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Spectroscopy of Even-Even Isotopes of Th, U and Pu Actinide Nuclei by Using the Harris Collective Model

M. Kotb
Physics Department, Faculty of Science,
Al-Azhar University, Cairo, Egypt.

(Corresponding author: M. Kotb)
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ABSTRACT: The systematic behaviors of excitation energies, energy ratios and moments of inertia of the ground rotational bands for the even-even deformed actinide ^{228,230,232}Th, ^{230,234,236,238}U and ^{236,238,240,242,244}Pu nuclides were analyzed by using the Harris four parameters collective model based on the cranking model. The parameters of the model were obtained by performing a computer simulated search program in order to obtain a minimum root mean square deviations between the calculated and the experimental excitation energies. Good agreement is found between the calculated and the experimental data. The systematic variational of the kinematic $J^{(1)}$ and dynamic $J^{(2)}$moments of inertia are investigated as a function of notational frequency $\hbar\omega$. Both $J^{(1)}$ and $J^{(2)}$ are concave and $J^{(1)}$ is found to be smaller than that of $J^{(2)}$ for all values of $\hbar\omega$. Identical bands are observed in ^{236}U and ^{238}U isotopes, this indicate that the phenomenon of identical bands is not restricted to super-deformed bands. Also the systematic variation of the energy ratios of successive levels of the rotational bands are studied. Deformations of the nuclear shape are analyzed and the nuclear shape evolution is studied by calculating the potential energy surfaces.

I. INTRODUCTION

An extensive experimental data were obtained for ground state rotational bands of actinide even-even nuclei [1]. The nuclei in this actinide region, with their rotational level schemes are characterized by large and permanent quadruple deformation. Theoretical studies based on microscopic and phenomenological approaches were performed to study the structure of these normal deformed actinide nuclei. A good description to the positive and negative parity bands was achieved by using the cluster model [2-4], the collective model [5], the alpha decay model [6] and the interacting vector boson model [7-9]. Theoretical studies based on macroscopic and microscopic approach were reported regarding the presence of negative parity bands of a stable axial octupole deformation in the ground state of some nuclei in the actinide region [10-15]. Development of microscopic approaches, which start with the effective nucleon-nucleon interaction treated within a self –consistent Hartree-Fock (HF) method were also used [16-19].

The purpose of this work is to focus on the spectroscopy of the ground state bands of the even-even actinide Th-U-Pu isotopes by using the phenomenological four parameters Harris model. In section 2 outline of model to be used is described and the moments of inertia are described. The energy ratios of the excitation energies are presented and discussed in section 3. Section 4 is devoted to study the concept of potential energy surfaces (PES'S). Numerical calculations and discussions for describing the structure of Th-U-Pu nuclei are presented in section 5. Finally in section 6 the conclusion of this work are made.

II. OUTLINE OF HARRIS ANALYTIC FORMULA

In framework of cranked shell model (CSM), the rotational energies E within a band of an axial symmetric nucleus exhibit a smooth dependence on angular frequency of rotation ω. For spins below band crossing, E can be expanded in powers of ω^2. In particular, for the ground rotational band (GRB) of an even-even deformed nucleus the energy take the Harris form [20]:

$$E_{rot} = A\omega^2 + B\omega^4 + C\omega^6 + D\omega^8 \quad (1)$$

With the rotational angular velocity and angular momentum defined through the relations:

$$\hbar\,\omega = \frac{dE}{d\hat{I}} \quad (2)$$

with $\hat{I} = [I(I+1)]^{1/2}$ (3)

In general the above Harris expansion converges faster than the Bohr-Mottelson expansion formula [21] and in most cases the two-parameters Harris formula is sufficient to obtain a good fit to the data.

