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Effect of gravity fluctuation on free convection flow

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ABSTRACT : Free convection and mass transfer flow of a viscous incompressible fluid past a uniformly moving infinite vertical porous plate in a porous medium has been generalized to include the effect of gravity fluctuation. The permeability and suction velocity have also been assumed to fluctuate with time. The effect of gravity variation on the velocity, temperature, and species concentration is discussed. Small increase in gravity fluctuation parameter shows significant changes in amplitude and phase of skin-friction. This parameter further shows an increase in the value of Nusselts number and decrease in phase of heat transfer.

Keywords : Porous medium, Magneto-hydrodynamics, Heat Transfer, Free convection, Gravity fluctuation, Variable suction.

I. INTRODUCTION

The time-dependent gravitational field is of interest in space laboratory experiments, crystal growth, large scale convection of atmosphere and other related applications. Fluctuating accelerations in a spacecraft may originate from crew activities, vibrations from on-board equipment and structural oscillations. Space technologies need special attention on forces involving vibrations that occur due to interaction of several phenomena. The nature of instability occurring in a differentially heated vertical slot under a modified gravity field $g = g_0 + g_1 \cos(\omega^* t^*)$ [1]. They concluded that gravity modulation can stabilize or destabilize the flow. The effect of gravity modulation on vertical slots with both horizontal and vertical stratification were analyzed [2]. The g-jitter convection of two diffusing miscible liquids under an oscillating vertical gravitational field with zero mean was studied by Duval and Jacqmin [3]. Jules *et al* [4] found that the International Space Station (ISS), is characterized by low mean accelerations which are $O(10^{-6})g_e$–the gravity on earth and fluctuations that are two or three order of magnitude above the mean.

Sharidan et al [5] analyzed the effect of fluctuating gravity induced mixed convection, on the flow and heat transfer associated with a stretching vertical surface. Siddavaram and Homsy [6] studied the effect of g-jitter on fluid mixing. They investigated the physical mechanism induced by g-jitter to affect the mixing characteristics of two miscible fluids. Jain [7] investigated the effect of gravity modulation and viscous heating on flow past a vertical plate. She has examined the cases in slip-flow regime with periodic temperature variations and oscillatory suction. Rajvanshi and Saini [8] have investigated the effect of fluctuating gravitational field on free convection MHD flow past a uniformly moving infinite vertical porous plate with time-dependent suction velocity in a porous medium. The governing equations have been solved by a perturbation technique.

In the present study, we examine the influence of combined heat and mass transfer, permeability variation, magnetic field and gravity modulation on free convection of a viscous fluid past a uniformly moving infinite vertical porous plate with periodic temperature. The permeability of the porous medium is taken in the form $K = K_0(1+\varepsilon e^{i\omega^* t^*})$. The suction velocity is defined as $V^* = -V_0(1+\varepsilon e^{i\omega^* t^*})$ where ω^* and t^* are frequency of oscillation and time respectively. It is further assumed that $V_0 > 0$ and $\varepsilon << 1$ is a positive constant. The gravity acceleration is assumed as $g = g_0 + g_1 \cos(\omega^* t^*)$. Perturbation method has been used to obtain the solution. The variation in gravity modulation and magnetic parameter make a significant change in skin-friction and heat transfer.

II. FORMULATION OF THE PROBLEM

We consider the flow of an incompressible, viscous and electrically conducting fluid past a vertical plate bounded by a porous medium of time dependent permeability. The flow of fluid is taken along x^*-axis which is taken in the vertically upward direction. y^*-axis is assumed normal to the plate. A uniform magnetic field is introduced normal to the direction of flow. The magnetic Reynolds number is taken to be very small so that the induced magnetic field can be neglected in comparison to the applied magnetic field. The temperature difference between the wall and the medium develops buoyancy force which induces the basic flow.

Initially the plate as well as fluid are assumed to be at the same temperature and the concentration of species is very low. For $t^* > 0$, the temperature of the plate is instantaneously raised or lowered to T^*_w and the concentration of species is raised or lowered to C^*_w.

