A Fixed Point Theorem for Continuous Mapping in Dislocated Quasi Metric Space

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ABSTRACT: In this paper we proved a fixed point theorem for Continuous contraction mapping in Dislocated Quasi Metric Space. Also we obtain a common fixed point theorem for a pair of mapping in Dislocated Metric Spaces.

Keywords: Contraction mapping, Dislocated Quasi Metric Space.

I. INTRODUCTION

Banach [1992] proved a fixed point theorem for contraction mapping in complete Metric space. It is well known as a Banach Fixed point theorem.

Every contraction mapping of a complete metric space X into itself has a unique fixed point (Bonsall 1962).

Aage and Salunke [4] proved the result on fixed point in Dislocated and Dislocated Quasi-Metric space.


Rohades [3] introduced a partial ordering for various definitions contractive mappings.

Hilzer and Seda introduced the notion of Dislocated Metric Space [8, 9] and generalized the Banach contraction principle in such spaces.


This object is to prove some fixed point theorem for continuous contraction mapping defined by Aage and Salunke [4] and Dass and Gupta [2] in Dislocated Quasi Metric Spaces.

II. PRELIMINARIES

Definition 1. Let X be a nonempty set and let $d : X \times X \to [0, \infty]$ be a function satisfying following conditions.

(i) $d(x, y) = d(y, x) = 0$ implies $y = x$

(ii) $d(x, y) < d(x, z) + d(z, y)$ for all $x, y, z \in X$.

Then d is called dislocated quasi Metric space on X, if d satisfies $d(x, y)$ then it is called dislocated metric space.

Definition 2. A sequence $[x_n]$ is dq Metric Space. (Dislocated Quasi Metric Space) $(X, d)$ is called Cauchy sequence if for $\varepsilon > 0,\exists n_0 \in N$, such that $\forall m, n > n_0$,

$d(x_m, x_n) < \varepsilon$ or $d(x_m, x_n) < \varepsilon$

i.e., $\min\{d(x_m, x_n), d(x_n, x_m)\} < \varepsilon$

Definition 3. A sequence $[x_n]$ dislocated Quasi convergence to x if

$Lt. \ n \to \infty d(x_n, x) = Lt. n \to \infty d(x, x_n) = 0$

In this case x is called a dq limit of $[X_n]$ we write $x_n \to x$.

Definition 4. Let $(X, d)$ be a dq Metric Space. A map $T : X \to X$ is called contraction if there exists $0 < \lambda < 1$ such that

$d(Ty, Tx) < \varepsilon$ or $d(Ty, Tx) < \varepsilon$

Definition 5. Let $(X, d)$ be a dq Metric Space. A map $T : X \to X$ is called contraction if there exists $0 < \lambda < 1$ such that

$d(Ty, Tx) < \lambda d(x, y) \forall x, y \in X$

Definition 6. A dq Metric Space $(X, d)$ is called complete if every cauchy sequence in it is a dq convergent.

III. MAIN RESULT

Let $(X, d)$ be a dq Metric Space and $f : X \to X$, is continuous contraction mapping. Satisfying the following condition:

$d(fx, f\delta) \leq \lambda \frac{d(y, f\delta)[1 + d(x, f\delta)]}{1 + d(x, y)}$

$+ \rho d(x, y) + \delta \frac{d(y, f\delta) + d(y, f\delta)}{1 + d(y, f\delta)} d(y, f\delta)$

$\forall x, y \in X, \lambda, \rho, \delta > 0$ and $\lambda + \rho + \delta < 1$

then f has a unique fixed point.
Proof
Let \([X_n]\) be sequence in \(X\), defined as follows :

Let \(x_0 \in X, f(x_0) = x_1, f(x_1) = x_2, \ldots, f(x_n) = x_{n+1}\)

Consider

\[
d(x_n, x_{n+1}) = d(fx_{n-1}, fx_n)
\]

\[
\leq \lambda d(x_n, fx_n) + \frac{1 + d(x_{n-1}, fx_{n-1})}{1 + d(x_{n-1}, x_n)}
\]

\[
\rho d(x_{n-1}, x_n) + \delta [d(x_n, fx_n) + d(x_{n-1}, fx_{n-1})]
\]

\[
\leq \lambda d(x_n, x_{n+1}) + \rho d(x_{n-1}, x_n)
\]

\[
d(x_n, x_{n+1}) - \lambda d(x_n, x_{n+1}) - \delta d(x_{n-1}, x_{n+1}) \leq \rho d(x_{n-1}, x_n)
\]

\[(1 - \lambda - \delta) d(x_n, x_{n+1}) \leq \rho d(x_{n-1}, x_n)\]

Let \(\alpha = \frac{\rho}{1 - \lambda - \delta}\) with \(0 \leq \alpha < 1\)

Then \(d(x_n, x_{n+1}) \leq \alpha d(x_{n-1}, x_n)\)

Similarly we get

\[
d(x_{n-1}, x_n) \leq \alpha d(x_{n-2}, x_{n-1})
\]

Then \(d(x_{n-1}, x_n) \leq \alpha^2 d(x_{n-2}, x_{n-1})\)

Continuing this process \(n\) time, then we get

\[
d(x_n, x_{n+1}) \leq \alpha^n d(x_{n-1}, x_n)
\]

Since \(0 \leq \alpha < 1, \alpha^n \to 0\) as \(n \to \infty\)

Hence \([X_n]\) is a dislocated quasi sequence converges to \(x_0\).

Thus \([X_n]\) is a dislocated quasi sequence converges to \(x_0\).

Since \(f\) is continuous then we have

\[
f(x_0) \text{Lt. } n \to \infty, f(x_n) = \text{Lt. } n \to \infty x_{n+1} = x_0
\]

Thus \(f(x_0) = x_0\)

Hence \(f\) has fixed point.

Uniqueness :
Let \(x\) be a fixed point of \(f\).

Then \(d(x, x) = d(f_x, f_x) \leq (\lambda + \rho + \delta)d(x, x)\)

Which gives \(d(x, x) = 0\), since \(0 \leq \lambda + \rho + \delta < 1\)

As \(x\) is fixed point.

Again let \(y\) be another fixed point of \(f\), i.e. \(f_y = y\)

\[
d(x, y) = d(f_x, f_y) \leq (\rho + \delta)d(x, y)
\]

Which gives \(d(x, y) \leq 0\), since \(0 \leq (\rho + \delta) < 1\)

But \(d(x, y) \geq 0\)

Hence \(d(x, y) = 0\), which implies \(x = y\).

Which is a contraction.

Thus fixed point of \(f\) is unique.

REFERENCES


