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The Innovative Execution of Fuzzy Automata Spaces in Developing Fuzzy Automata Inverse Semi-Groups

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ABSTRACT: In this paper, we developed concepts of fuzzy automata inverse semi groups and fuzzy automata inverse sub-semi groups using the concept of fuzzy spaces introduced by K. A. Dib. In addition, the concept of the fuzzy normal sub-semi group of a fuzzy automata inverse semi-group is introduced. Finally, we study their relationships between fuzzy automata inverse sub-semi groups and classical definitions of fuzzy automata inverse sub-semi groups.

Keywords: Fuzzy automata space; fuzzy automata inverse semi group; fuzzy automata inverse sub-semi group.

I. INTRODUCTION

A nonempty set X, a fuzzy subset of X is, by definition, an arbitrary mapping A: X → [0,1], where [0,1] is the usual interval of real numbers. The important concept of fuzzy automata set position onwards by Zadeh has opened up keen insights and applications in a wide range of scientific fields [1]. It offers tools and a new approach to model imprecision and uncertainty present in phenomena that do not have sharp boundaries. Since then, a series of research on fuzzy automata sets has come out expounding the importance of the concept and its applications to logic, set theory, algebra theory, real analysis, topology, etc. [3]. Fleck used the notion of a fuzzy subsets of a set to introduce the notion of fuzzy group of a group, Rosenfeld's paper motivated the development of fuzzy algebras [10]. Following the formulation of fuzzy subgroups by Rosenfeld, Dib introduced the concept of a fuzzy automata space as a replacement for the concept of universal set in the ordinary case. Recently, some basic concepts of fuzzy algebras such as fuzzy homomorphism's were introduced and discussed by using the new approach of fuzzy space and fuzzy automata groups introduced. In this paper we introduce concepts of fuzzy automata inverse semi groups and redefine fuzzy automata inverse sub-semi groups using the concept of fuzzy spaces introduced by Dib [2].

II. NOTATIONS AND PRELIMINARIES

Throughout this paper we adopt the following notations, definitions and some results.

I: The lattice [0,1] with the usual order of real numbers.

L, K, N: An arbitrary completely distributive lattices with least and greatest element by Li and Pedryz [6].

$J = L \otimes K$: The vector lattice *L × K* with the partial order defined by

(a) $(r_1, r_2) \le (s_1, s_2)$ iff $r_1 \le s_1, r_2 \le s_2$ *whenever* $s_1 \ne 0, s_2 \ne 0$;

(b) $(s_1, s_2) = (0,0)$ *whenever* $s_1 = 0$ *or* $s_2 = 0$.

$J^* : The\ lattice\ I \otimes I.$

X always denotes a nonempty ordinary set and A, B, C are fuzzy subsets of X. When we speak of an L-fuzzy automata subset we mean that the associated membership function takes its values from the lattice L [4], [6].

Definition: A nonempty set L closed under two binary operations as follows:
∧ And ∨, is called a lattice (L, ∧, ∨) provided following *ax*ioms hold:
Idempotent law, Commutative law, Associative law and Absorption law [7].

Definition: A fuzzy space, denoted by X_L, is the set of all ordered pairs
$(x,L) x \in X$, *i.e.*, $X_L = \{(x,L) : x \in X\}$, *where* (x,L) *is called a fuzzy element of the fuzzy space* X_L

Definition: A fuzzy subspace U of the fuzzy space X_L and Y_K is a collection of ordered pairs (y, L_y), where $y \in U_0$ for a given subset U_0 if X and L_y is an M-sub lattice of L and denoted by $U = \{(y, L_y) : y \in U_0\}$ where (y, L_y) is called a fuzzy automata element of the fuzzy automata subspace U.

Definition: A fuzzy automata Cartesian product of the two fuzzy automata spaces [11], [12] X_L and Y_K is the fuzzy automata space $X_L \times Y_K \equiv (X \times Y)_J = \{((x,y),J):(x,y) \in X \times Y\}$. for the two fuzzy subspaces $U = \{(x,L_x): x \in U_0\} and V = \{(y,L_y): y \in V_0\}$ of the fuzzy spaces X_L and Y_K respectively, their fuzzy Cartesian product is the fuzzy automata subspace $U \times V = \{((x,y), L_x \otimes K_y):(x,y) \in U_0 \times V_0\}$ of the fuzzy automata space $X_L \times Y_K$.

Definition: A fuzzy automata function $\tilde{F} = (F, f_{xy}): X \times Y \to Z$ is called a uniform of its co membership functions f_{xy} are identical i.e., $f_{xy}(r,s) = f(r,s) \text{for all } (x,y) \in X \times Y$.