The suction velocity on the vertical plate is imposed in the form

$$V^* = -V_0^*(1+\varepsilon e^{i\omega^* t^*}) \quad \ldots(1)$$

The permeability of the porous medium is taken in the form

$$K^* = K_0(1+\varepsilon e^{i\omega^* t^*}) \quad \ldots(2)$$

The time-dependent gravitational acceleration is assumed in the form $g = g_0 + g_1\cos(\omega^* t^*)$, where g_0 is the constant gravity level in the environment, g_1 is the amplitude of the oscillating component of acceleration and ω^* is the frequency of oscillation. The gravitational acceleration is rewritten in the form

$$g = g_0 + g_1 e^{i\omega^* t^*} \quad \ldots(3)$$

where V_0^* is the suction velocity constant of the fluid through the porous surface, K_0 is the permeability constant of porous medium, U^* is the velocity of the moving vertical porous plate, C^* is the species concentration in the fluid, D is the chemical molecular diffusivity, ρ is the density of the fluid and ν is kinematic viscosity.

The physical variables are non-dimensionalized by using the following quantities

$$y = \frac{y^* V_0^*}{\upsilon}, \quad t = \frac{t^* V_0^{*2}}{4\upsilon}, \quad \omega = \frac{4\upsilon\omega^*}{V_0^{*2}}, \quad u = \frac{u^*}{V_0^*},$$

$$U = \frac{U^*}{V_0^*}, \quad C = \frac{C^* - C_\infty^*}{C_w^* - C_\infty^*}, \quad \theta = \frac{T^* - T_\infty^*}{T_w^* - T_\infty^*},$$

Grashof Number $Gr = g_0 \beta \upsilon^* \dfrac{(T_w^* - T_\infty^*)}{V_0^{*3}}$,

Prandtl Number $Pr = \dfrac{\upsilon}{(\kappa/\rho C_p)}$, Magnetic parameter $M = \dfrac{B_0}{V_0^*}\sqrt{\dfrac{\sigma\upsilon}{\rho^*}}$

Modified Grashof Number $Gc = g_0 \beta^* \nu \dfrac{(C_w^* - C_\infty^*)}{V_0^{*3}}$,

Schmidt number $Sc = \dfrac{\upsilon}{D}$,

Eckert number $Ec = \dfrac{V_0^{*2}}{C_p(T_w^* - T_\infty^*)}$, Permeability parameter $K = \dfrac{V_0^{*2} K^*}{\upsilon^2}$,

where B_0 is the magnetic field intensity, β is the coefficient of thermal expansion, β^* is the coefficient of thermal expansion with concentration, κ is the thermal conductivity, μ is the viscosity, σ is the electric permeability, T^* is the temperature, T^*_∞ is the temperature of fluid in free stream, T^*_w is the wall temperature and C_p is the specific heat at constant pressure.

The (*) stands for dimensional quantities. The subscripts ∞ and w denote the free stream condition and wall conditions respectively.

Governing equations assume the following non-dimensional form

$$\frac{1}{4}\frac{\partial u}{\partial t} - (1+\varepsilon e^{i\omega t})\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + (1 - i\varepsilon\alpha e^{i\omega t})(\theta\, Gr + C\, Gc) - M^2 u - \frac{u}{K_0(1+\varepsilon e^{i\omega t})} \quad \ldots(4)$$

$$\frac{1}{4}\frac{\partial \theta}{\partial t} - (1+\varepsilon e^{i\omega t})\frac{\partial \theta}{\partial y} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} + Ec\left(\frac{\partial u}{\partial y}\right)^2 \quad \ldots(5)$$

$$\frac{1}{4}\frac{\partial C}{\partial t} - (1+\varepsilon e^{i\omega t})\frac{\partial C}{\partial y} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} \quad \ldots(6)$$

where $g_1 = \varepsilon\alpha\, g_0$

The relevant boundary conditions are

$$u = U, \quad \theta = 1 + \varepsilon e^{i\omega t}, \quad C = 1 + \varepsilon e^{i\omega t} \quad \text{at} \quad y = 0,$$
$$u \to 0, \quad \theta \to 0, \quad C \to 0 \quad \text{as} \quad y \to 0 \quad \ldots(7)$$

III. SOLUTION OF THE GOVERNING EQUATIONS

For small amplitude oscillation ($1 < \varepsilon << 1$), the flow variables u, θ and C are represented as

$$(u,\theta,C)(y,t) = (u_0,\theta_0,C_0)(y) + \varepsilon(u_1,\theta_1,C_1)(y)e^{i\omega t} \quad \ldots(8)$$

