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Intuitionistic fuzzy sg-irresolute mappings

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ABSTRACT : The aim of this paper is to extend the concept of fuzzy sg- Irresolute mappings due to Tapi, Thakur and Rathore [14] for intuitionistic fuzzy topological spaces and some of its basic properties are studied.

Keywords : Intuitionistic fuzzy sg-closed sets and Intuitionistic fuzzy sg-open sets, Intuitionistic fuzzy sg- continuous mappings,Intuitionistic fuzzy sg- connectedness, Intuitionistic fuzzy sg-compactness.

I. INTRODUCTION

After the introduction of fuzzy sets by Zadeh [22] in 1965 and fuzzy topology by Chang [5] in 1967, several researches were conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. The concept of intuitionistic fuzzy sets was introduced by Atanassov [1] as a generalization of fuzzy sets. In the last 25 years various concepts of fuzzy mathematics have been extended for intuitionistic fuzzy sets. In 1997 Coker [6] introduced the concept of intuitionistic fuzzy topological spaces. Recently many fuzzy topological concepts such as fuzzy compactness [8], fuzzy connectedness [18], fuzzy separation axioms [4], fuzzy continuity [9] and fuzzy multifunctions [11], fuzzy g-closed[16]and fuzzy g-continuity[17] have been generalized for intuitionistic fuzzy topological spaces. The concept of sg-closed sets in fuzzy topological spaces has been discussed in [12]. Recently authors extended this concept for intuitionistic fuzzy topological spaces. In the present paper we introduce the concepts of intuitionistic fuzzy sg-irresolute mappings and obtain some of their characterization and properties.

II. PRELIMINARIES

Let X be a nonempty fixed set. An intuitionistic fuzzy set A in X is an object having the form $A = \{<x, \mu_A(x), \gamma_A(x)> : x \in X\}$, where the functions $\mu_A : X \to [0,1]$ and $\Upsilon_A : X \to [0,1]$ denotes the degree of membership $\mu_A(x)$ and the degree of non membership $\gamma_A(x)$ of each element $x \in X$ to the set A respectively and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each $x \in X$. The intuitionistic fuzzy sets $\tilde{\mathbf{0}} = \{< x, 0, 1> : x \in X\}$ and $\tilde{\mathbf{1}} = \{<x, 1, 0> : x \in X\}$ are respectively called empty and whole intuitionistic fuzzy set on X. An intuitionistic fuzzy set $A = \{<x, \mu_A(x), \gamma_A(x)> : x \in X\}$ is called a subset of an intuitionistic fuzzy set $B = \{<x, \mu_B(x), \gamma_B(x)> : x \in X\}$ (for short $A \subseteq B$) if $\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for each $x \in X$. The complement of an intuitionistic fuzzy set $A = \{< x, \mu_A(x), \gamma_A(x)> : x \in X\}$ is the intuitionistic fuzzy set $A^c = \{ < \gamma_A(x), \mu_A(x) >: x \in X\}$. The intersection (resp. union) of any arbitrary family of intuitionistic fuzzy sets $A_i = \{<x, \mu_{Ai}(x), \gamma_{Ai}(x) >: x \in X, (i \in Ù)\}$ of X be the intuitionistic fuzzy set $A_i = \{<x, \wedge \mu_{Ai}(x), \vee \gamma_{Ai}(x) > : x \in X\}$ (resp. $\cup A_i = \{< x, \vee \mu_{Ai}(x), \wedge \gamma_{Ai}(x)>: x \in X\}$). Two intuitionistic fuzzy sets $A = \{<x, \mu_A(x), \gamma_A(x)> : x \in X\}$ and $B = \{<x, \mu_B(x), \gamma_B(x) > : x \in X\}$ are said to be q-coincident ($A_q B$ for short) if and only if $\exists$ an element $x \in X$ such that $\mu_A(x) > \gamma_B(x)$ or $\gamma_A(x) < \mu_B(x)$. A family $\Im$ of intuitionistic fuzzy sets on a non empty set X is called an intuitionistic fuzzy topology [6] on X if the intuitionistic fuzzy sets $\tilde{\mathbf{0}}$, $\tilde{\mathbf{1}} \in \Im$, and $\Im$ is closed under arbitrary union and finite intersection. The ordered pair $(X, \Im)$ is called an intuitionistic fuzzy topological space and each intuitionistic fuzzy set in $\Im$ is called an intuitionistic fuzzy open set. The compliment of an intuitionistic fuzzy open set in X is known as intuitionistic fuzzy closed set. The intersection of all intuitionistic fuzzy closed sets which contains A is called the closure of A. It denoted cl(A). The union of all intuitionistic fuzzy open subsets of A is called the interior of A. It is denoted int(A) [6].

