



# Some random fixed point theorems for random multivalued operators on polish space

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**ABSTRACT :** The objective of this paper is to obtain some fixed point theorems for one and two multivalued operators defined on a Polish space.

**Mathematics subject classification:** 47H10, 54H25

**Keywords :** Polish Space, Random Multivalued Operators, Random Fixed Point.

## I. INTRODUCTION/PRELIMINARIES

Let  $(X, d)$  be a polish space that is a separable complete metric space and  $(\Omega, q)$  be measurable space. Let  $2^X$  be a family of all subsets of  $X$  and  $CB(X)$  denote the family of all non-empty bounded closed subsets of  $X$ . A mapping  $T : \Omega \rightarrow 2^X$  is called measurable if for any open subset  $C$  of  $X$ ,  $T^{-1}(C) = \{ \omega \in \Omega : T(\omega) \cap C \neq \emptyset \} \in q$ . A mapping  $\xi : \Omega \rightarrow X$  is said to be measurable selector of a measurable mapping  $T : \Omega \rightarrow 2^X$ , if  $\xi$  is measurable and for any  $x \in X$ ,  $T(x)$  for any open subsets  $\omega$  (open subsets  $X$ ) denote the family of all non-empty bounded closed subsets of  $A \in \Omega, \xi(\omega) \in T(\omega)$ . A mapping  $f : \Omega \times X \rightarrow X$  is called random operator, if for any  $x \in X, f(\cdot; x)$  is measurable. A mapping  $T : \Omega \times X \rightarrow CB(X)$  is a random multivalued operator, if for every  $x \in X, T(\cdot; x)$  is measurable. A measurable mapping  $\xi : \Omega \rightarrow X$  is called random fixed point of a random multivalued operator  $T : \Omega \times X \rightarrow CB(X)$  ( $f : \Omega \times X \rightarrow X$ ), if for every  $\omega \in \Omega, \xi(\omega) \in T(\omega, \xi(\omega))$  ( $f(\omega, \xi(\omega)) = \xi(\omega)$ ). Let  $T : \Omega \times X \rightarrow CB(X)$  be a random operator and  $\{\xi_n\}$  a sequence of measurable mappings  $\xi_n : \Omega \rightarrow X$ .

## II. MAIN RESULT

**Theorem 2.1.** Let  $X$  be a Polish space. Let  $T : \Omega \times X \rightarrow CB(X)$  be a continuous random multivalued operator. If there exists measurable mapping  $a, b : \Omega \rightarrow (0, 1)$  such that

$$H(T(\omega, x), T(\omega, y)) \leq \frac{a(\omega)d(y, T(\omega, y))[1 + d(x, T(\omega, x))]}{1 + d(x, y)} + b(\omega)[d(x, T(\omega, x)) + d(y, T(\omega, y))] + c(\omega)d(x, y) \quad \dots(A)$$

for each  $x, y \in X, \omega \in \Omega$  &  $a, b, c \in R^+$  with  $a(\omega) + 2b(\omega) + c(\omega) < 1$ , then there exists a common random fixed point of  $T$  (Here  $H$  represents the Hausdorff metric on  $CB(X)$  introduced by the metric  $d$ ).

**Proof:** Let  $\xi_0 : \Omega \rightarrow X$  be an arbitrary measurable mappings and choose a measurable mapping  $\xi : \Omega \rightarrow X$  such that  $\xi_1(\omega) \in T(\omega, \xi_0(\omega))$  for each  $\omega \in \Omega$ . Then for each  $\omega \in \Omega$ ,

$$H(T(\omega, \xi_0(\omega)), T(\omega, \xi_1(\omega))) \leq \frac{a(\omega)d(\xi_1(\omega), T(\omega, \xi_1(\omega)))[1 + d(\xi_0(\omega), T(\omega, \xi_0(\omega)))]}{1 + d(\xi_0(\omega), \xi_1(\omega))} + b(\omega)[d(\xi_0(\omega), T(\omega, \xi_0(\omega))) + d(\xi_1(\omega), T(\omega, \xi_1(\omega)))] + c(\omega)d(\xi_0(\omega), \xi_1(\omega))$$

