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Effects of Blood Partial Pressure on the Human Respiratory System

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ABSTRACT: Two nonlinear coupled ordinary differential equations associated to arterial pressure and venous pressure are considered and the heart rate and ventilation function for controlling the blood pressure of the human respiratory system are used as main factors in this model. The solutions of non linear coupled ordinary differential equations are obtained numerically using fourth order Runge kutta method.

Keywords: Cardiovascular system, respiratory system, blood pressure.

I. INTRODUCTION

Gas diffuses from areas of high pressure to areas of low pressure and the rate of diffusion depends on the concentration gradient and the nature of the barrier between the two areas. When a mixture of gases is in contact with and permitted to equilibrate with a liquid, each gas in the mixture dissolves in the liquid to an extent determined by its partial pressure and its solubility in the fluid. The partial pressure of a gas in a liquid is that pressure which in the gaseous phase in equilibrium with the liquid would produce the concentration of gas molecules found in the liquid. The composition of dry air is 20.98% O_2, 0.04% CO_2, 78.06% N_2 and 0.92% other inert constituents such as argon and helium. The barometric pressure (PB) at sea level is 760 mm Hg (1 atmosphere). The partial pressure (indicated by the symbol P) of O_2 in dry air is therefore 0.21 × 760, or 160mm Hg at sea level. The partial pressure of N_2 and the other inert gases is 0.79 × 760, or 600mm Hg; and the PCO_2 is 0.0004 ×760, or 0.3mm Hg. The water vapor in the air in most climates reduces these percentages, and therefore the partial pressures, to a slight degree. Air equilibrated with water is saturated with water vapor, and inspired air is saturated by the time it reaches the lungs. The PH_2O at body temperature (37^{o}C) is 47 mm Hg. Therefore, the partial pressures at sea level of the other gases in the air reaching the lungs are PO_2, 149 mm Hg; PCO_2, 0.3 mm Hg ; and PN_2 (including the other gases), 564 mm Hg. A number of models of blood pressure of the human respiratory system have been developed since last six decades. Millhorn *et al.* (1965, 1971) developed a model for steady state simulation of the human respiratory system. Onopchuk *et al.* (1993] gave the conceptual model of respiratory system control and their analysis by mathematical modeling. Kuboyama *et al.* (1997) gave the idea of changes in cerebral blood flow velocity in healthy young men during overnight sleep and while awake . Pfeiffer *et.al* (1997) gave the analysis of heart rate and blood pressure. Ikarashi *et.al* (1998) discussed the effect of respiratory mode on human middle ear pressure. Huang *et.al* (1998) has discussed about the oxygen arterial blood gases and pressure support ventilation. Koh *et al*. (1998) discussed the tidal lung inflation to arterial pressure fluctuations. Yamakage *et al.* (1999] discussed about the changes in respiratory pattern and arterial blood gases during sedation with propofal or midiazolam in spinal anesthesia. Aittokallio *et al.* (2002) discussed the adjustment of the human respiratory system to increased upper airways resistance during sleep. Wang *et al.* (2004) discussed about the ventilator dynamic response to carbon dioxide perturbations. Ntaganda *et al.* (2007) developed a model for blood partial pressure of the human respiratory system. Ben-Tal and Smith (2008) developed a model for control breathing in mammals.

In this paper, two nonlinear coupled ordinary differential equations associated to arterial pressure and venous pressure are considered. Study of blood pressure of the human respiratory system is an important topic of human health. Blood pressure is a force exerted by circulating blood on the walls of blood vessels during each heart beat. We have used the heart rate and ventilation function for controlling the blood pressure of the human respiratory system. In the present study we deal with the problem of blood partial pressure of our body system mainly the arterial system and venous system. The solutions of the system of non linear coupled ordinary differential equations are obtained numerically using fourth order Runge kutta method. Breathing consists of two phases, inspiration and expiration. During inspiration, the diaphragm and the intercostals muscles contract. During expiration the diaphragm and muscles relax. Kappel *et al.* (1993, 1997) developed a model for human respiratory control system. They have discussed about the control of the cardiovascular – respiratory system during orthostatic stress induced by lower body negative pressure.

Kleinstreuer *et al.* (2007) developed a model for combined inertial and gravitational deposition of micro particle in small model airways of a human respiratory system.