Definition 2.1[8]**:** A family $\{ G_i : i \in \wedge\}$ of intuitionistic fuzzy sets in X is called an intuitionistic fuzzy open cover of X if $\cup\{ G_i : i \in \wedge \} = \mathbf{1}$ and a finite subfamily of an intuitionistic fuzzy open cover $\{ G_i : i \in \wedge\}$of X which also an intuitionistic fuzzy open cover of X is called a finite sub cover of $\{ G_i : i \in \wedge\}$.

Definition 2.2 [8]**:** An intuitionistic fuzzy topological space $(X, \Im)$ is called fuzzy compact if every intuitionistic fuzzy open cover has a finite sub cover.

Definition 2.3[16]**:** An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space $(X,\Im)$ is called:

(a) intuitionistic fuzzy g-closed if $cl(A) \subseteq O$ whenever $A \subseteq O$ and O is intuitionistic fuzzy open.

(b) intuitionistic fuzzy g open if its complement A^c is intuitionistic fuzzy g-closed.

Remark 2.1[16]**:** Every intuitionistic fuzzy closed set is intuitionistic fuzzy g-closed but its converse may not be true.

Definition 2.4[9]**:** An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space $(X,\Im)$ is called intuitionistic fuzzy semi open (resp. intuitionistic fuzzy semi closed) if there exists a intuitionistic fuzzy open (resp.

intuitionistic fuzzy closed) U such that U ⊆ A ⊆ cl(A) (resp.int(U) ⊆ A ⊆ U)

Definition 2.5[9]**:** The semi interior and semi closure of an intuitionistic fuzzy set A of a intuitionistic fuzzy topological space (X,ℑ) respectively denoted by sint (A) and scl(A) are defined as follows:

sint (A) = ∪ { V : V⊆ A , V is intuitionistic fuzzy semi open}

scl (A) =∩ { F: A⊆ F, F is intuitionistic fuzzy semi closed}

Definition 2.6[9]**:** Let (X,ℑ) and (Y, s) be two intuitionistic fuzzy topological spaces and let *f*: X→Y be a function. Then *f* is said to be

(*a*) Intuitionistic fuzzy continuous if the pre image of each intuitionistic fuzzy open set of Y is an intuitionistic fuzzy open set in X.

(*b*) Intuitionistic fuzzy semi continuous if the pre image of each intuitionistic fuzzy open set of Y is an intuitionistic fuzzy semi open set in X.

(*c*) Intuitionistic fuzzy irresolute if the pre image of each intuitionistic fuzzy semi open set in Y is an intuitionistic fuzzy semi open set in X.

(*d*) Intuitionistic fuzzy open if image of each intuitionistic fuzzy open in X is intuitionistic fuzzy open in Y.

(*e*) Intuitionistic fuzzy semi open if image of each intuitionistic fuzzy semi open in X is intuitionistic fuzzy semi open in Y.