It further implies [2 & 1, Lemma 2.3] then there exists a measurable mapping  $\xi_2 : \Omega \rightarrow X$  such that for any  $\omega \in \Omega, \xi_2(\omega) \in T(\omega, \xi_1(\omega))$  and

$$d(\xi_1(\omega), \xi_2(\omega)) = H(T(\omega, \xi_0(\omega)), T(\omega, \xi_1(\omega))) d(\xi_1(\omega), \xi_2(\omega)) \leq \frac{a(\omega)d(\xi_2(\omega), T(\omega, \xi_2(\omega)))[1 + d(\xi_0(\omega), T(\omega, \xi_0(\omega)))]}{1 + d(\xi_0(\omega), \xi_1(\omega))} + b(\omega)[d(\xi_0(\omega), T(\omega, \xi_0(\omega))) + d(\xi_1(\omega), T(\omega, \xi_1(\omega)))] + c(\omega)d(\xi_0(\omega), \xi_1(\omega))$$

$$= \frac{a(\omega)d(\xi_2(\omega), \xi_2(\omega))[1 + d(\xi_0(\omega), \xi_1(\omega))]}{1 + d(\xi_0(\omega), \xi_1(\omega))} + b(\omega)[d(\xi_0(\omega), \xi_1(\omega)) + d(\xi_1(\omega), \xi_2(\omega))] + c(\omega)d(\xi_0(\omega), \xi_1(\omega)) = [a(\omega) + b(\omega)]d(\xi_0(\omega), \xi_1(\omega)) + b(\omega) + c(\omega)d(\xi_0(\omega), \xi_1(\omega))$$

$$\begin{aligned} &\Rightarrow [1 - (a(\omega) + b(\omega))]d(\xi_1(\omega), \xi_2(\omega)) \\ &\leq [b(\omega) + c(\omega)]d(\xi_1(\omega), \xi_2(\omega)) \\ &\Rightarrow d(\xi_1(\omega), \xi_2(\omega)) \leq \frac{[b(\omega) + c(\omega)]}{1 - [a(\omega) + b(\omega)]} d(\xi_1(\omega), \xi_2(\omega)) \\ &d(\xi_1(\omega), \xi_2(\omega)) \leq Kd(\xi_1(\omega), \xi_2(\omega)) \\ &\text{where } K = \frac{[b(\omega) + c(\omega)]}{1 - [a(\omega) + b(\omega)]} \\ &< 1 [As a(\omega) + b(\omega) + c(\omega) < 1] \\ &\text{By above lemma in the same manner, there exists a measurable mapping } \xi_3: \Omega \rightarrow X \text{ such that for any } \\ &\omega \in \Omega, \xi_3(\omega) \in T(\omega, \xi_2(\omega)) \text{ and} \\ &d(\xi_2(\omega), \xi_3(\omega)) = H(T(\omega, \xi_2(\omega)), T(\omega, \xi_2(\omega))) \\ &d(\xi_2(\omega), \xi_3(\omega)) \\ &\leq \frac{a(\omega)d(\xi_2(\omega), T(\omega, \xi_2(\omega)))[1 + d(\xi_2(\omega), T(\omega, \xi_2(\omega)))]}{1 + d(\xi_2(\omega), \xi_3(\omega))} \\ &+ b(\omega)[d(\xi_2(\omega), T(\omega, \xi_2(\omega))) + d(\xi_2(\omega), T(\omega, \xi_2(\omega)))] \\ &+ c(\omega)d(\xi_2(\omega), \xi_3(\omega)) \\ &= \frac{a(\omega)d(\xi_2(\omega), \xi_3(\omega))[1 + d(\xi_2(\omega), \xi_3(\omega))]}{1 + d(\xi_2(\omega), \xi_3(\omega))} \\ &+ b(\omega)[d(\xi_2(\omega), \xi_3(\omega)) + d(\xi_2(\omega), \xi_3(\omega))] \\ &+ c(\omega)d(\xi_2(\omega), \xi_3(\omega)) \\ &= [a(\omega) + b(\omega)]d(\xi_2(\omega), \xi_3(\omega)) \\ &+ [b(\omega) + c(\omega)]d(\xi_2(\omega), \xi_3(\omega)) \\ &\Rightarrow [1 - (a(\omega) + b(\omega))]d(\xi_2(\omega), \xi_3(\omega)) \\ &\leq [b(\omega) + c(\omega)]d(\xi_2(\omega), \xi_3(\omega)) \\ &\Rightarrow d(\xi_2(\omega), \xi_3(\omega)) \leq \frac{[b(\omega) + c(\omega)]}{[1 - (a(\omega) + b(\omega))]} d(\xi_2(\omega), \xi_3(\omega)) \\ &\Rightarrow d(\xi_2(\omega), \xi_3(\omega)) \leq Kd(\xi_2(\omega), \xi_3(\omega)) \\ &\Rightarrow d(\xi_2(\omega), \xi_3(\omega)) \leq K^2 d(\xi_2(\omega), \xi_3(\omega)), \end{aligned}$$