Using the definition of dynamical moment of inertia $J^{(2)}$, yield

$$\frac{J^{(2)}}{\hbar^2} = \frac{1}{\hbar^2}\frac{1}{\omega}\frac{dE}{d\omega} = 2A + 4B\omega^2 + 6C\omega^4 + 8\,D\omega^6 \quad (4)$$

Which leads to expression for nuclear spin I ,

$$\hbar\hat{I} = \int d\omega\, J^{(2)} = 2A\omega + \frac{4}{3}B\omega^3 + \frac{6}{5}C\omega^5 + \frac{8}{7}D\omega^7 \quad (5)$$

Which leads finally to expression for the kinematic moment of inertia $J^{(1)}$:

$$\frac{J^{(1)}}{\hbar^2} = \frac{\hat{I}}{\hbar\omega} = 2A + \frac{4}{3}B\omega^2 + \frac{6}{5}C\omega^4 + \frac{8}{7}D\omega^6 \quad (6)$$

If we truncate the expansion of energy at the second term only, we may obtain:

$$E = A\omega^2 + B\omega^4 \quad (7)$$

$$J^{(2)} = 2A + 4B\omega^2 \quad (8)$$

$$\hat{I} = 2A\omega + \frac{4}{3}B\omega^3 \quad (9)$$

$$J^{(1)} = 2A + \frac{4}{3}B\omega^2 \quad (10)$$

Elimination ω from equations (7) and (9), we get a cubic equation for the energy of the rotational band levels:

$$\frac{64}{9}\frac{B^2}{A^3}\frac{E^3}{I(I+1)} - \frac{32}{3}\frac{B}{A}\frac{E^2}{I(I+1)} + 4A\frac{E^2}{I(I+1)} + 12\frac{B}{A^2}E - 1 - \frac{9}{4}\frac{B}{A^3}I(I+1) = 0 \quad (11)$$

Putting

$$x = \frac{1}{12}\frac{B}{A^3}I(I+1) \quad (12)$$

We have

$$E = \frac{\hbar^2}{4A}\, I(I+1)(1-x) \quad (13)$$

If the third and the fourth terms in the master equation (1) are taken into account, then we have

$$E = \frac{\hbar^2}{4A}\, I(I+1)[1 - x + 4x^2 - 24x^3\,] \quad (14)$$

One can determine the initial values of the two parameters A and B not from all the levels but from the two low levels $E(2_1^+)$ and $E(4_1^+)$. It is easily to verify that

$$A = \frac{105}{100\,E(2_1^+) - 9E(4_1^+)} \quad (15)$$

$$B = 6A^3\,\frac{10\,E(2_1^+) - 3E(4_1^+)}{100\,E(2_1^+) - 9E(4_1^+)} \quad (16)$$

In order to obtain the rotational frequency ω, we must calculate the derivative of E will represent to spin $\hat{I}$, and for this purpose we have employed an approximate description in which E is filled with a linear function of $\hat{I}^2$ over the energy intervals between successive members of the rotational band. Thus, we obtain

$$\omega(I) = 2\,\hat{I}\,\frac{dE}{dI^2} \quad (17)$$

That is,

$$\omega^2 \cong 2\left[I_i\,(I_i+1) + I_f\,(I_f+1)\right]\left[\frac{E(I_f) - E(I_i)}{I_f\,(I_f+1) - I_i\,(I_i+1)}\right]^2 \quad (18)$$

For $I_i = I-2$ and $I_f = I$, yield

$$\omega^2 = (\,I^2 - I + 1)\left[\frac{E(I) - E(I-2)}{(2I-1)}\right]^2 \quad (19)$$

For spin $I \geq 6$, the rotational frequency take the form $\hbar\,\omega \cong [E(I) - E(I-2)]/2$

Also for kinematic moment of inertia, yields

$$J^{(1)} = \frac{1}{2}\,\left(\frac{dE}{d\hat{I}^2}\right)^{-1} = \frac{1}{2}\left[\frac{I_f\,(I_f+1) - I_i\,(I_i+1)}{E(I_f) - E(I_i)}\right]^2 \quad (20)$$

Also for $I_i = I-2$ and $I_f = I$, one get

$$2J^{(1)} = \frac{4I-2}{E(I)-E(I-2)} \qquad (21)$$

III. SUCCESSIVE ENERGY RATIOS

The energy ratios were a useful tool to describe the structure of nuclei. Knowing the excitation energies E(I) of levels with spin I, one can form the ratios

$$R((I+2)/I) = \frac{E(I+2)}{E(I)} \qquad (22)$$

It was pointed [22] that ground state bands in even-even nuclei exhibited a regular behavior when the energy ratios $E(I)/E(2_1^+)$ were plotted against $E(4_1^+)/E(2_1^+)$. The data spanned the interval $1 \leq R \leq 3.333$
For vibrational limit