Definition 2.7[17]**:** Let(X,ℑ) and (Y, s) be two intuitionistic fuzzy topological spaces and let *f*:X→Y be a function. Then *f* is said to be g-continuous if pre image of every intuitionistic fuzzy closed set in Y is intuitionistic fuzzy g –closed in X.

Remark 2.2[17]**:** Every intuitionistic fuzzy continuous mapping is intuitionistic fuzzy g-continuous, but the converse may not be true.

Definition 2.8[18]**:** An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X,ℑ) is called an intuitionistic fuzzy semi generalized closed (written as intuitionistic fuzzy sg-closed) if scl(A) ⊆ O whenever A ⊆ O and O is intuitionistic fuzzy semi open.

Remark 2.3[18]: Every intuitionistic fuzzy semi closed set is intuitionistic fuzzy sg-closed but its converse may not be true.

Definition 2.9[18]**:** A collection { A_i : i∈ ?} of intuitionistic fuzzy sg- open sets in intuitionistic fuzzy topological space (X,ℑ) is called intuitionistic fuzzy sg -open cover of intuitionistic fuzzy set B of X if B ⊆ ∪{ A_i : i∈ ?}.

Definition 2.10[18]**:** An intuitionistic fuzzy topological space (X,ℑ) is said to be intuitionistic fuzzy sg -compact if every intuitionistic fuzzy sg -open cover of X has a finite sub cover.

Definition 2.11[18] **:** An intuitionistic fuzzy topological space X is intuitionistic fuzzy sg – connected if there is no proper intuitionistic fuzzy set of X which is both intuitionistic fuzzy sg- open and intuitionistic fuzzy sg- closed.

Definition 2.7[19]**:** Let(X,ℑ) and (Y, s) be two intuitionistic fuzzy topological spaces and let *f*: X→Y be a function. Then *f* is said to be intuitionistic fuzzy sg-continuous if pre image of every intuitionistic fuzzy closed set in Y is intuitionistic fuzzy sg –closed in X.

III. INTUITIONISTIC FUZZY SG-IRRESOLUTE MAPPINGS

Definition 3.1: A mapping *f* : (X,ℑ). →(Y, s) is intuitionistic fuzzy sg- irresolute if inverse image of every intuitionistic fuzzy sg-closed set of Y is intuitionistic fuzzy sg-closed set in X.

Theorem 3.1: A mapping *f* : (X,ℑ). →(Y, s) is intuitionistic fuzzy sg- irresolute if and only if the inverse image of every intuitionistic fuzzy open set of Y is intuitionistic fuzzy sg-open in X.

Proof: It is obvious because $f^{-1}(U^c) = (f^{-1}(U))^c$ for every intuitionistic fuzzy set U of Y.

Remark 3.1: Since every intuitionistic fuzzy semi closed set is intuitionistic fuzzy sg-closed, it is clear that every intuitionistic fuzzy sg-irresolute mapping is intuitionistic fuzzy sg- continuous, but converse may not be true. For,

Example 3.1: Let X= {a, b} and Y={x, y} and ℑ ={ $\tilde{0}$, $\tilde{1}$,U}and s ={ $\tilde{0}$, $\tilde{1}$ } be intuitionistic fuzzy topologies on X and Y respectively where U={< a, 0.7, 0.3> <b , 0.5, 0.5>}.Then the mapping *f*: (X,ℑ). →(Y, s) defined by f(a) = x, and f(b) = y is intuitionistic fuzzy continuous and hence intuitionistic fuzzy sg-continuous but not intuitionistic fuzzy sg-irresolute.

Example 3.2: Let X= {a, b} and Y={x, y} and ℑ = { $\tilde{0}$, $\tilde{1}$,}and s ={ $\tilde{0}$, $\tilde{1}$,V} be intuitionistic fuzzy topologies on X and Y respectively where V={< a, 0.5, 0.5> <b ,0.3,0.1>}.Then the mapping *f*: (X,ℑ). →(Y, s) defined by f(a) = x, and f(b) = y is intuitionistic fuzzy sg-irresolute but not intuitionistic fuzzy continuous.