Substituting (8) in (4) to (6) and separating steady and unsteady components, we have

$$u_0'' + u_0' - \left(M^2 + \frac{1}{K_0}\right)u_0 = -Gr\theta_0 - GcC_0 \quad \ldots(9)$$

$$u_1'' + u_1' - \left(\frac{i\omega}{4} + \frac{1}{K_0} + M^2\right)u_1 = -u_0'' - 2u_0' - (\theta_0 + \theta_1)Gr - (C_0 + C_1)Gc + i\alpha(\theta_0 Gr + C_0 Gc) + M^2 u_0 \quad \ldots(10)$$

$$\theta_0'' + \Pr\theta_0' = -Ec\Pr(u_0')^2 \quad \ldots(11)$$

$$\theta_1'' + \Pr\theta_1' - \frac{i\omega\Pr}{4}\theta_1 = -\Pr\theta_0' - 2Ec\Pr u_0' u_1' \quad \ldots(12)$$

$$C_0'' + ScC_0' = 0 \quad \ldots(13)$$

$$C_1'' + ScC_1' - \frac{i\omega\, Sc}{4}C_1 = -Sc\, C_0' \quad \ldots(14)$$

where prime denotes derivative with respect to y.

The corresponding boundary conditions (7) reduce to the form

$$u_0 = U, \quad u_1 = 0, \theta_0 = 1, \theta_1 = 1, C_0 = 1, C_1 = 1, \text{ at } y = 0$$
$$u_0 \to 0, \quad u_1 \to 0, \theta_0 \to 0, \theta_1 \to 0, \quad C_0 \to 0, \quad C_1 \to 0,$$
$$\text{as } y \to \infty \quad \ldots(15)$$

The equations (9) to (12) are still coupled for the variables u_0, u_1 and θ_0, θ_1. These equations have been solved for $Ec << 1$ using perturbation technique in the following form

$$(u_0, \theta_0, u_1, \theta_1) = (u_{00}, \theta_{00}, u_{10}, \theta_{10}) + Ec(u_{01}, \theta_{01}, u_{11}, \theta_{11}) \quad \ldots(16)$$

Finally, the expression for the velocity, temperature and concentration profiles up to first order terms in Ec are put in the following form

$$u(y,t) = u_{00}(y) + u_{01}(y)Ec + \varepsilon[u_{10}(y) + u_{11}(y)Ec]e^{i\omega t} \quad \ldots(17)$$

$$\theta(y,t) = \theta_{00}(y) + \theta_{01}(y)Ec + \varepsilon[\theta_{10}(y) + \theta_{11}(y)Ec]e^{i\omega t} \quad \ldots(18)$$

$$C(y,t) = C_0(y) + \varepsilon\, C_1(y)\, e^{i\omega t} \quad \ldots(19)$$

The analytical expressions have not been recorded here for the sake of brevity.

IV. RESULTS AND DISCUSSION

In this section, velocity field, temperature field, concentration field and skin-friction and heat transfer at the plate are discussed by assigning numerical values to various parameters appearing in the solution.

The values of Prandtl number Pr are taken as 0.71 and 11.4. The values of Schmidt number Sc for Hydrogen, Oxygen, Ammonia, Helium, Water-vapor and Propyl benzene are taken as 0.22, 0.66, 0.78, 0.30, 0.60 and 2.62 respectively. In gravity modulation $g = g_0 + g_1 \cos(\omega^* t^*)$, $g_1/g_0 = \varepsilon\alpha = 10$ is considered. The value of Eckert number Ec is taken 0.001 and $\varepsilon = 0.005$. The values of Grashof number Gr, Modified Grash of number, Gc and permeability parameter K_0 are selected arbitrarily.

A. Velocity Profiles

Fig.1 depicts the velocity profiles for various values of Sc, U, $\varepsilon\alpha$ and M. The velocity component near the plate increases with increase in gravity modulation parameter ($g_1/g_0 = \varepsilon\alpha$), and after attaining a maximum value, it start decreasing gradually. It also increases with increase in the value of U. The magnitude decreases with increase in Schmidt number Sc and magnetic parameter M. There is a shift in the point of maxima (away from the plate) with decrease in magnetic parameter M. The velocity component shows more variation in the vicinity of the plate and then decreases exponentially far away from the plate.

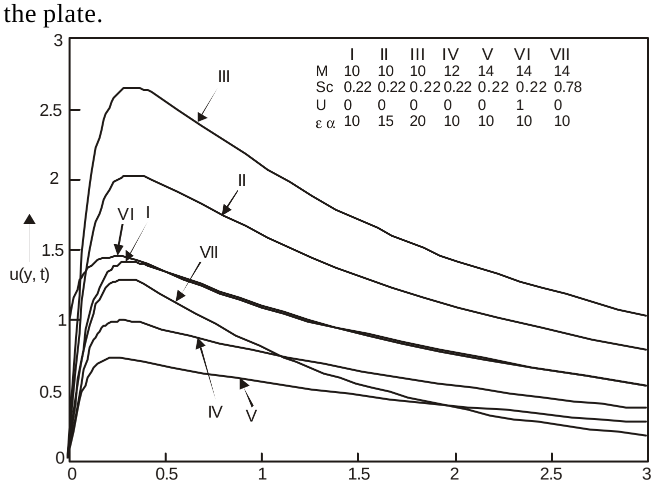

Fig. 1. Velocity profiles for Pr = 0.71, Gr = 10, Gc = 10, k_0 = 10 and ωt = π/4.