Lemma: Every fuzzy automata function $\tilde{F} = (F, f_{xy}): X \times Y \to Z$ with onto co membership functions $f_{xy}: J \to N$ defines a function, denoted again by $\tilde{F}$, from the fuzzy space $X_L \times Y_K$ to the fuzzy automata space Z_N and its acts as $\tilde{F}((x,y),J) = (F(x,y), f_{xy}(J)) \equiv (xF_y, N)$.

Definition: A fuzzy binary function $\tilde{F} = (F, f_{xy})$ on the fuzzy space X_L is a fuzzy automata function $\tilde{F}$ from $X \times X$ to X with co membership functions $f_{xy}: L \otimes L \to L$ that satisfy:

(i) $f_{xy}(r,s) = 0$ if and only if r = 0 or s=0;

(ii) f_{xy} are onto, i.e., $f_{xy}(L \otimes L) = L \ for\ all\ (x,y) \in X \times X$.

Definition: A fuzzy space X_L with a fuzzy automata binary operation $\tilde{F} = (F, f_{xy})$ is said to be a fuzzy automata groupoid and denoted by (X_1, F).

A fuzzy semi group is a fuzzy groupoid that is associative, i.e., $((x,I)\tilde{F}(y,I)\tilde{F}(z,I) = (x,I)\tilde{F}((y,I)\tilde{F}(z,I))\ for\ any\ (x,I),(y,I)\ and\ (z,I) \in X_1$.

Lemma: Associated to each fuzzy automata semi group $(X_1, \tilde{F})$ there is an ordinary semi group (X ,F) and they are isomorphic to each other by the correspondence $(x,I) \leftrightarrow x$.

Definition: A fuzzy subspace

$$i) U = \{(x,L_x): x \in U_0\} \text{ we have } (x,L_x)\tilde{F}(y,L_y) = (xF_y, L_{xFy}) \in U;$$

$$ii) (U, \tilde{F}) \text{ satisfies the condition of a semigroup.}$$

Lemma: A fuzzy subspace U of the fuzzy semi group $(X_1, \tilde{F})$ is a fuzzy sub-semi group of $(X_1, \tilde{F})$ if and only if

(i) (U_0, F) is a sub-semi group automata of the ordinary semi group (X, F);

(ii) $f_{xy}(L_x \otimes L_y) = L_{xFy}, for\ all\ x,y \in U_0$. Concerning the induced fuzzy subspace H(A) by the fuzzy automata subset A of X.

Theorem: An induced fuzzy automata subspace H(A) by A of the fuzzy automata semi group $(X_1, \tilde{F})$ is a fuzzy sub-semi group if and only if:

(I) (*A*0, F) is a sub-semi group automata of the ordinary semi group (*X, F*).

(i) $f_{xy}(A(x), A(y)) = A(xFy), for\ all\ x,y \in U_0$.

Finally, concerning the classical fuzzy automata inverse sub-semi group, we mentioned the subsequent definition.

Definition: A fuzzy sub-semi group A of an (ordinary) inverse semi group (*X, F*) is called fuzzy inverse automata if and only if for any x ε $x \in X$ there exists unique $x^{-1} \in I_x = \{x^{-1} \in X : xFx^{-1}Fx = x, x^{-1}FxFx^{-1} = x^{-1}\}\, such\, that\, A(x^{-1}) \geq A(x).$

II. FUZZY AUTOMATA INVERSE SEMI GROUPS

Commencing now on, we think about the fuzzy automata space X_I with the lattice I = [0,1] by Benjamin Steinberg [9].

Definition: A fuzzy automata semi group $(X_1, \tilde{F})$ is called fuzzy automata inverse if, for each fuzzy automata element (x,I) of X_I, there exists a unique fuzzy automata element $(x^{-1}, I)\, in\, X_I\, such that\, (x,I) = (x,I)\tilde{F}((x^{-1},I)\tilde{F}(x,I)\, and\, (x^{-1},I) = (x^{-1},I)\tilde{F}(x,I)\tilde{F}(x^{-1},I)(*)$.

Here we observe that each fuzzy automata semi group $(X_1, \tilde{F})$ is isomorphic [10] to the ordinary semi group (X, F).The equality $(*)$ can be written as: $(x,I) = (xFx^{-1}Fx, I)\, and\, (x^{-1},I) = (x^{-1}FxFx^{-1}, I)$. From the preceding discussion, we get the following theorem.