Similarly proceeding in the same way: by induction we produce sequence of measurable mapping  $\xi_n: \Omega \rightarrow X$  such that for any  $\omega \in \Omega$ ,

$$\begin{aligned} &\xi_{n+1}(\omega) \in T(\omega, \xi_n(\omega)) \text{ when } n = 0, 1, 2, \dots \text{ and} \\ &d(\xi_n(\omega), \xi_{n+1}(\omega)) \leq Kd(\xi_{n-1}(\omega), \xi_n(\omega)) \dots \\ &\leq K^n d(\xi_0(\omega), \xi_1(\omega)) \\ &\text{Now we shall prove that for } \omega \in \Omega, \{\xi_n\} \text{ is a Cauchy} \\ &\text{sequence. Further more } m > n \\ &d(\xi_n(\omega), \xi_m(\omega)) \leq d(\xi_n(\omega), \xi_{n+1}(\omega)) + d(\xi_{n+1}(\omega), \xi_{n+2}(\omega)) + \dots \\ &+ d(\xi_{m-1}(\omega), \xi_m(\omega)) \\ &\leq K^n d(\xi_0(\omega), \xi_1(\omega)) + K^{n+1} d(\xi_0(\omega), \xi_1(\omega)) + \dots \\ &+ K^{m-1} d(\xi_0(\omega), \xi_1(\omega)) \\ &= (K^n + K^{n+1} + \dots + K^{m-1})d(\xi_0(\omega), \xi_1(\omega)) \\ &= (1 + K + K^2 + \dots + K^{m-n-1})K^n d(\xi_0(\omega), \xi_1(\omega)) \\ &\Rightarrow d(\xi_n(\omega), \xi_m(\omega)) \leq \frac{K^n}{1 - K} d(\xi_0(\omega), \xi_1(\omega)) \end{aligned}$$

as  $n, m \rightarrow \infty, d(\xi_n(\omega), \xi_m(\omega)) \rightarrow 0$  it follows that for  $\omega \in \Omega, \{\xi_n(\omega)\}$  is a Cauchy sequence and there exists a measurable mapping  $\xi: \Omega \rightarrow X$  such that  $\xi_n(\omega) \rightarrow \xi(\omega)$  for each  $\omega \in \Omega$ . It further implies that

$$\xi_n(\omega) \rightarrow \xi(\omega) \text{ and } \xi_{n+1}(\omega) \rightarrow \xi(\omega)$$

