3}}{\Gamma(2\alpha+4)} - 36x\frac{t^{3\alpha+1}}{\Gamma(3\alpha+2)} - 36x\frac{t^{3\alpha+3}}{\Gamma(3\alpha+4)} - 36x\frac{t^{3\alpha+1}}{\Gamma(3\alpha+2)} - 36x\frac{t^{3\alpha+3}}{\Gamma(3\alpha+4)} + 6x^3\frac{t^{3\alpha+1}}{\Gamma(3\alpha+2)} + (x^3 - 6x)\frac{6t^{3\alpha+3}}{\Gamma(3\alpha+4)}, \quad (32a)$$

$$\overline{u}_3(x,t,r) = (1.5 - 0.5r)\left(1 - \frac{t^\alpha}{\Gamma(\alpha+1)} + \frac{t^{2\alpha}}{\Gamma(2\alpha+1)} - \frac{t^{3\alpha}}{\Gamma(3\alpha+1)}\right) + 6x^3\frac{t^{\alpha+1}}{\Gamma(\alpha+2)} + (x^3 - 6x)\frac{6t^{\alpha+3}}{\Gamma(\alpha+4)} + 36x\frac{t^{2\alpha+1}}{\Gamma(2\alpha+2)} + 36x\frac{t^{2\alpha+3}}{\Gamma(2\alpha+4)} - 6x^3\frac{t^{2\alpha+1}}{\Gamma(2\alpha+2)} - (x^3 - 6x)\frac{6t^{2\alpha+3}}{\Gamma(2\alpha+4)} - 36x\frac{t^{3\alpha+1}}{\Gamma(3\alpha+2)} - 36x\frac{t^{3\alpha+3}}{\Gamma(3\alpha+4)} - 36x\frac{t^{3\alpha+1}}{\Gamma(3\alpha+2)} - 36x\frac{t^{3\alpha+3}}{\Gamma(3\alpha+4)} + 6x^3\frac{t^{3\alpha+1}}{\Gamma(3\alpha+2)} + (x^3 - 6x)\frac{6t^{3\alpha+3}}{\Gamma(3\alpha+4)}, \quad (32b)$$

In the view of the ADM the first few component of above problem are derived as follows,

$$\underline{u}(x,t,r) = (0.5+0.5r)\left(1-\frac{t^{\alpha}}{\Gamma(\alpha+1)}+\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}-\frac{t^{3\alpha}}{\Gamma(3\alpha+1)}\right)+6x^3\frac{t^{\alpha+1}}{\Gamma(\alpha+2)}+(x^3-6x)\frac{6t^{\alpha+3}}{\Gamma(\alpha+4)}+36x\frac{t^{2\alpha+1}}{\Gamma(2\alpha+2)}+36x\frac{t^{2\alpha+3}}{\Gamma(2\alpha+4)}-6x^3\frac{t^{2\alpha+1}}{\Gamma(2\alpha+2)}-(x^3-6x)\frac{6t^{2\alpha+3}}{\Gamma(2\alpha+4)}-36x\frac{t^{3\alpha+1}}{\Gamma(3\alpha+2)}-36x\frac{t^{3\alpha+3}}{\Gamma(3\alpha+4)}-36x\frac{t^{3\alpha+1}}{\Gamma(3\alpha+2)}-36x\frac{t^{3\alpha+3}}{\Gamma(3\alpha+4)}+6x^3\frac{t^{3\alpha+1}}{\Gamma(3\alpha+2)}+(x^3-6x)\frac{6t^{3\alpha+3}}{\Gamma(3\alpha+4)}+36x\frac{t^{4\alpha+1}}{\Gamma(4\alpha+1)}+36x\frac{t^{4\alpha+3}}{\Gamma(4\alpha+4)}+36x\frac{t^{4\alpha+1}}{\Gamma(4\alpha+2)}+36x\frac{t^{4\alpha+3}}{\Gamma(4\alpha+4)}+36x\frac{t^{4\alpha+1}}{\Gamma(4\alpha+2)}+36x\frac{t^{4\alpha+3}}{\Gamma(4\alpha+4)}-6x^3\frac{t^{4\alpha+1}}{\Gamma(4\alpha+2)}-(x^3-6x)\frac{6t^{4\alpha+3}}{\Gamma(4\alpha+4)} \quad (33a)$$