$$R((I+2)/I)_{vib} = \frac{I+2}{I} \qquad (23)$$

For rotational limit

$$R((I+2)/I)_{rot} = \frac{(I+2)(I+3)}{I(I+1)} \qquad (24)$$

In vibrational and rotational limits the R((I+2)/I) energy ratios are decreasing with increasing I. the same is true for the difference

$$R((I+2)/I)_{rot} - R((I+2)/I)_{vib} = \frac{2(I+2)}{I(I+1)} \qquad (25)$$

The systematic of energy ratios of successive levels of rotational bands in even-even actinide nuclei are studied in terms of an energy ratio represents the deviation from the vibrational and rotational limits [23]:

$$r((I+2)/I) = \frac{R((I+2)/I)_{exp} - R((I+2)/I)_{vib}}{R((I+2)/I)_{rot} - R((I+2)/I)_{vib}} = \frac{R((I+2)/I)_{exp} - ((I+2)/I)}{\left(\frac{2(I+2)}{I(I+1)}\right)} \qquad (26)$$

IV. POTENTIAL ENERGY SURFACE (PES)

The collective properties of nuclei can be illustrated by calculating the potential energy surface (PES) which describes all deformation effects of the nuclei. We determine the potential energy for all the nuclei of present interest as:

$$E(N,\beta) = \frac{A_2\beta^2 + A_3\beta^3 + A_4\beta^4}{(1+\beta^2)^2} + A_o \qquad (27)$$

With A_2,A_3,A_4 and A_o are adjustable parameters and depending on the total number of the valence pairs of nucleons N. Then the equilibrium value of β is determine by vanish the first derivative

$$\frac{\partial E(N,\beta)}{\partial \beta} = 0 \qquad (28)$$

To yield $2A_2 + 3A_3\beta + (4A_4 - 2A_2)\beta^2 - A_3\beta^3 = 0$ (29)

For pure rotator the relation between the parameters are $A_2 : A_3 : A_4 \equiv 8 : 4\sqrt{2} : 1$

and in this case equation (29) becomes

$$16 + 12\sqrt{2}\beta - 12\beta^2 - 4\sqrt{2}\beta^3 = 0 \qquad (30)$$

and a deep minimum occur at $\beta = \sqrt{2}$.

V. NUMERICAL CALCULATIONS AND DISCUSSIONS

The calculations of the energies of nuclear excited states are performed with the aid of Harris expansion. In order to determine the best sets of the parameters A, B, C and D, chi-square χ^2 fits are made to the experimental levels using a computer simulated search program to minimize χ^2

$$\chi^2 = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{E^{cal}(I_i) - E^{exp}(I_i)}{\Delta E^{exp}(I_i)}\right)^2 \quad (31)$$

Where N is the number of data points considered and ΔE^{exp} is the experimental errors in the excitation energies. Our results for the best adopted model parameters are presented in table (1). The systematic behavior of excitation energies E(I), the kinematic $J^{(1)}$ and dynamic $J^{(2)}$ moments of inertia are very useful to understand the properties of ground state rotational bands. The calculated E(I) , $J^{(1)}$ and $J^{(2)}$ moments of inertia for the yrast rotational bands of our selected nuclei using the Harris model are calculated and listed in table (2). $J^{(1)}$ and $J^{(2)}$ as a function of rotational frequency $\hbar\omega$ illustrated in figure(1). The calculated energy spectra with the optimized parameters are compared with the observed ground rotational bands. A good agreement is obtained. The $J^{(1)}$ moment of inertia is found to be smaller than that of $J^{(2)}$ for all values of $\hbar\omega$. Both $J^{(1)}$ and $J^{(2)}$ are concave and each display the characteristic rise with increasing $\hbar\omega$. We see that there is a quite similarity in excitation energies and in dynamical moments of inertia $J^{(2)}$ in the ground state bands of the isotopes ^{236}U and ^{238}U up to high spin, this indicate that phenomenon of identical bands is not restricted to superdeformed nuclei. One of the best signature of a shape description is the behavior of the ratio between the excitation energy of the first excited 4_1^+ state to the excitation energy of the first excited 2_1^+ state R(4/2)= $E(4_1^+)/E(2_1^+)$ along the isotopic chain. R(4/2) varies from 2 to 3.33 corresponding to vibrational nuclides and well deformed rotors respectively. The systematic of the energy ratios R(I/2) and R(6/4) for ^{228}Th ,^{230}U and ^{244}Pu rotational nuclei are illustrated in figure(2a). To investigate the dependence of the energy ratios on the spin, the energy ratios r(I+2/I) are examined in figure (2b). A regular pattern of energy ratios are seen in rotational region. Figure (3) represents the calculated PES's, E(N, β_2) for thorium, uranium and plutonium isotopic series obtained from equation(27) plotted as function of the quadrapole deformation parameter β_2 . The figure show two wells on the prolate and oblate sides which indicate that all studied nuclei are deformed and have rotational like characters. In ^{244}Pu, the secondary potential minimum take place at β_2=1.65 while for ^{228}Th the local minimum is at β_2=0.85. The adjusted parameters of the PES's are listed in table(3).