II. FORMULATION OF PROBLEM

The problem is divided in three cases

CASE-I In this case we consider the walking stage of a person. A mathematical model is developed for determining the blood partial pressure with respect to heart rate and alveolar ventilation related to the properties of the human respiratory system. The blood flow in our body system is driven by pressure and this pressure is generated due to heart rate and pulse rate. Our body system is controlled by the level of carbon dioxide and oxygen. If H is the heart rate and VA is the alveolar ventilation, is the blood partial pressure in arterial system and P_v is the blood partial pressure in venous system. Following Ntaganda (2007) System is defined as-

$$\frac{dP_A}{dt} = -P_A + (P_V)^{\alpha} \exp\left(2.681\, V_A^{-0.0479} + 3.4921\, H^{-0.0943}\right) \quad \ldots(1)$$

$$\frac{dP_V}{dt} = -P_V + (P_A)^{\beta} \exp\left(H^{-30.7207} - 0.0981\right) \quad \ldots(2)$$

Where α and β are constants. For system stability we take $0 < \alpha\beta < 1$, α =–0.0112, β =–0.172 . Equation (1) and (2) arise from straightforward development of mass balance between systemic arterial and systemic venous compartments. These equations are obtained from basic law relating to the concentration of the gas in the solution of partial pressure. The right hand side of equations (1) and (2) use the two functions of heart rate and alveolar ventilation. These functions play an important role for controlling both the blood pressures (blood pressure in arterial system and blood pressure in venous system).

CASE- II

In this case we consider jogging state of a person. Our body system is controlled by the level of oxygen and carbon dioxide. The blood flow in our body system is induced by pressure and this pressure is generated by the heart and pulse rate. If P_{as} is the blood partial pressure in arterial system, P_{vs} is the blood partial pressure in venous system, H is the heart rate and $\dot{V}_A$ alveolar ventilation. Then the model is as follows

$$\frac{dP_{as}}{dt} = -P_{as} + (P_{vs})^{\alpha}[1.6331\dot{V}_A^{2} - 0.1091\dot{V}_A \times H - 0.0160\,] \quad \ldots(3)$$

$$\frac{dP_{vs}}{dt} = -P_{vs} + (P_{as})^{\beta}\left[0.2402(\dot{V}_A - 2.1853)^{0.6390} + H^{0.3722}\right] \quad \ldots(4)$$

Where α and β are two constants. For system stability take $0 < \alpha\beta < 1$, $\alpha = -$ 0.0112, β = –0.172. Equation (3) and (4) arise from straightforward development of mass balance between systemic arterial and systemic venous compartments. These equations are obtained from basic law relating to the concentration of the gas in the solution of partial pressure. The right hand side of equations (3) and (4) use the two functions of heart rate and alveolar ventilation.

CASE- III

In this case we consider jogging state of a person. If H is the heart rate and V_A is the alveolar ventilation, p_A is the blood partial pressure in arterial system and p_V is the blood partial pressure in venous system. The mathematical model is as follows.

$$\frac{dp_A}{dt} = -p_A + (p_V)^{\alpha} \exp\left(0.5472V_A^{0.382} + 0.7518H^{0.2846}\right) \quad (5)$$

$$\frac{dp_V}{dt} = -p_V + (p_A)^{\beta} V_A \exp\left(H^{-0.0985} - 1.7440\right) \quad (6)$$

Where α and β are constants, For system stability we take $0 < \alpha\beta < 1$, $\alpha = -$ 0.0112, $\beta = -$ 0.172.

Equation (5) and (6) arise from straightforward development of mass balance between systemic arterial and systemic venous compartments. These equations are obtained from basic laws relating to the concentration of the gas in the solution of partial pressure. The right hands side of equations (5) and (6) have two functions of heart rate and ventilation.

III. METHOD OF SOLUTION

Fourth order Runge Kutta method is used to solve above system of equation for each case. The initial values used in the model are given in table 1.