**Existence of random fixed point.** For  $\omega \in \Omega$

$$\begin{aligned} &d(\xi(\omega), T(\omega, \xi(\omega))) \leq d(\xi(\omega), \xi_{n+1}(\omega)) \\ &+ d(\xi_{n+1}(\omega), T(\omega, \xi(\omega))) \\ &= d(\xi(\omega), \xi_{n+1}(\omega)) + d(T(\omega, \xi(\omega)), T(\omega, \xi(\omega))) \\ &\leq d(\xi(\omega), \xi_{n+1}(\omega)) \\ &+ \frac{a(\omega)d(\xi(\omega), T(\omega, \xi(\omega)))[1 + d(\xi(\omega), T(\omega, \xi(\omega)))]}{1 + d(\xi(\omega), \xi(\omega))} \\ &+ b(\omega)[d(\xi(\omega), T(\omega, \xi(\omega))) + d(\xi(\omega), T(\omega, \xi(\omega)))] \\ &+ c(\omega)d(\xi(\omega), \xi(\omega)) \\ &= d(\xi(\omega), \xi_{n+1}(\omega)) \\ &+ \frac{a(\omega)d(\xi(\omega), T(\omega, \xi(\omega)))[1 + d(\xi(\omega), \xi_{n+1}(\omega))]}{1 + d(\xi(\omega), \xi(\omega))} \\ &+ b(\omega)[d(\xi(\omega), \xi_{n+1}(\omega)) + d(\xi(\omega), T(\omega, \xi(\omega)))] \\ &+ c(\omega)d(\xi(\omega), \xi(\omega)) \end{aligned}$$

$$\begin{aligned}
 & d(\xi(\omega), T(\omega, \xi(\omega))) \leq d(\xi_0(\omega), \xi_1(\omega)) \\
 & + \frac{a(\omega)d(\xi_1(\omega), T(\omega, \xi_1(\omega)))[1+d(\xi_1(\omega), \xi_2(\omega))]}{1+d(\xi_1(\omega), \xi_2(\omega))} \\
 & + b(\omega)[d(\xi_1(\omega), \xi_2(\omega)) + d(\xi_2(\omega), T(\omega, \xi_2(\omega)))] \\
 & + c(\omega)d(\xi_2(\omega), \xi_3(\omega)) \quad [As \ n \rightarrow \infty] \\
 & = a(\omega)d(\xi_1(\omega), T(\omega, \xi_1(\omega))) + b(\omega)d(\xi_1(\omega), T(\omega, \xi_2(\omega))) \\
 & \Rightarrow [1 - (a(\omega) + b(\omega))]d(\xi_1(\omega), T(\omega, \xi_1(\omega))) \leq 0 \\
 & \Rightarrow d(\xi_1(\omega), T(\omega, \xi_1(\omega))) = 0 \quad [As \ a(\omega) + b(\omega) < 1]
 \end{aligned}$$

Hence  $\xi(\omega) \in T(\omega, \xi(\omega))$  for  $\omega \in \Omega$ .

This completes the proof of the theorem.

**Theorem: 2.2** Let  $X$  be a Polish space. Let  $T, S : \Omega \times X \rightarrow CB(X)$  be two continuous random multivalued operators. If there exists measurable mapping  $\xi : \Omega \rightarrow (0, 1)$  such that

$$\begin{aligned}
 & H(S(\omega, x), T(\omega, y)) \leq \delta \max\{d(x, y), \\
 & [d(x, S(\omega, x)) + d(y, T(\omega, y))], \\
 & [d(x, T(\omega, y)) + d(y, S(\omega, x))]\} \dots \dots \dots (1)
 \end{aligned}$$

for each then  $x, y \in X, \omega \in \Omega$  and  $0 < \delta < \frac{1}{2}$   $S, T$  have a common fixed point on  $X$  (Here  $H$  represents the Hausdroff metric on  $CB(X)$  introduced by the metric  $d$ )

**Proof :** Let  $\xi_0 : \Omega \rightarrow X$  be an arbitrary measurable mapping  $\xi : \Omega \rightarrow X$  & choose a measurable mapping such that  $\xi_1(\omega) \in S(\omega, \xi_0(\omega))$  for each  $\omega \in \Omega$ ,

$$\begin{aligned}
 & H(S(\omega, \xi_0(\omega)), T(\omega, \xi_1(\omega))) \leq \delta \max\{d(\xi_0(\omega), \xi_1(\omega)), \\
 & [d(\xi_0(\omega), S(\omega, \xi_0(\omega))) + d(\xi_1(\omega), T(\omega, \xi_1(\omega)))], \\
 & [d(\xi_0(\omega), T(\omega, \xi_1(\omega))) + d(\xi_1(\omega), S(\omega, \xi_0(\omega)))]\}
 \end{aligned}$$