Table 1: Values of the model parameters fitted to the yrast energies for some selected actinide nuclei.

Nucleus	A (ħ^2 MeV^{-1})	B (ħ^4 MeV^{-3})	C (ħ^6 MeV^{-5})	D (ħ^8 MeV^{-7})
^{228}Th	2.58692 × 10^1	3.86898x10^2	7.00733x10^3	8.62694x10^5
^{230}Th	2.84682 × 10^1	2.60226x10^2	3.28536x10^3	3.90011x10^5
^{232}Th	3.07987 × 10^1	2.63652x10^2	3.41209x10^3	4.69473x10^5
^{230}U	2.89348 × 10^1	3.29247x10^2	4.40103x10^3	3.53619x10^5
^{234}U	3.47087 × 10^1	3.27982x10^2	4.41870x10^3	5.86313x10^5
^{236}U	3.32796 × 10^1	2.30799x10^2	2.43785x10^3	2.77524x10^5
^{238}U	3.36447 × 10^1	2.30118x10^2	2.34700x10^3	2.54702x10^5
^{236}Pu	3.39964 × 10^1	2.02496x10^2	2.66842x10^3	5.40827x10^5
^{238}Pu	3.39314 × 10^1	2.33910x10^2	7.57355x10^3	-2.13390x10^5
^{240}Pu	3.49064 × 10^1	2.82721x10^2	1.02000x10^3	-3.13739x10^5
^{242}Pu	3.3749 × 10^1	2.18715x10^2	1.56377x10^3	5.22912x10^5
^{244}Pu	3.26673x10^1	2.25732x10^2	1.52175x10^3	1.16090x10^5

Table (2a): The Calculated kinematic $J^{(1)}$ and dynamic $J^{(2)}$ moments of inertia for $^{228\text{-}230\text{-}232}$Th.

^{228}Th			^{230}Th			^{232}Th		
ħω (MeV)	$J^{(1)}$ ($\hbar^2$ MeV^{-1})	$J^{(2)}$ ($\hbar^2$ MeV^{-1})	ħω (MeV)	$J^{(1)}$ ($\hbar^2$ MeV^{-1})	$J^{(2)}$ ($\hbar^2$ MeV^{-1})	ħω (MeV)	$J^{(1)}$ ($\hbar^2$ MeV^{-1})	$J^{(2)}$ ($\hbar^2$ MeV^{-1})
0.0286	52.3112	55.6684	0.0261	57.2562	59.1427	0.0242	61.8748	63.5132
0.0646	53.1783	63.9202	0.0600	58.3192	63.8334	0.0557	62.8005	67.6029
0.0958	57.3570	76.4292	0.0913	60.2106	71.6127	0.0853	64.4658	74.4837
0.1220	61.4459	89.9159	0.1192	62.8804	82.6809	0.1121	66.8639	84.7008
0.1443	65.8543	102.1033	0.1434	66.2200	96.4622	0.1357	69.9658	98.7532
0.1638	70.1692	136.5747	0.1641	70.0388	110.6041	0.1560	73.7023	116.4754
0.1785	75.6160		0.1822	74.0631	121.5990	0.1732	77.9427	135.7865

Table (2b): The same as Table (2a) but for $^{230-234}U$.