$$\overline{u}(x,t,r) = (1.5-0.5r)\left(1-\frac{t^{\alpha}}{\Gamma(\alpha+1)}+\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}-\frac{t^{3\alpha}}{\Gamma(3\alpha+1)}\right)+6x^3\frac{t^{\alpha+1}}{\Gamma(\alpha+2)}+(x^3-6x)\frac{6t^{\alpha+3}}{\Gamma(\alpha+4)}+36x\frac{t^{2\alpha+1}}{\Gamma(2\alpha+2)}+36x\frac{t^{2\alpha+3}}{\Gamma(2\alpha+4)}-6x^3\frac{t^{2\alpha+1}}{\Gamma(2\alpha+2)}-(x^3-6x)\frac{6t^{2\alpha+3}}{\Gamma(2\alpha+4)}-36x\frac{t^{3\alpha+1}}{\Gamma(3\alpha+2)}-36x\frac{t^{3\alpha+3}}{\Gamma(3\alpha+4)}-36x\frac{t^{3\alpha+1}}{\Gamma(3\alpha+2)}-36x\frac{t^{3\alpha+3}}{\Gamma(3\alpha+4)}+6x^3\frac{t^{3\alpha+1}}{\Gamma(3\alpha+2)}+(x^3-6x)\frac{6t^{3\alpha+3}}{\Gamma(3\alpha+4)}+36x\frac{t^{4\alpha+1}}{\Gamma(4\alpha+1)}+36x\frac{t^{4\alpha+3}}{\Gamma(4\alpha+4)}+36x\frac{t^{4\alpha+1}}{\Gamma(4\alpha+2)}+36x\frac{t^{4\alpha+3}}{\Gamma(4\alpha+4)}+36x\frac{t^{4\alpha+1}}{\Gamma(4\alpha+2)}+36x\frac{t^{4\alpha+3}}{\Gamma(4\alpha+4)}-6x^3\frac{t^{4\alpha+1}}{\Gamma(4\alpha+2)}-(x^3-6x)\frac{6t^{4\alpha+3}}{\Gamma(4\alpha+4)}. \quad (33b)$$

Now in the view of NIM the first few iteration is given by

$$u(x,y,z,t)=\sum_{k=0}^{m-1}h_k(x,y,z)\frac{t^k}{k!}+I_t^{\alpha}C+I_t^{\alpha}B+I_t^{\alpha}A = f+N(u)$$

where $f=\sum_{k=0}^{m-1}h_k(x,y,z)\frac{t^k}{k!}+I_t^{\alpha}C+I_t^{\alpha}B$ and $N(u)=I_t^{\alpha}A$ and $u_0=f, u_{n+1}=N(u_n), n=0,1,2,\cdots$

$$\underline{u}_0(x,t,r)=(0.5+0.5r)+6x^3\frac{t^{\alpha+1}}{\Gamma(\alpha+2)}+(x^3-6x)\frac{6t^{\alpha+3}}{\Gamma(\alpha+4)}, \quad (34a)$$

$$\overline{u}_0(x,t,r)=(1.5-0.5r)+6x^3\frac{t^{\alpha+1}}{\Gamma(\alpha+2)}+(x^3-6x)\frac{6t^{\alpha+3}}{\Gamma(\alpha+4)}, \quad (34b)$$

$$\underline{u}_1(x,t,r)=-(0.5+0.5r)\frac{t^{\alpha}}{\Gamma(\alpha+1)}+36x\frac{t^{2\alpha+1}}{\Gamma(2\alpha+2)}+36x\frac{t^{2\alpha+3}}{\Gamma(2\alpha+4)}-6x^3\frac{t^{\alpha+1}}{\Gamma(\alpha+2)}+(x^3-6x)\frac{6t^{\alpha+3}}{\Gamma(\alpha+4)}, \quad (35a)$$

