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A Fixed Point Theorem for Continuous Mapping in Dislocated Quasi Metric Space

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ABSTRACT : In this paper we proved a fixed point theorem for Continuous contraction mapping in Dislocated Quasi Metric Space Also we obtain a common fixed point theorem for a pair of mapping in Dislocated Metric Spaces.

Keywords : Contraction mapping, Dislocated Quasi Metric Space.

I. INTRODUCTION

Banach [1992] Proved a fixed point theorem for contraction mapping in complete Metric space. It is well known as a Banach Fixed point theorem.

Every contraction mapping of a complete metric space X into itself has a unique fixed point (Bonsall 1962).

Aage and Salunke [4] proved the result on fixed point in Dislocated and Dislocated Quasi-Metric space.

Dass and Gupta [2] generalized Banach's contraction principle in Metric Space.

Rohades [3] introduced a partial ordering for various definitions contractive mappings.

Hilzer and Seda introduced the notion of Dislocated Metric Space [8, 9] and generalized the Banach contraction principle in such spaces.

Zeyada *et al.* [6] generalized the result of Hitzler and Seda. Also Zoto [7] gives some new results in Dislocated and Dislocated Quasi Metric Space.

This object is to prove some fixed point theorem for continuous contraction mapping defined by Aage and Salunke [4] and Dass and Gupta [2] in Dislocated Quasi Metric Spaces.

II. PRELIMINARIES

Definition 1. Let X be a nonempty set and let $d : X \times X \to [0,\infty]$ be a function satisfying following conditions.

(i) $d(x, y) = d(y, x) = 0$ implies $y = x$

(*ii*) $d(x, y) < d(x, z) + d(z, y) \ \forall\, x, y, z \in X$.

Then d is called dislocated quasi Metric space on X, if d satisfies $d(x, y)$ then it is called dislocated metric space.

Definition 2. A sequence $[X_n]$ is dq Metric Space. (Dislocated Quasi Metric Space) (X, d) is called Cauchy sequence if for $\varepsilon > 0, \exists a n_0 \in N$, such that $\forall m, n > n_0$.

$$\Rightarrow d(x_m, x_n) < \varepsilon \text{ or } d(x_n, x_m) < \varepsilon$$

i.e., $$\min\{d(x_m, x_n), d(x_n, x_m)\} < \varepsilon$$

Definition 3. A sequence $[x_n]$ dislocated Quasi convergence to x if

$$\text{Lt. } n \to \infty d(x_n, x) = \text{Lt. } n \to \infty d(x, x_n) = 0$$

In this case x is called a dq limit of $[X_n]$ we write $x_n \to x$.

Definition 5. Let (X, d) be a dq Metric Space. A map $T : X \to X$ is called contraction if there exists $0 < x < 1$ such that

$$d(Ty, Tx) < \lambda d(x, y) \forall x, y \in X$$

Definition 6. A dq Metric Space (X, d) is called complete if every cauchy sequence in it is a dq convergent.

III. MAIN RESULT

Let (X, d) be a dq Metric Space and $f : X \to X$, is continuous contraction mapping. Satisfying the following condition :

$$d(f_x, f_y) \le \lambda \frac{d(y, f_y).[1 + d(x, f_x)]}{1 + d(x, y)} + \rho d(x, y) + \delta \frac{d(y, f_y) + d(y, f_x)}{1 + d(y, f_y).d(y, f_x)}$$

$$\forall x, y \in X, \lambda, \rho, \delta > 0 \text{ and } \lambda + \rho + \delta < 1$$

then f has a unique fixed point.

Proof

Let $[X_n]$ be sequence in X, defined as follows :

Let $x_0 \in X, f(x_0) = x_1, f(x_1) = x_2, ..., f(x_n) = x_{n+1}$

Consider

$$d(x_n, x_{n+1}) = d(fx_{n-1}, fx_n)$$

$$\leq \lambda d(x_n, fx_n)\frac{[1 + d(x_{n-1}, fx_{n-1})]}{1 + d(x_{n-1}, x_n)} +$$

$$\rho d(x_{n-1}, x_n) + \frac{\delta[d(x_n, fx_n(+d(x_n, fx_{n-1})]}{1 + d(x_n, fx_n).d(x_n, fx_{n-1})}$$

$$\leq \lambda d(x_n, x_{n+1})\frac{[1 + d(x_{n-1}, x_n)]}{1 + d(x_{n-1}, x_n)} +$$

$$\rho d(x_{n-1}, x_n) + \delta\frac{[d(x_n, x_{n+1}) + d(x_n, x_n)]}{1 + d(x_n, x_{n+1}).d(x_n, x_n)}$$

$$\leq \lambda d(x_n, x_{n+1}) + \rho d(x_{n-1}, x_n) + \delta d(x_n, x_{n+1})$$

$$d(x_n, x_{n+1}) - \lambda d(x_n, x_{n+1}) - \delta d(x_n, x_{n+1}) \leq \rho d(x_{n-1}, x_n)$$

$$(1 - \lambda - \delta)d(x_n, x_{n+1}) \leq \rho d(x_{n-1}, x_n)$$

$$(1 - \lambda - \delta)d(x_n, x_{n+1}) \leq \frac{\rho}{(1 - \lambda - \delta)}d(x_{n-1}, x_n)$$

Let $\alpha = \frac{\rho}{(1 - \lambda - \delta)}$ with $0 \leq \alpha < 1$

Then $d(x_n, x_{n+1}) \leq \alpha d(x_{n-1}, x_n)$

Similarly we get

$$d(x_{n-1}, x_n) \leq \alpha d(x_{n-2}, x_{n-1})$$

Then $d(x_n, x_{n+1}) \leq \alpha^2 d(x_{n-2}, x_{n-1})$

Continuing this process n time, then we get

$$d(x_n, x_{n+1}) \leq \alpha^n d(x_{n-1}, x_n)$$

Since $0 \leq \alpha < 1, \alpha^n \to 0$ as $n \to \infty$

Hence $[X_n]$ is a *dq* sequence in the complete dislocated Quasi Metric Space X.

Thus $[X_n]$ is a Dislocated Quasi sequence converges to x_0.

Since f is continuous then we have

$f(x_0)$Lt. $n \to \infty$, $f(x_n) =$ Lt. $n \to \infty x_{n+1} = x_0$

Thus $f(x_0) = x_0$

Hence f has fixed point.

Uniqueness :

Let x be a fixed point of f.

Then $d(x, x) = d(f_x, f_x) \leq (\lambda + \rho + \delta)d(x, x)$

Which gives $d(x, x) = 0$, since $0 \leq \lambda + \rho + \delta < 1$

As x is fixed point f.

Again let y be another fixed point of f,

i.e. $f_y = y$

$$d(x, y) = d(f_x, f_y) \leq (\rho + \delta)d(x, y)$$

Which gives $d(x, y) \leq 0,$ since $0 \leq (\rho + \delta) < 1$

But $d(x, y) \geq 0$

Hence $d(x, y) = 0$, which implies $x = y$.

Which is a contraction.

Thus fixed point of f is unique.

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