^{230}U			^{234}U		
$\hbar\omega$ (MeV)	$J^{(1)}$ ($\hbar^2$ MeV^{-1})	$J^{(2)}$ ($\hbar^2$ MeV^{-1})	$\hbar\omega$ (MeV)	$J^{(1)}$ ($\hbar^2$ MeV^{-1})	$J^{(2)}$ ($\hbar^2$ MeV^{-1})
0.0257	58.2603	60.5464	0.0215	69.6896	71.2859
0.0587	59.5450	66.1266	0.0495	70.5929	75.2374
0.0890	61.7810	74.9133	0.0761	72.2148	81.7444
0.1157	64.8107	86.0622	0.1006	74.5311	91.1390
0.1389	68.3647	97.3472	0.1225	77.5045	103.5732
0.1595	72.0978	107.0950	0.1418	81.0524	118.6169
0.1781	75.7658	121.1276	0.1587	85.0423	134.5080

Table (2c): The same as Table (2a) but for $^{236-238}U$.

^{236}U			^{238}U		
$\hbar\omega$ (MeV)	$J^{(1)}$ ($\hbar^2$ MeV^{-1})	$J^{(2)}$ ($\hbar^2$ MeV^{-1})	$\hbar\omega$ (MeV)	$J^{(1)}$ ($\hbar^2$ MeV^{-1})	$J^{(2)}$ ($\hbar^2$ MeV^{-1})
0.0224	66.7675	67.9925	0.0222	67.4930	68.6848
0.0518	67.4620	71.0164	0.0513	68.1689	71.6281
0.0800	68.7126	76.0152	0.0792	69.3874	76.4628
0.1063	70.5192	83.3125	0.1054	71.1429	83.4846
0.1303	72.8751	93.3184	0.1293	73.4282	93.0362
0.1517	75.7615	106.4197	0.1508	76.2220	105.4685
0.1705	79.1391	122.7295	0.1698	79.4875	120.9555

Table 2: (2d) The same as Table (2a) but for $^{236-238-240}Pu$.

^{236}Pu			^{238}Pu			^{240}Pu		
$\hbar\omega$ (MeV)	$J^{(1)}$ ($\hbar^2$ MeV^{-1})	$J^{(2)}$ ($\hbar^2$ MeV^{-1})	$\hbar\omega$ (MeV)	$J^{(1)}$ ($\hbar^2$ MeV^{-1})	$J^{(2)}$ ($\hbar^2$ MeV^{-1})	$\hbar\omega$ (MeV)	$J^{(1)}$ ($\hbar^2$ MeV^{-1})	$J^{(2)}$ ($\hbar^2$ MeV^{-1})
0.0220	68.1678	69.2125	0.0220	68.0642	69.2293	0.0214	70.0427	71.3699
0.0509	68.7609	71.8352	0.0509	68.7251	71.9592	0.0494	70.7950	74.4712
0.0787	69.8479	76.3285	0.0787	69.8669	76.0311	0.0762	72.0890	78.3116
0.1049	71.4660	83.1652	0.1050	71.4108	81.0766	0.1018	73.6496	86.6907
0.1289	73.6471	92.8893	0.1296	73.2493	86.6138	0.1249	76.0584	90.5615
0.1505	76.3665	105.5408	0.1527	75.2691	92.4428	0.1469	78.2387	100.4167
0.1694	79.658	118.8562	0.1744	77.3993	99.6884	0.1669	80.8853	

Table (2e): The same as Table (2a) but for $^{242-244}Pu$.

^{242}Pu			^{244}Pu		
$\hbar\omega$ (MeV)	$J^{(1)}$ ($\hbar^2$ MeV^{-1})	$J^{(2)}$ ($\hbar^2$ MeV^{-1})	$\hbar\omega$ (MeV)	$J^{(1)}$ ($\hbar^2$ MeV^{-1})	$J^{(2)}$ ($\hbar^2$ MeV^{-1})
0.0222	67.4915	68.6141	0.0228	65.5451	66.7712
0.0513	68.1285	71.3292	0.0528	66.2402	69.7192
0.0794	69.2586	75.6315	0.0815	67.4643	74.3275
0.1058	70.8506	81.4912	0.1084	69.1674	80.4942
0.1303	72.8533	88.7646	0.1332	71.2790	87.9468
0.1529	75.1975	97.0991	0.1560	73.7084	96.4134
0.1735	77.7972	106.0501	0.1767	76.3730	106.1683

Table 3: Values of the parameters A_2,A_3,A_4,A_o and the total number of valence pair nucleons describing the PES's for Th/ U/ Pu isotopic chains.