Remark 3.2: Example (3.1) and Example (3.2) assert that concept of intuitionistic fuzzy sg- irresolute and intuitionistic fuzzy continuous mappings are independent.

Theorem 3.2: If *f*: (X,ℑ). →(Y, s) is bijective intuitionistic fuzzy semi open and intuitionistic fuzzy sg-continuous then f is intuitionistic fuzzy sg-irresolute.

Proof: Let A is intuitionistic fuzzy sg-closed in Y and let $f^{-1}(A)$ ⊆ G where G is intuitionistic fuzzy semi open in X. Then A ⊆ *f* (G). Since *f* is intuitionistic fuzzy semi open and G is intuitionistic fuzzy semi open in X, therefore *f*(G) is intuitionistic fuzzy semi open in Y. Now A is intuitionistic fuzzy sg-closed set in Y such that A ⊆ *f* (G) , therefore scl(A) ⊆ *f* (G) which implies $f^{-1}(scl(A)) \subseteq G$. Since f is intuitionistic fuzzy sg-continuous and scl(A) is intuitionistic fuzzy closed in Y then $f^{-1}(scl(A))$ is intuitionistic fuzzy sg-closed in X such that $f^{-1}(scl(A)) \subseteq G$ where G is intuitionistic fuzzy semi open in X. Therefore $scl(f^{-1}(scl(A))) \subseteq G$ and so $scl(f^{-1}((A))) \subseteq G$. Therefore $f^{-1}((A))$ is intuitionistic fuzzy sg-closed in X. Hence f is intuitionistic fuzzy sg-irresolute.

Theorem 3.3: If $f: (X,\Im) \to (Y, s)$ is intuitionistic fuzzy sg-irresolute and $g: (Y, s) \to (Z, \mu)$ be two intuitionistic fuzzy sg-continuous mapping, then $gof: (X,\Im) \to (Z,\mu)$ is intuitionistic fuzzy sg-continuous**.**

Proof: Let A is an intuitionistic fuzzy closed set in Z. then $g^{-1}(A)$ is intuitionistic fuzzy sg-closed in Y because g is intuitionistic fuzzy sg-continuous. Therefore $(gof)^{-1}(A) = f^{-1}(g^{-1}(A))$ is intuitionistic fuzzy sg – closed in X because f is intuitionistic fuzzy sg-irresolute. Hence gof is intuitionistic fuzzy sg–continuous.

Theorem 3.4: If $f: (X,\Im) \to (Y, s)$ and $g: (Y, s) \to (Z, \mu)$ be two intuitionistic fuzzy sg-irresolute mapping, then $gof: (X,\Im) \to (Z,\mu)$ is intuitionistic fuzzy sg-irresolute.

Proof : Let A is an intuitionistic fuzzy sg-closed set in Z. then $g^{-1}(A)$ is intuitionistic fuzzy sg-closed in Y, because g is intuitionistic fuzzy sg-irresolute. Therefore $(gof)^{-1}(A) = f^{-1}(g^{-1}(A))$ is intuitionistic fuzzy sg – closed in X ,because f is intuitionistic fuzzy sg-irresolute . Hence gof is intuitionistic fuzzy sg – irresolute.

Theorem 3.5: Let $f: (X,\Im) \to (Y, s)$ is intuitionistic fuzzy sg-irresolute mapping and if B is fuzzy sg-compact relative to X, then image f(B) is intuitionistic fuzzy sg- compact relative to Y.

Proof: Let $\{A_i : i \in ?\}$ be an intuitionistic fuzzy sg open set of Y such that $f(B) \subseteq \cup\{A_i : i \in ?\}$ then $B \subseteq \cup\{f^{-1}(A_i) : i \in ?\}$. By using the assumption , there exists a finite sub set $?_o$ of ? such that $B \subseteq \cup\{f^{-1}(A_i) : i \in ?_0\}$.Therefore $f(B) \cup\{A_i : i \in ?_o\}$ which shows that f(B) is intuitionistic fuzzy sg- compact relative to Y.