It further implies [2 & 1, Lemma 2.3] then there exists a measurable mapping  $\xi_2 : \Omega \rightarrow X$  such that for any  $\omega \in \Omega, \xi_2(\omega) \in T(\omega, \xi_1(\omega))$  and

$$\begin{aligned}
 & d(\xi_1(\omega), \xi_2(\omega)) = H(S(\omega, \xi_0(\omega)), T(\omega, \xi_1(\omega))) \\
 & d(\xi_1(\omega), \xi_2(\omega)) \leq \delta \max\{d(\xi_0(\omega), \xi_1(\omega)), \\
 & [d(\xi_0(\omega), \xi_1(\omega)) + d(\xi_1(\omega), \xi_2(\omega))], \\
 & [d(\xi_0(\omega), \xi_2(\omega)) + d(\xi_1(\omega), \xi_1(\omega))]\} \\
 & = \delta \max\{d(\xi_0(\omega), \xi_1(\omega)), [d(\xi_0(\omega), \xi_1(\omega)) \\
 & + d(\xi_1(\omega), \xi_2(\omega))], [d(\xi_0(\omega), \xi_2(\omega))]\} \\
 & = \delta [d(\xi_0(\omega), \xi_1(\omega)) + d(\xi_1(\omega), \xi_2(\omega))]
 \end{aligned}$$

$$\begin{aligned}
 & \Rightarrow (1 - \delta)d(\xi_1(\omega), \xi_2(\omega)) \leq \delta d(\xi_0(\omega), \xi_1(\omega)) \\
 & \Rightarrow d(\xi_1(\omega), \xi_2(\omega)) \leq \frac{\delta}{(1 - \delta)} d(\xi_0(\omega), \xi_1(\omega)) \\
 & \Rightarrow d(\xi_1(\omega), \xi_2(\omega)) \leq K d(\xi_0(\omega), \xi_1(\omega))
 \end{aligned}$$

$$\text{where } K = \frac{\delta}{(1 - \delta)} < 1$$

By above lemma in the same manner there exists measurable mapping  $\xi_3 : \Omega \rightarrow X$  such that for any  $\omega \in \Omega, \xi_3(\omega) \in S(\omega, \xi_2(\omega))$  and

$$\begin{aligned}
 & d(\xi_2(\omega), \xi_3(\omega)) = d(\xi_3(\omega), \xi_2(\omega)) \\
 & = H(S(\omega, \xi_2(\omega)), T(\omega, \xi_3(\omega))) \\
 & \leq \delta \max\{d(\xi_2(\omega), \xi_3(\omega)), [d(\xi_2(\omega), S(\omega, \xi_2(\omega))) \\
 & + d(\xi_3(\omega), T(\omega, \xi_3(\omega))], \\
 & [d(\xi_2(\omega), T(\omega, \xi_3(\omega))) + d(\xi_3(\omega), S(\omega, \xi_2(\omega)))]\} \\
 & \Rightarrow d(\xi_2(\omega), \xi_3(\omega)) \leq \delta \max\{d(\xi_2(\omega), \xi_3(\omega)), \\
 & [d(\xi_2(\omega), \xi_3(\omega)) + d(\xi_2(\omega), \xi_2(\omega))], \\
 & [d(\xi_2(\omega), \xi_2(\omega)) + d(\xi_3(\omega), \xi_3(\omega))]\} \\
 & = \delta \max\{d(\xi_2(\omega), \xi_3(\omega)), [d(\xi_2(\omega), \xi_2(\omega)) + d(\xi_3(\omega), \xi_3(\omega))], \\
 & [d(\xi_3(\omega), \xi_3(\omega))]\} \\
 & = \delta [d(\xi_2(\omega), \xi_3(\omega)) + d(\xi_2(\omega), \xi_3(\omega))] \\
 & \Rightarrow (1 - \delta)d(\xi_2(\omega), \xi_3(\omega)) \leq \delta d(\xi_2(\omega), \xi_3(\omega)) \\
 & \Rightarrow d(\xi_2(\omega), \xi_3(\omega)) \leq \frac{\delta}{1 - \delta} d(\xi_2(\omega), \xi_3(\omega)) \\
 & = K d(\xi_2(\omega), \xi_3(\omega))
 \end{aligned}$$