$$\overline{u}_1(x,t,r)=-(1.5-0.5r)\frac{t^{\alpha}}{\Gamma(\alpha+1)}+36x\frac{t^{2\alpha+1}}{\Gamma(2\alpha+2)}+36x\frac{t^{2\alpha+3}}{\Gamma(2\alpha+4)}-6x^3\frac{t^{\alpha+1}}{\Gamma(\alpha+2)}+(x^3-6x)\frac{6t^{\alpha+3}}{\Gamma(\alpha+4)}, \quad (35b)$$

$$\underline{u}_2(x,t,r)=(0.5+0.5r)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}-36x\frac{t^{2\alpha+1}}{\Gamma(2\alpha+2)}-36x\frac{t^{2\alpha+3}}{\Gamma(2\alpha+4)}-36x\frac{t^{3\alpha+1}}{\Gamma(3\alpha+2)}-36x\frac{t^{3\alpha+3}}{\Gamma(3\alpha+4)}+6x^3\frac{t^{2\alpha+1}}{\Gamma(2\alpha+2)}+(x^3-6x)\frac{6t^{2\alpha+3}}{\Gamma(2\alpha+4)}, \quad (36a)$$

$$\overline{u}_2(x,t,r)=(1.5-0.5r)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}-36x\frac{t^{2\alpha+1}}{\Gamma(2\alpha+2)}-36x\frac{t^{2\alpha+3}}{\Gamma(2\alpha+4)}-36x\frac{t^{3\alpha+1}}{\Gamma(3\alpha+2)}-36x\frac{t^{3\alpha+3}}{\Gamma(3\alpha+4)}+6x^3\frac{t^{2\alpha+1}}{\Gamma(2\alpha+2)}+(x^3-6x)\frac{6t^{2\alpha+3}}{\Gamma(2\alpha+4)}, \quad (36b)$$

$$\underline{u}_3(x,t,r)=-(0.5+0.5r)\frac{t^{3\alpha}}{\Gamma(3\alpha+1)}+36x\frac{t^{3\alpha+1}}{\Gamma(3\alpha+2)}+36x\frac{t^{3\alpha+3}}{\Gamma(3\alpha+4)}+36x\frac{t^{3\alpha+1}}{\Gamma(3\alpha+2)}+36x\frac{t^{3\alpha+3}}{\Gamma(3\alpha+4)}+36x\frac{t^{4\alpha+1}}{\Gamma(4\alpha+2)}+36x\frac{t^{4\alpha+3}}{\Gamma(4\alpha+4)}-6x^3\frac{t^{3\alpha+1}}{\Gamma(3\alpha+2)}-(x^3-6x)\frac{6t^{3\alpha+3}}{\Gamma(3\alpha+4)}, \quad (37a)$$

$$\overline{u}_3(x,t,r)=-(1.5-0.5r)\frac{t^{3\alpha}}{\Gamma(3\alpha+1)}+36x\frac{t^{3\alpha+1}}{\Gamma(3\alpha+2)}+36x\frac{t^{3\alpha+3}}{\Gamma(3\alpha+4)}+36x\frac{t^{3\alpha+1}}{\Gamma(3\alpha+2)}+36x\frac{t^{3\alpha+3}}{\Gamma(3\alpha+4)}+36x\frac{t^{4\alpha+1}}{\Gamma(4\alpha+2)}+36x\frac{t^{4\alpha+3}}{\Gamma(4\alpha+4)}-6x^3\frac{t^{3\alpha+1}}{\Gamma(3\alpha+2)}-(x^3-6x)\frac{6t^{3\alpha+3}}{\Gamma(3\alpha+4)}, \quad (37b)$$

and so on.

IV. CONCLUSION

The article consist of three methods namely VIM, ADM, and NIM is explore to calculate the fractional order fuzzy Klein Gordon Fock equation. The result we obtained is accomplish by the three method is in infinite series form, which can be articulated by an inherent form with proper fuzzy IC, the final result is shown by graphically, how approximation the method is.

V. FUTURE SCOPE

We will try to extend in three dimensional fractional partial differential Klein–Gordon–Fock equation to Trapezoidal fuzzy fractional partial differential equation with these methods.

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