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Fuzzy g*-closed sets

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ABSTRACT : In 2000 Veera Kumar [9] introduced the concepts of g*-closed sets in general topology. The present paper extends the concepts of g*-Closed sets in fuzzy topology and explore their study.

I. PRELIMINARIES

Let X be a nonempty set and I =[0,1]. A fuzzy set on X is a mapping from X Ito I. The null fuzzy set 0 on X into I which assumes only the values 0 and the whole fuzzy set 1 is a mapping from X on to I which takes the values 1 only .The union (resp. intersection) of family $\{A_a : a \in \wedge\}$ of fuzzy set of X is defined to be the mapping sup A_a (resp. inf A_a). A fuzzy set A of X is contained in a fuzzy set B of X if $A(x) \leq B(x)$ for each $x \in X$. A fuzzy point x_β in X is a fuzzy set defined by $x_\beta(y) = \beta$ for y = x and x(y) = 0 for $y \neq x$, $\beta \in [0,1]$ and $y \in X$. A fuzzy point x_β is said to be quasi-coincident with the fuzzy set A denoted by $x_{\beta q}A$ if and only if $\beta + A(x) > 1$.A fuzzy set A is quasi coincident with a fuzzy set B is denoted by $A_q B$ if and only if there exists a point $x \in X$ such that $A(x)+B(x)>1$. $A \leq B$ if and only if $\rceil A_q B^c$ [6].

A family τ of fuzzy set of X is called the fuzzy topology [4] on X if 0 and 1 belongs to τ and τ is closed with respect to arbitrary union and finite intersection. The member of τ are called fuzzy open sets and their compliment are fuzzy closed sets. For a fuzzy set A of X the closure of A (denoted by cl(A)) is the intersection of all the fuzzy closed superset of A and the interior of A (denoted by int(A)) is the union of all fuzzy open subsets of A.

Definition 1.1: A fuzzy set A of a fuzzy topological space (X,τ) is called :

(a) Fuzzy g-closed if $cl(A) \leq G$ whenever $A \leq G$ and $G \in \tau$ [7].

(b) Fuzzy g-open if 1-A is fuzzy g*-closed [7].

Definition 1.2: A mapping f: $(X,\tau) \to (Y,\Gamma)$ is said to be gc –irresolute if f(O) is fuzzy g-closed in X for every fuzzy g-closed set O in Y [8].

2. FUZZY g^*-CLOSED SETS

Definition 2.1 : A fuzzy set A of a fuzzy topological space $(X,\Im)$ is called a fuzzy g^*- closed if $cl(A) \leq O$ whenever $A \leq O$ and O is fuzzy g-open.

Remark 2.1: Every fuzzy closed set is fuzzy g^*-closed and every fuzzy g^*-closed set is fuzzy g-closed but the converses may not be true.

Example 2.1: Let X = {a, b} and $\Im = \{\mathbf{0}, \mathbf{1}, U\}$ be an fuzzy topology on X, where U(a) = 0.5,U(b) = 0.6. Then the fuzzy set defined by A(a) = 0.3, A(b) = 0.4, is fuzzy g^*-closed and the fuzzy set B defined by B(a) = 0.6,B(b) = 0.6, is fuzzy g-closed but it is not fuzzy g^*-closed.

Theorem 2.1: Let $(X,\Im)$ be a fuzzy topological space and A is a fuzzy set of X. Then A is fuzzy g^*-closed if and only if $\rceil(AqF) \Rightarrow \rceil(cl(A)qF)$ for every fuzzy g-closed set F of X.

Proof: Necessity : Let F be an fuzzy g-closed subset of X, and $\rceil(AqF)$. Then by Lemma (1.1.A), $A \leq 1\text{-}F$ and 1-F fuzzy g-open in X. Therefore $cl(A) \leq 1\text{-} F$ because A is fuzzy g^*-closed. Hence by $\rceil(cl(A)qF)$.

Sufficiency: Let O be a fuzzy g-open set of X such that $A \leq O$. Then $\rceil(A_q(1\text{-}O)$ and 1-O is an fuzzy g-closed set in X. Hence by hypothesis $\rceil(cl(A)_q(1\text{-}O)$. Therefore $cl(A) \leq O$. Hence A is fuzzy g^*-closed in X.

Theorem 2.2: Let A and B are two fuzzy g*-closed sets in a fuzzy topological space $(X,\Im)$, then $A \cup B$ is fuzzy g^*-closed.

Proof: Let O be an fuzzy g-open set in X, such that $A \cup B \leq O$. Then $A \leq O$ and $B \leq O$ and $cl(A) \leq O$ and $cl(B) \leq O$. Therefore $cl(A) \cup cl(B) = cl(A \cup B) \leq O$. Hence $A \cup B$ is fuzzy g^*-closed.

Remark 2.2: The intersection of two fuzzy g^*-closed sets in a fuzzy topological space $(X,\Im)$ may not be fuzzy g^*-closed. For

Example 2.2: Let X = {a, b} and U, A and B be the fuzzy sets of X defined as follows:

U (a)= 0.7, U(b)=0.6

A(a)= 0.6, A(b)= 0.7,

B(a)=0.8, B(b)=0.5

Let $\Im = \{\mathbf{0}, \mathbf{1}, U\}$ be an fuzzy topology on X. Then A and B are fuzzy g^*-closed in $(X,\Im)$ but $A \cap B$ is not fuzzy g-closed.