B. Temperatures Profiles

Fig. 2 exhibits the variation of temperature profiles for externally cooled plate for different values of Prandtl number Pr, $\varepsilon\alpha$, M and U. With increase in Prandtl number Pr temperature profiles decrease more rapidly for water (Pr = 11.4) in comparison to air (Pr = 0.71). Fluid temperature decreases with increase in U and M, while it increases with increase in gravity modulation parameter $\varepsilon\alpha$. There is a little effect of the velocity of the moving vertical plate on fluid temperature.

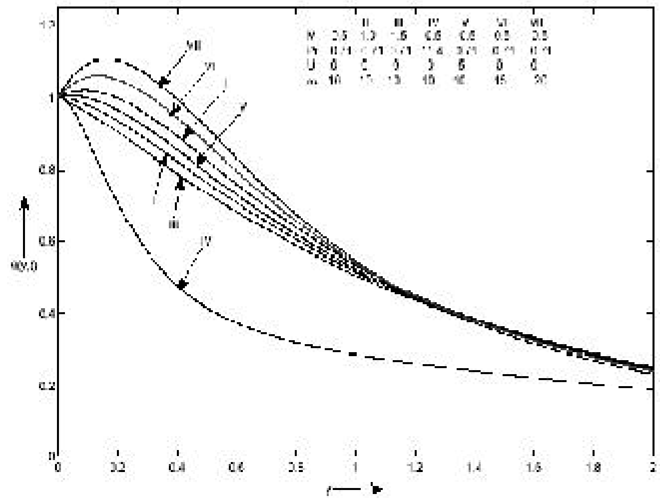

Fig. 2. Temperature profiles for 3c = 0.22, Ec = 0.001, Gr = 10, Gc = 10 and ωt = π/4.

C. Concentration profiles

Concentration profiles for different values of Schmidt number Sc at time $t = \pi/4\omega$ are shown by Fig. 3. It is observed that concentration profiles decreases with increase in Schmidt number Sc, the concentration field falls slowly and steadily. The gravity modulation parameter and magnetic parameter M do not have any impact on the concentration profiles.

D. Skin-friction

The skin-friction in the non-dimensional form on the plate $y = 0$ is given by

$$\tau = \frac{\tau_x^*}{\rho V_0^{*2}} = \tau_m + \varepsilon|N|\cos(\omega t + \phi) \quad \ldots(20)$$

where τ_m is the mean skin friction; $\varepsilon|N|$ and ϕ are amplitude and phase difference of the fluctuating component.

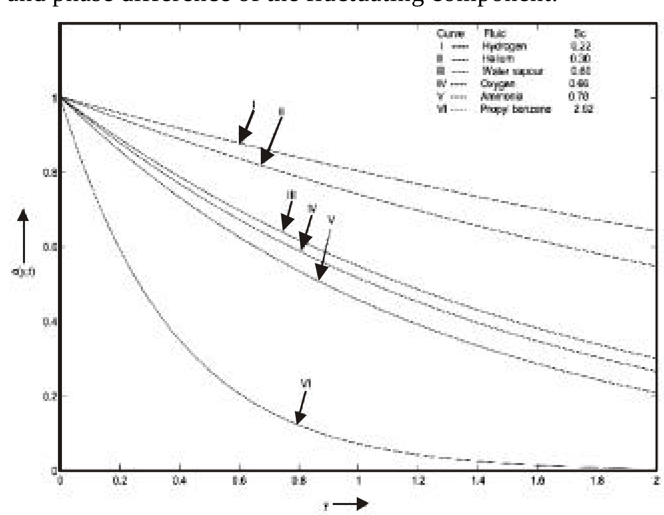

Fig. 3. Concentration profiles for ω = 30 c = 0.006 and ωt = π/4.