Theorem: To each fuzzy automata inverse semi group $(X_1, \tilde{F})$ there is an associated ordinary inverse automata semi group (X, F) and they are isomorphic by the correspondence (x, I) ↔ x. Therefore, the inverse automata semi group (X, F) is isomorphic to every fuzzy automata inverse semi group $(X_1, \tilde{F})$. Where $\tilde{F} = (F, f_{xy})$ for arbitrary choice of onto co-membership functions f_{xy}.

Definition: A fuzzy finite automata inverse semi group $(X_1, \tilde{F})$, where $\tilde{F} = (F, f_{xy})$, is called a uniform if the co-membership functions f_{xy} are identical, i.e., $f_{xy} = f$ for all $x, y \in X$.

Remark: In the snappish case, where I is replaced by {0,1}, the fuzzy automata Cartesian product space $(X \times X)_{J*}$ is identified with ordinary Cartesian product $X \times X$ and the fuzzy automata binary $\tilde{F} = (F, f_{xy})$ is reduced to the ordinary binary operation F. Since every element x in X can be identified with (x, {0, 1}), hence the fuzzy automata inverse semi group $(X_1, \tilde{F})$ can be identified with the ordinary inverse automata semi group (X, F) By Park [11].

Definition: If $(U, \tilde{F})$ is called a fuzzy automata inverse sub-semi group of $(X_1, \tilde{F})$

i) U is closed under the fuzzy automata binary operation $\tilde{F}$, i.e., $(x, L_x)\tilde{F}(y, L_y) = (xF_y, L_{xFy}) \in U\; for\, all (x, U_x), (y, L_y) \in U;$

ii) $(U, \tilde{F})$ Satisfies the conditions of the ordinary inverse automata semi group.

Theorem: A fuzzy subspace U is a fuzzy sub-semi group of $(X_1, \tilde{F})$ if and only if the following conditions are satisfied (i) (U_0, F) is an ordinary inverse sub-semi group of (X, F);

(ii) $f_{xy}(L_x \otimes L_y) = L_{xFy,}\; for\, all\, x, y \in U_0.$

Proof: If (i) and (ii) are satisfied, we have (a) $\tilde{F}$ is closed on the fuzzy automata subspace U; (b) $(U, \tilde{F})$ satisfies the conditions of an ordinary semi group. Moreover, Let $(x, L_x) \in \mathrm{U}$. Then, by (i) and (ii), there exists unique $(x^{-1}, L_{x^{-1}}) \in \mathrm{U}$, where

$$x^{-1} \in I_x = \{x^{-1} \in U_0 : (x, I) = (x F x^{-1} F x, I) \, and \, (x^{-1}, I) = (x^{-1} F x F x^{-1}, I), such\, that$$

$$(x, L_x) = (x, L_x)\tilde{F}(x^{-1}, L_{x^{-1}})\tilde{F}(x, L_x) \, and \, (x^{-1}, L_{x^{-1}}) = (x^{-1}, L_{x^{-1}})\tilde{F}(x, L_x)\tilde{F}(x^{-1}, L_{x^{-1}}).$$

We have thus shown that $(U, \tilde{F})$ is a fuzzy automata inverse sub-semi group of $(X_1, \tilde{F})$.

Let $(U, \tilde{F})$ be a fuzzy automata inverse sub-semi group of $(X_1, \tilde{F})$. Then, it is easy to see that condition (ii) can be obtained directly, because $(U, \tilde{F})$ satisfies the conditions of the ordinary inverse automata semi group. Concerning the induced fuzzy automata subspace H(A) by the fuzzy automata subset A of X.

Theorem: An induced fuzzy automata subspace H (A) by A of the fuzzy automata $(X_1, \tilde{F})$ is a fuzzy inverse automata sub-semi group if and only if the following conditions are satisfied: [8]
(i) (A_0, F) is an ordinary inverse sub-semi group of (X, F);
(ii) For all x, y ε U_0, we have $f_{x,y}(A(x), A(y)) = A(x F_y)$.

Remark: For any onto co membership functions $f_{x,y}$, the fuzzy inverse automata semi group $(X_1, \tilde{F})$, where $\tilde{F} = (F, f_{xy})$, is isomorphic to the ordinary inverse semi group (X, F) .Therefore, if $(U, \tilde{F})$ is a fuzzy automata inverse sub-semi group of $(X_1, \tilde{F})$, then we can say that $(U, \tilde{F})$ is a fuzzy automata inverse sub-semi group of the ordinary inverse semi group (X, F) by Park C. H. [11].

Definition: An element e in the inverse semi group (X, F) is idempotent if $eFe = e$. Let E_X denotes the set of al idempotent elements of (X, F)

Definition: A fuzzy sub-semi group $(U, \tilde{F})$ of the fuzzy inverse of the fuzzy automata inverse semi group $(X_1, \tilde{F})$ is called a fuzzy full if U_0 contains all the fuzzy idempotent of (X, F), i.e., $E_X \subset U_0$ [13].