Theorem 3.6: A intuitionistic fuzzy sg – irresolute image of a intuitionistic fuzzy sg-compact space is intuitionistic fuzzy sg -compact.

Proof: Let. $f: (X,\Im) \to (Y, s)$ is intuitionistic fuzzy sg-irresolute map from a intuitionistic fuzzy sg-compact space $(X,\Im)$ onto a intuitionistic fuzzy topological space (Y, s). Let $\{A_i : i \in ?\}$ be an intuitionistic fuzzy open cover of Y then $\{f^{-1}(A_i) : i \in ?\}$ is a intuitionistic fuzzy sg-open cover of X. Since X is intuitionistic fuzzy sg compact it has finite intuitionistic fuzzy sub cover say $\{f^{-1}(A_1), f^{-1}(A_2)$,———— —$f^{-1}(An)$ } since f is onto $\{A_1, A_2,$ __________$A_n\}$ is an intuitionistic fuzzy sg-open cover of Y and so (Y, s) is intuitionistic fuzzy sg-compact.

Theorem 3.7 Let $(X \times Y, \Im \times s)$ be the intuitionistic fuzzy product space of non empty intuitionistic fuzzy topological spaces $(X, \Im)$ and (Y, s). Then the projection mapping P: XxY'!X is an intuitionistic fuzzy sg-irresolute.

Proof: Let F be any intuitionistic fuzzy sg-closed set of X. Then $F \times 1$ (= $P^{-1}(F)$)) is intuitionistic fuzzy sg-closed and hence P is an intuitionistic sg-irresolute.

Theorem 3.8: If the product space $(X \times Y, \Im \times s)$ of two non empty intuitionistic fuzzy topological spaces $(X, \Im)$ and (Y, s) is intuitionistic fuzzy sg-compact ,then each factor space is intuitionistic fuzzy sg-compact.

Proof : Let $(X \times Y, \Im \times s)$ is intuitionistic fuzzy sg-compact, then by Corollary (3.3), we obtain that the intuitionistic fuzzy sg-irresolute image of $p(X \times Y) = X$ is intuitionistic fuzzy sg-compact.

Theorem 3.9: If $f: (X, \Im). \to (Y, s)$ is intuitionistic fuzzy sg-irresolute surjection and $(X, \Im)$ is intuitionistic fuzzy sg-connected then (Y, s) is intuitionistic fuzzy sg-connected.

Proof: Suppose Y is not intuitionistic fuzzy sg-connected. Then there exists a proper intuitionistic fuzzy set G of Y which is both intuitionistic fuzzy sg-open and intuitionistic fuzzy sg-closed. Therefore $f^{-1}(G)$ is a proper intuitionistic fuzzy set of X, which is both intuitionistic fuzzy sg- open and intuitionistic fuzzy sg–closed, because f is intuitionistic fuzzy sg-irresolute surjection. Hence X is not intuitionistic fuzzy sg-connected, which is a contradiction. Hence Y is intuitionistic fuzzy sg-connected.

Theorem 3.10: If the product space $(X \times Y, \Im \times s)$ of two non empty intuitionistic fuzzy topological spaces $(X, \Im)$ and (Y, s) is intuitionistic fuzzy sg-connected, then each factor space is intuitionistic fuzzy sg-connected.

Proof: If $(X \times Y, \Im \times s)$ is intuitionistic fuzzy sg-connected, then mapping P: $X \times Y \to X$ is sg-irresolute. Hence by theorem 3.6 the intuitionistic fuzzy sg-irresolute image $P(X \times Y) = X$ of an intuitionistic fuzzy sg-connected space $X \times Y$ is an intuitionistic fuzzy sg-connected.

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