IV. RESULTS AND DISCUSSION

The numerical values of the results obtained are shown from figure 1 to figure 6. From figure it is clear the pressure in arterial system with respect to time increases in the first two minutes and in the interval from two to six minutes, the pressure in arterial system increasing rate is slow. After six minutes we see that the blood pressure in systemic arterial region is constant indicating an equilibrium stage. In figure 2 we see that the blood pressure in systemic venous region increases rapidly in first three minutes. After three minutes pressure increases slowly. After six minutes pressure is constant which represents the equilibrium stage. Figure 3 shows a relation between blood pressure in arterial system (mm Hg) and time (minute). We see that the pressure in arterial system with respect to time increases rapidly in first two minutes and in the interval from two to six minutes the pressure in arterial system increases very slowly. After five minutes the pressure in arterial system is constant i.e it is at the equilibrium stage. In figure 4 we see that the blood pressure in systemic venous region decreases in first three minutes and after that from three to six minutes pressure decreases but slowly. Pressure is constant after six minute i.e it is at equilibrium stage. From figure 5 we observed that the pressure in arterial system with respect to time increases rapidly in the first two minutes and in the interval from two to six minutes the pressure in arterial system increases very slowly. After five minutes we see that the pressure in arterial system is constant, i.e. it is in equilibrium stage. From figure 6 it is clear that the blood pressure in systemic venous region decreases rapidly in first three minutes and then from three to six minutes pressure again decreases slowly and finally pressure becomes constant.

V. CONCLUSION

In this study it is observed that heart rate and ventilation rate depends are controlling factor of the blood pressure in systemic arterial region and venous region. The model may be helpful for determining the optimum breathing capacity and in improving the treatment of the respiratory diseases.

Table 1

Name and Unit	Initial value Case-I	Initial value Case-II	Initial value Case-III
Ventilation(l/min)	8.5	15	25
Heart Rate(beats/min)	85	140	180
Arterial Pressure(mm hg)	110	135	170
Venous Pressure(mm hg)	3.47	3.28	3.23

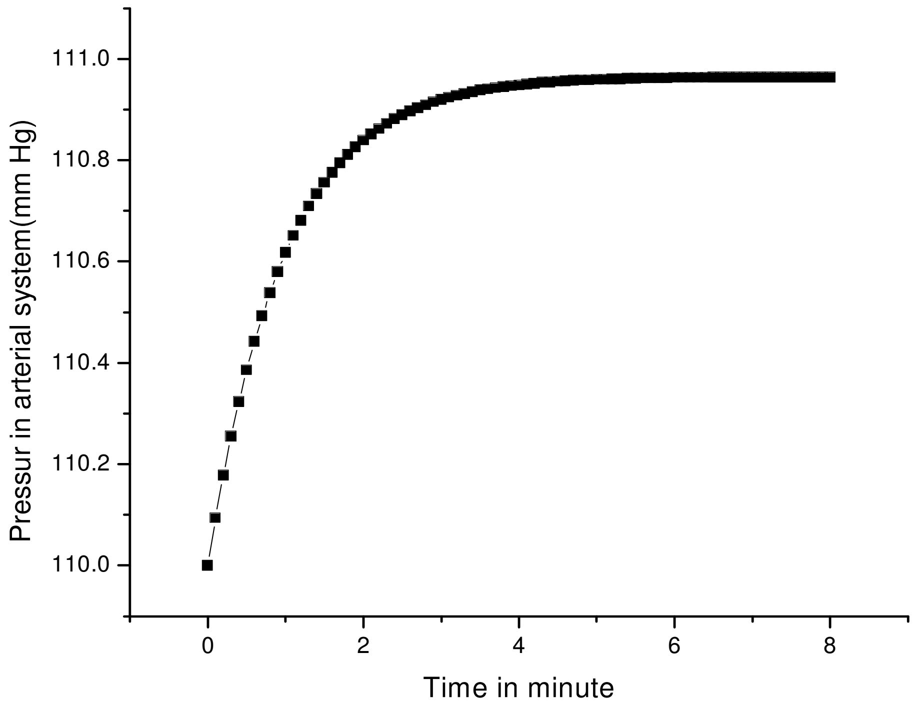

Fig. 1. Pressure in arterial system in walking stage.