Definition: A fuzzy sub-semi group $(U, \tilde{F})$ of the fuzzy inverse semi group $(X_1, \tilde{F})$ is fuzzy

Self-conjugate, in the sense that $(y, L_y) \in U \Rightarrow (\forall (x, I) \in \mathrm{X}_1)(x^{-1}, I)\tilde{F}(y, L_y)\tilde{F}(x, I) \in U.$

Definition: A fuzzy sub-semi group $(U, \tilde{F})$ of the fuzzy inverse automata semi group $(X_1, \tilde{F})$ is called normal if it is fuzzy full and fuzzy self-conjugate.
Remark: The exceeding introduced concepts give means for attacking many problems in fuzzy inverse semi groups which go parallel with ordinary inverse semi groups theory [11],[14], [15].

III. THE RELATIONSHIP BETWEEN FUZZY AUTOMATA INVERSE SUB-SEMI GROUPS AND THE CLASSICAL ONES

We recall that fuzzy inverse semi group $(X_1, \tilde{F})$, where $\tilde{F} = (F, f_{xy})$, is said to be a uniform if $\tilde{F}$ is uniform fuzzy binary operation on X_1, i.e., $f_{xy} = f$ for all $x, y \in X$.

Theorem: Let $(X_1, \tilde{F})$ be uniform fuzzy inverse automata semi group, where $\tilde{F} = (F, \wedge)\, and\, "\wedge"$ the minimum function from $J^* \,\mathrm{in\, to\, I}$. Then every fuzzy subset A of which induces a fuzzy inverse sub-semi group $(H(A), \tilde{F})$ of the fuzzy inverse automata semi group $(X_1, \tilde{F})$ is a classical fuzzy inverse sub-semi group of the ordinary inverse automata semi group (X, F)

Proof: Suppose that A is a fuzzy automata subset of X such that $(H(A), \tilde{F})$ is a fuzzy inverse automata sub-semi group of the uniform fuzzy inverse semi group(X_1, F), where $\tilde{F} = (F, \wedge)\, and\, "\wedge"$.

(i) (A_0, F) is an ordinary inverse automata sub-semi group of (X, F);

$$x, y \in A_{0,}\, we\, have\, A(x) \wedge A(y) = A(xFy). Thus, for\, any\, x \in X, if A(x) \neq 0, then, by$$

$$(i)\, there\, exists\, unique\, x^{-1} \in A_0\, such\, that\, x = xFx^{-1}, and\, so\, we\, have, by\, (ii),$$

$$\begin{aligned}
\mathrm{A}(x) &= \mathrm{A}(xFx^{-1}Fx) = A(xFx^{-1}) \wedge \mathrm{A}(x) \\
&= (\mathrm{A}(x) \wedge A(x^{-1})) \wedge \mathrm{A}(x) \\
&= \mathrm{A}(x) \wedge A(x^{-1}),
\end{aligned}$$

$$\begin{aligned}
A(x^{-1}) &= A(x^{-1}FxFx^{-1}) \\
&= A(x^{-1}Fx) \wedge A(x^{-1}) \\
&= (A(x^{-1}) \wedge \mathrm{A}(x) \wedge A(x^{-1}) \\
&= A(x^{-1}) \wedge \mathrm{A}(x),
\end{aligned}$$

$$\text{which means that } A(x^{-1}) \geq \mathrm{A}(x) \text{ and } \mathrm{A}(x) \geq A(x^{-1}),$$

$$\text{i.e., } A(x^{-1}) = \mathrm{A}(x). \text{If } \mathrm{A(x)} = 0, \text{then, by the inversive property of } (X, F)$$

(ii) For all

$$x^{-1} \in X\, such\, that\, A(x^{-1}) \geq A(x). Hence, xFx^{-1}Fx = x, x^{-1}FxFx^{-1} = x^{-1}$$

$$and\, it\, is\, clear\, that\; A(x^{-1}) \geq A(x). which\, means\, that\, A\, is\, a\, classical\;\, fuzzy\, inverse\, sub-semi\, group\, of\, (X, F).$$

Theorem: Suppose that (Y,F) is an inverse sub-semi group of the inverse semi group (X,F) and A is a fuzzy subset of X such that A_0 = Y. Then there is a fuzzy inverse semi group $(X_1, \tilde{F})$ such that the induced fuzzy sub space H (A0 by A is a fuzzy inverse sub-semi group of $(X_1, \tilde{F})$.