$$\text{where } K = \frac{\delta}{1 - \delta} < 1 \quad [As \ 2\delta < 1]$$

$$\Rightarrow d(\xi_2(\omega), \xi_3(\omega)) \leq K^2 d(\xi_0(\omega), \xi_1(\omega))$$

Similarly, proceeding in the same way: by induction, we produce a sequence of measurable mapping  $\xi_n : \Omega \rightarrow X$  such that for  $\gamma > 0$  and any  $\omega \in \Omega$ ,

$$\begin{aligned}
 & \xi_{2\gamma+1}(\omega) \in S(\omega, \xi_{2\gamma}(\omega)), \xi_{2\gamma+2}(\omega) \in T(\omega, \xi_{2\gamma+1}(\omega)) \\
 & \text{and } d(\xi_n(\omega), \xi_{n+1}(\omega)) \leq K d(\xi_{n-1}(\omega), \xi_n(\omega)) \leq \dots \\
 & \leq K^n d(\xi_0(\omega), \xi_1(\omega))
 \end{aligned}$$

Now we shall prove that for  $\omega \in \Omega, \{\xi_n(\omega)\}$  is a Cauchy sequence, further more for  $m > n$

$$\begin{aligned}
 d(\xi_n(\omega), \xi_m(\omega)) &\leq d(\xi_n(\omega), \xi_{n+1}(\omega)) + d(\xi_{n+1}(\omega), \xi_{n+2}(\omega)) + \dots \\
 &+ d(\xi_{n-1}(\omega), \xi_n(\omega)) \\
 &\leq K^n d(\xi_0(\omega), \xi_1(\omega)) + K^{n+1} d(\xi_0(\omega), \xi_1(\omega)) + \dots \\
 &+ K^{m-1} d(\xi_0(\omega), \xi_1(\omega)) \\
 &= (K^n + K^{n+1} + \dots + K^{m-1}) d(\xi_0(\omega), \xi_1(\omega)) \\
 &= (1 + K + K^2 + \dots + K^{m-n-1}) K^n d(\xi_0(\omega), \xi_1(\omega)) \\
 &\Rightarrow d(\xi_n(\omega), \xi_m(\omega)) \leq \frac{K^n}{1-K} d(\xi_0(\omega), \xi_1(\omega))
 \end{aligned}$$

$$\begin{aligned}
 &\Rightarrow d(\xi(\omega), S(\omega, \xi(\omega))) \leq d(\xi(\omega), \xi(\omega)) + \delta \max\{d(\xi(\omega), \xi(\omega)), \\
 &[d(\xi(\omega), \xi(\omega)) + d(\xi(\omega), S(\omega, \xi(\omega))], \\
 &[d(\xi(\omega), S(\omega, \xi(\omega))) + d(\xi(\omega), \xi(\omega))]\} \\
 &\leq \delta d(\xi(\omega), S(\omega, \xi(\omega))) \\
 &\Rightarrow (1-\delta)d(\xi(\omega), S(\omega, \xi(\omega))) \leq 0 \\
 &\Rightarrow d(\xi(\omega), S(\omega, \xi(\omega))) = 0 \text{ [As } \delta < \frac{1}{2}\text{]}
 \end{aligned}$$

Hence  $\xi(\omega) \in S(\omega, \xi(\omega))$  for  $\omega \in \Omega$ .