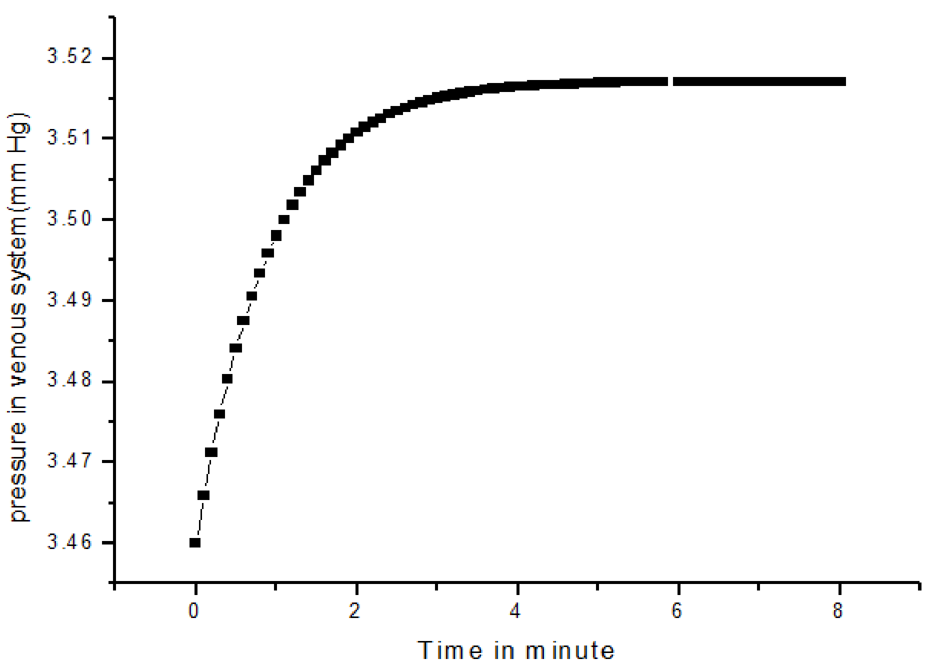

Fig. 2. Pressure in venous system in walking stage.

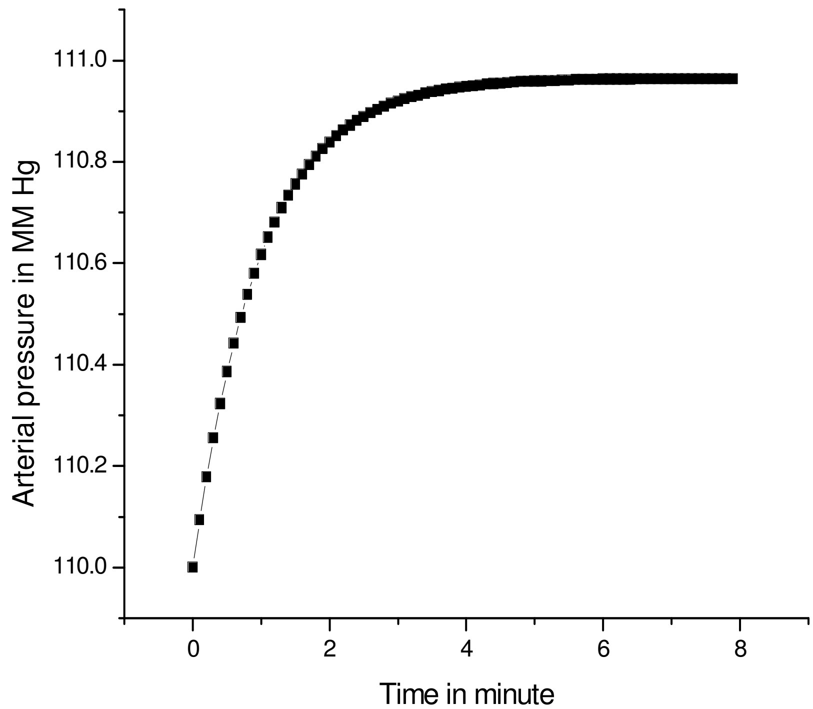

Fig. 3. Pressure in arterial system in jogging.