Proof: Suppose that (Y, F) is an inverse sub-semi group of the inverse semi group (X,F) and A is a fuzzy sub set of X such that A_0 = Y [15].

Now, we construct fuzzy automata inverse semi group $(X_1, \tilde{F})$ as follows:

$$f : J^* \to I\, be\; a\; given\; t-norm.\, And\, take\; \tilde{F} = (F, f_{xy}),\, where\, f_{xy} = h_{xy}(f(r,s))$$

$$such\, that\, if\;\, f(A(x), A(y)) \neq 0, then$$

Let $$h_{xy}(t) = \{ \frac{A(xFy)}{f(A(x), A(y)} t, \text{if } t \leq \mathrm{f(A(x), A(y))}$$

$$\{1 + \frac{1 - A(xFy)}{1 - f(A(x), A(y)}(t-1), if\;\, t > f(\mathrm{A(x), A(y)})$$

And if $f(A(x), A(y) = 0, \text{then } \mathrm{h}_{xy} = t\, for\, all\, 0 \leq t \leq 1.$ It is obvious that $f_{xy} x, y \in X$ are onto co membership functions and $f_{x,y}(r,s) = 0$ if and only if r = 0 or s = 0. Therefore, $(X_1, \tilde{F})$, then from the definition of $f_{x,y}$, and noticing that f is t-norm, we have, for all

$x, y \in X, f_{xy}(A(x), A(y)) = h_{xy}(f(A(x), \mathrm{A(y)})) = \mathrm{A(xFy)}$.

$\text{Which implies that } (\mathrm{H(A)}, \tilde{F})\ is\ a\ fuzzy\ automata\ inverse\ sub-semi\ group\ of\ (X_1, \tilde{F})$.

Corollary: Every classical fuzzy automata inverse sub-semi group A of the ordinary inverse automata semi group (X,F) induces a fuzzy inverse sub-semi group relative to some fuzzy inverse automata $(X_1, \tilde{F})$.

III. CONCLUSION

Using the concept of fuzzy automata spaces, we have defined fuzzy automata inverse semi groups and fuzzy automata inverse sub-semi groups. The aim that to establish those definitions is threefold. Firstly, to generalize fundamental results from inverse automata semi group theory; Secondly, to find out some new results; thirdly, to clarify the links between our approach and the classical one.

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REFERENCES

[1]. Zadeh, L.A. (1965). “Fuzzy sets” *Inform. Control,* 8, 338-353 (1965).

[2]. Dib, K.A. (1991). “Fuzzy Cartesian product, Fuzzy relations and fuzzy Functions”, **41**, 299-315.

[3]. Srivastava, A.K. and Shukla, W. (1986). “A Topology for Automata II”, *Internat. Jour. Math. and Math. Sc.,* **9**: 425-428.

[4]. Li, Y.M. (2006). “Finite Automata theory with membership values in Lattices, *Information Sciences”*,**176**, 3232-3255.

[5]. Meng, F. and Chen, X. (2015). "A new method for a triangular fuzzy compare wise judgment matrix process based on consistency analysis,” *International Journal of Fuzzy Systems,* Vol. **309,** PP. 119-1375.

[6]. Li, Y.M. and Pedrycz, W. (2005). “Fuzzy finite automata and Regular Expressions with membership values in Lattice ordered monoids”, *Fuzzy sets and systems,* **156**, 68-92.

[7]. Peeva, K. (2004). “Finite L-fuzzy machines”, *Fuzzy sets and systems*, **141**, 415-437.

[8]. Santos, E.S. (1968). “Fuzzy automata and Languages”, *Information Sciences,* **10**, 367-377.

[9]. Benjamin Steinberg (2002). Invrse Automata nd Profinite Topologies on a free group. *Journal of Pure and Applied Algebra,* **167**, 341-354.

[10]. Fleck, A.C. (1962). Isomorphism groups of automata. *J. ACM,* **9**, 469-476.

[11]. Park, C.H. (1993). Some Remarks on the Automata-Homorphisms Comm. *Korean Math. Soc.,* **8**, 499-809.

[12]. Hall, M. (1950). A topology for free groups and related groups. *Annals of Mathematics*, **52**, 127-139.

[13]. Ribes, L. and Zalesskil, P.A. (1994). “The pro-p topology of a free group and algorithmic problem in semigrops, *Inter. J. Algebra and Comput*. **4**, 359-374.

[14]. Qiu, D.W. (2004). “Characterizations of fuzzy finite automata”, *Fuzzy sets and systems*, **141**, 391-414.

[15]. Wang, X.P. and Liu, W.J. (1983). Fuzzy regular subsemigroups in semigroups. *Inform. Sci.,* **68**: 225-231.

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