Fig.4 shows the effect of M, Sc, U and $\varepsilon\alpha$ on skin friction τ for $Pr = 0.71$, $Gr = 5$, $Gc = 3$, $Ec = 0.001$, $\omega = 30$ and $\varepsilon = 0.005$ at $\omega t = \pi/4$ versus K_0. It is seen that τ decreases with increase in Schmidt number Sc, magnetic parameter M and the velocity of the moving vertical porous plate U. Skin friction coefficient τ increases with increase in gravity modulation parameter $\varepsilon\alpha$. With increase in permeability parameter K_0, there is a decrease in τ. There is a significant difference in the value of with small change in magnetic parameter M.

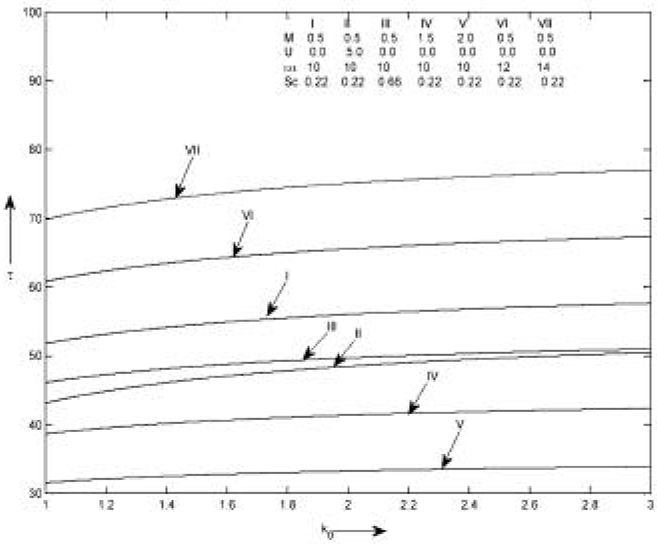

Fig. 4. Skin-friction for Gr = 5, Gc = 3, Pr = 0.71, Ec = 0.001, $\varepsilon = 0.005$, $\omega = 10$ and $\omega t = \pi/4$.

E. Coefficient of heat transfer

Heat transfer coefficient on the vertical plate in dimensionless form is given by

$$Nu = \frac{q_w^{*}\nu}{\kappa V_0^{*}(T_w^{*} - T_\infty^{*})} = -\left.\frac{\partial\theta}{\partial y}\right|_{y=0} \qquad \ldots(21)$$

The detailed analytical expression is not recorded here for the sake of brevity.

Fig. 5 show the variation of Nusselt number for different values of M, Sc, U and $\varepsilon\alpha$ with $Ec = 0.001$ and $\varepsilon = 0.005$ at $\omega t = \pi/4$ versus K_0. Nusselt number increases with increase in the value of schmidt number Sc, U and magnetic parameter M. The increase in gravity modulation parameter $\varepsilon\alpha$ leads to decrease in Nusselt number Nu. Increase in permeability parameter K_0, shows decreasing effect on Nu.

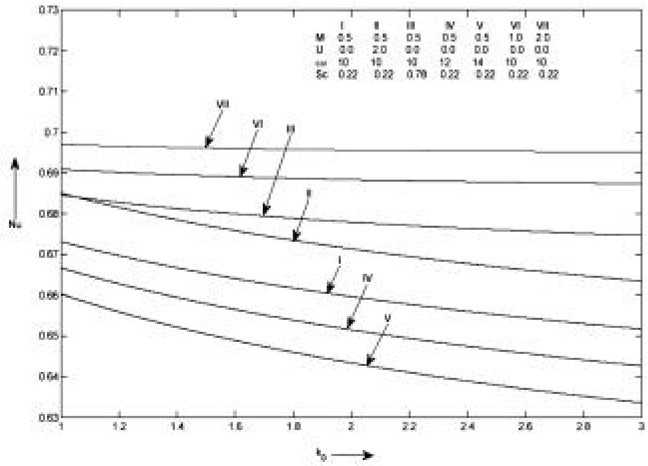

Fig. 5. Heat transfer for Gr = 5, Gc = 3, Ec = 0.001, Pr = 0.71, $\varepsilon = 0.005$, $\omega = 30$ and $\omega t = \pi/4$.

V. CONCLUSIONS

It is seen that fluid velocity and temperature increases with increase in gravity modulation parameter $\varepsilon\alpha$, while they decreases with increase in magnetic parameter M. It is also observed that skin friction coefficient τ increases with increase in gravity modulation parameter $\varepsilon\alpha$ while it decreases with increase in magnetic parameter M and the velocity of the moving vertical porous plate U. The heat transfer coefficient, Nusselt number Nu increases with increase in gravity modulation parameter $\varepsilon\alpha$. It is also seen that increase in permeability parameter K_0, Nusselt number Nu decreases.

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