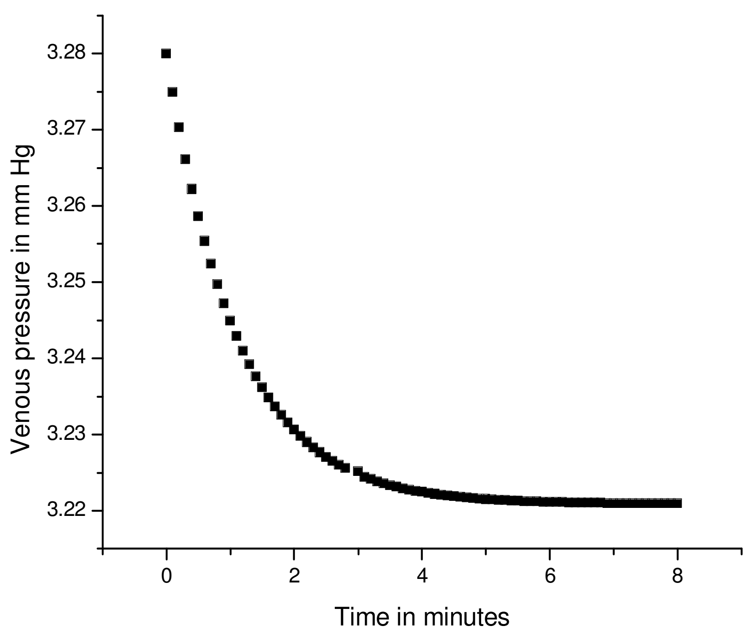

Fig. 4. Pressure in venous system in jogging stage.

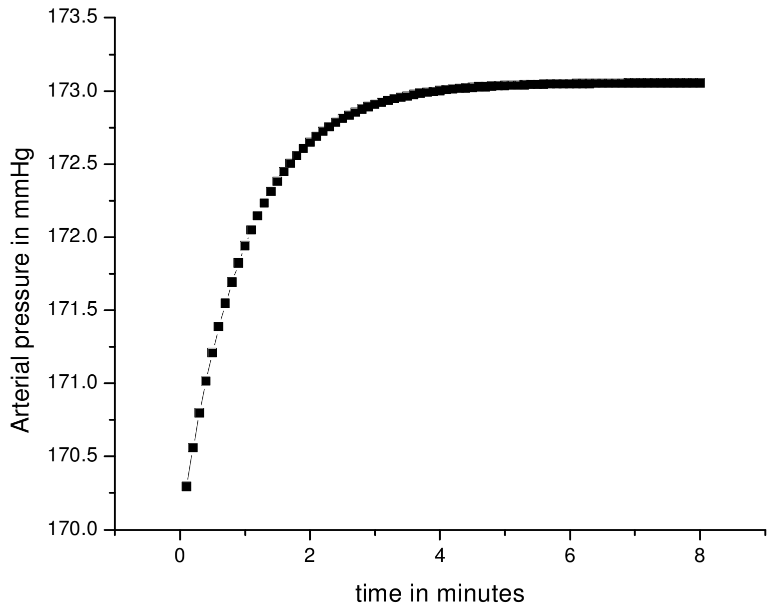

Fig. 5. Pressure in arterial system in fast running.

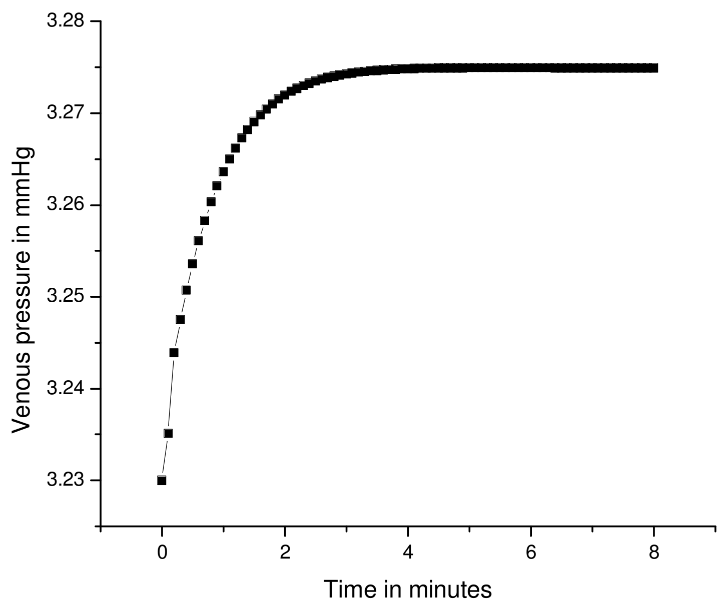

Fig. 6. Pressure in venous system in fast running.

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