as  $n, m \rightarrow \infty, d(\xi_n(\omega), \xi_m(\omega)) \rightarrow 0$  it follows that for  $\omega \in \Omega \{\xi_n(\omega)\}$  is a Cauchy sequence and there exists a measurable mapping  $\xi: \Omega \rightarrow X$  such that  $\xi_n(\omega) \rightarrow \xi(\omega)$  for each  $\omega \in \Omega$  it further implies that  $\xi_{2\gamma+1}(\omega) \rightarrow \xi(\omega)$  and  $\xi_{2\gamma+2}(\omega) \rightarrow \xi(\omega)$

**Existence of fixed point.** For  $\omega \in \Omega$

$$\begin{aligned}
 d(\xi(\omega), S(\omega, \xi(\omega))) &\leq d(\xi(\omega), \xi_{2\gamma+2}(\omega)) \\
 &+ d(\xi_{2\gamma+2}(\omega), S(\omega, \xi(\omega))) \\
 &= d(\xi(\omega), \xi_{2\gamma+2}(\omega)) + H(T(\omega, \xi_{2\gamma+1}(\omega)), S(\omega, \xi(\omega))) \\
 &\leq d(\xi(\omega), \xi_{2\gamma+2}(\omega)) + \delta \max\{d(\xi_{2\gamma+1}(\omega), \xi(\omega)), \\
 &[d(\xi_{2\gamma+1}(\omega), T(\omega, \xi_{2\gamma+1}(\omega))) + d(\xi(\omega), S(\omega, \xi(\omega))]\} \\
 &[d(\xi_{2\gamma+1}(\omega), S(\omega, \xi(\omega))) + d(\xi(\omega), T(\omega, \xi_{2\gamma+1}(\omega)))]\} \\
 &= d(\xi(\omega), \xi_{2\gamma+2}(\omega)) + \delta \max\{d(\xi_{2\gamma+1}(\omega), \xi(\omega)), \\
 &[d(\xi_{2\gamma+1}(\omega), \xi_{2\gamma+2}(\omega)) + d(\xi(\omega), S(\omega, \xi(\omega))]\}, \\
 &[d(\xi_{2\gamma+1}(\omega), S(\omega, \xi(\omega))) + d(\xi(\omega), \xi_{2\gamma+2}(\omega))]\}
 \end{aligned}$$

As  $\{\xi_{2\gamma+1}(\omega)\}, \{\xi_{2\gamma+2}(\omega)\}$  are sub sequences of  $\{\xi_{2\gamma}(\omega)\}$  as  $\gamma \rightarrow \infty, \xi_{2\gamma+1}(\omega) \rightarrow \xi(\omega), \xi_{2\gamma+2}(\omega) \rightarrow \xi(\omega)$

Similarly we can prove that

$$\xi(\omega) \in T(\omega, \xi(\omega)) \text{ for } \omega \in \Omega.$$

This completes the proof of the theorem.

**Corollary 2.3.** Let  $X$  be a Polish space. Let  $T, S: \Omega \times X \rightarrow CB(X)$  be two continuous random multivalued operators. If there exists measurable mapping  $a, b: \Omega \rightarrow (0, 1)$  such that

$$\begin{aligned}
 H(T(\omega, x), T(\omega, y)) &\leq \delta \max\{d(x, y), \\
 [d(x, T(\omega, x)) + d(y, T(\omega, y))], \\
 [d(x, T(\omega, y)) + d(y, T(\omega, x))]\}
 \end{aligned}$$

for each  $x, y \in X, \omega \in \Omega$  and  $0 < \delta < \frac{1}{2}$  then  $T$  has a common fixed point on  $X$  (Here  $H$  represents the Hausdorff metric on  $CB(X)$  introduced by the metric  $d$ )

**III. ACKNOWLEDGEMENT**

The authors would like to thank Professor S. S. Pagey (Institute for Excellence in Higher Education, Bhopal (M.P.) India) for constant encouragement and helpful discussions in the preparation of this paper.

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