Theorem 2.3: Let and A be a fuzzy g^*-closed set in fuzzy topological space $(X,\Im)$ and $A \le B \le cl(B)$. Then B is fuzzy g^*- closed in X.

Proof: Let O be fuzzy g-open set such that $B \le O$. Then $A \le O$. Since A is fuzzy g^*-closed, $cl(A) \le O$. Now $B \le cl(A) \Rightarrow cl(B) \le cl(A) \le O$. Consequently B is fuzzy g^*-closed.

Theorem 2.4: Let $(Y, \Im_Y)$ be a subspace of a fuzzy topological space $(X,\Im)$ and A be fuzzy set in Y. If A is fuzzy g^*-closed in X then A is fuzzy g^*-closed in Y.

Proof: Let $A \le O_Y$ where O_Y is fuzzy g-open in Y. Then there exists fuzzy g-open set O in X such that $O_Y = O \cap Y$. Therefore $A \subseteq O$ and since A is fuzzy g^*-closed in X, $cl(A) \le O$. It follows that $cl_Y(A) = cl(A) \cap Y \le O \cap Y = O_Y$. Hence A is fuzzy g^*-closed in Y.

Definition 2.2: A fuzzy set A of fuzzy topological space $(X,\Im)$ is called fuzzy g^*-open if its complement $1 - A$ is fuzzy g^*-closed.

Remark 2.2: Every fuzzy open set is fuzzy g^*-open and every fuzzy g^*-open set is fuzzy g- open. But the converses may not be true.

Theorem 2.5: An fuzzy set A of a fuzzy topological space $(X,\Im)$ is fuzzy g^*-open if and only if and $F \le int(A)$ whenever F is fuzzy g-closed and $F \le A$.

Proof: Obvious.

Theorem 2.6: Let A and B are q-separated fuzzy g^*-open subsets of a fuzzy topological space $(X,\Im)$, then $A \cup B$ is fuzzy g^*-open.

Proof: Let F be a fuzzy g-closed subset of X and $F \le A \cup B$.

Then $F \cap cl(A) \le A \cup B) \cap cl(A) = (A \cap cl(A)) \cup (B \cap cl(A))$ £ Int(A). Similarly $F \cap cl(B)$ int(B).

Now $F = F \cap (A \cup B) \le (F \cap cl(A)) \cup (F \cap cl(B))$ £ $int(A) \cup int(B) \le int(A \cup B)$. Hence $F \le int(A \cup B)$ and by theorem 2.5, $A \cup B$ is fuzzy g^*-open.

Theorem 2.7: Let A and B be two fuzzy g^*-closed sets of a fuzzy topological space $(X,\Im)$ and suppose that 1 - A and 1 - B are q-separated, then $A \cap B$ is fuzzy g*-closed.

Proof: Since A^c and B^c are q-separated fuzzy g^*-open sets, by Theorem 2.6, $1 - (A \cap B) = 1 - A \cup (1 - B)$ is fuzzy g^*-open. Hence $A \cap B$ is fuzzy g*-closed.

Theorem 2.8: Let A be fuzzy g^*-open set of fuzzy topological space $(X,\Im)$ and $Int(A) \le B \le A$. Then B is fuzzy g^*-open.

Proof: Since $1-A \le 1-B \le cl(1-A)$ and 1-A is fuzzy g^*-closed it follows from theorem 2.3 that 1-B is fuzzy g-closed. Hence B is fuzzy g^*-open.

Definition 2.3: A fuzzy topological space $(X,\Im)$ is said to be fuzzy g^*-compact if every fuzzy g^*-open cover of X has a finite sub cover.

Theorem 2.9: Let $(X,\Im)$ be a fuzzy g-compact space and suppose that Y is fuzzy g^*-closed crisp subset of X, then $(Y, \Im_Y)$ is g-compact.

Proof: Let **V** be a $\Im_Y$- fuzzy g^*-open covering of Y and let $G = \{V \in \Im : V \cap Y \in \mathbf{V}\}$. Then Y £ $\cup$ G. Since Y is fuzzy g^*-closed, $cl(Y) \le \cup G$. Therefore $G \cup 1-(cl(Y))$ is a $\Im$- fuzzy g-open cover of X. Since X is fuzzy g-compact, $G \cup 1-(cl(Y))$ has a finite sub cover $\{V_1, V_2, \ldots, V_n, 1-(cl(Y))\}$. But then, $\{V_1 \cap Y, V_2 \cap Y, \ldots\ldots, V_n \cap Y\}$ is a finite sub cover of **V**.

Theorem 2.10: Let A be a fuzzy g-closed set in fuzzy topological space $(X,\Im)$, and $f: (X,\Im) \to (Y,\Im^*)$ is fuzzy gc-irresolute and fuzzy closed mapping then f(A) is an g-closed set in Y.

Proof: If $f(A) \le G$ where G is fuzzy g-open in Y then $A \le f^{-1}(G)$ and hence $cl(A) \le f^{-1}(G)$. Thus $f(cl(A)) \le G$ and $f(cl(A))$ is fuzzy closed set. It follows that $cl(A) \le cl(f(cl(A))) = f(cl(A)) \le G$. Hence $cl(f(A)) \le G$ and f(A) is fuzzy g^*-closed.

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