Nucleus	N	A_2	A_3	A_4	A_o
^{228}Th	10	-375.250	-1909.188	1987.250	-375
^{230}Th	11	-732.875	-2333.452	2154.625	-412.5
^{232}Th	12	-1159.500	-2800.142	2305.500	-450
^{230}U	11	-749.375	-2333.452	2138.125	-412.5
^{232}U	12	-1177.500	-2800.142	2287.500	-450
^{234}U	13	-1689.025	-3309.259	2405.975	-487.5
^{236}U	14	-2192.750	-3860.803	2584.750	-525
^{238}U	15	-2790.375	-4454.772	2722.125	-562.5
^{236}Pu	14	-2884	-4118.189	2212	-560
^{238}Pu	15	-3570	-4751.757	2310	-600
^{240}Pu	16	-4320	-5430.580	2400	-640
^{242}Pu	17	-5134	-6154.657	2482	-680
^{244}Pu	18	-6012	-6923.989	2556	-720

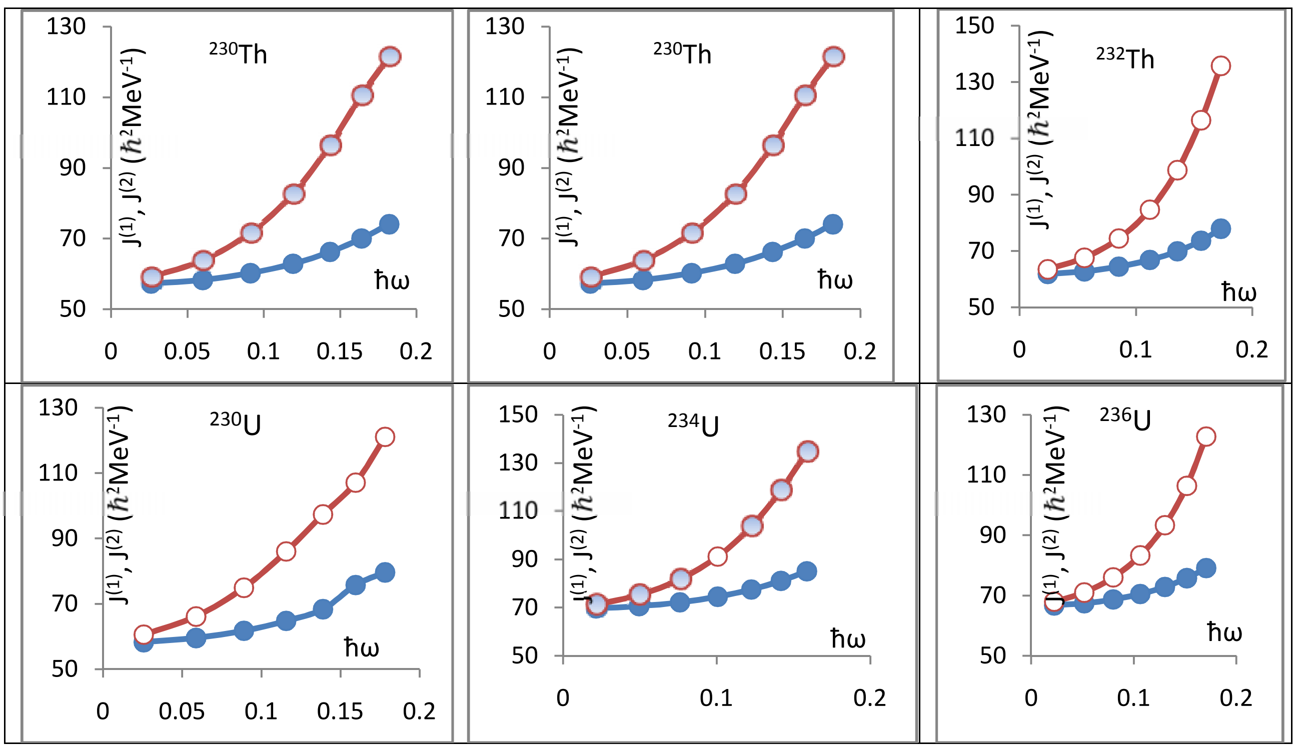

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Fig. 1. Kinematic $J^{(1)}$ (closed circle) and dynamic $J^{(2)}$ (open circle) moments of inertia are platted a function of rotational frequency ħω for the ground state bands of even-even Th-U-Pu isotopes.

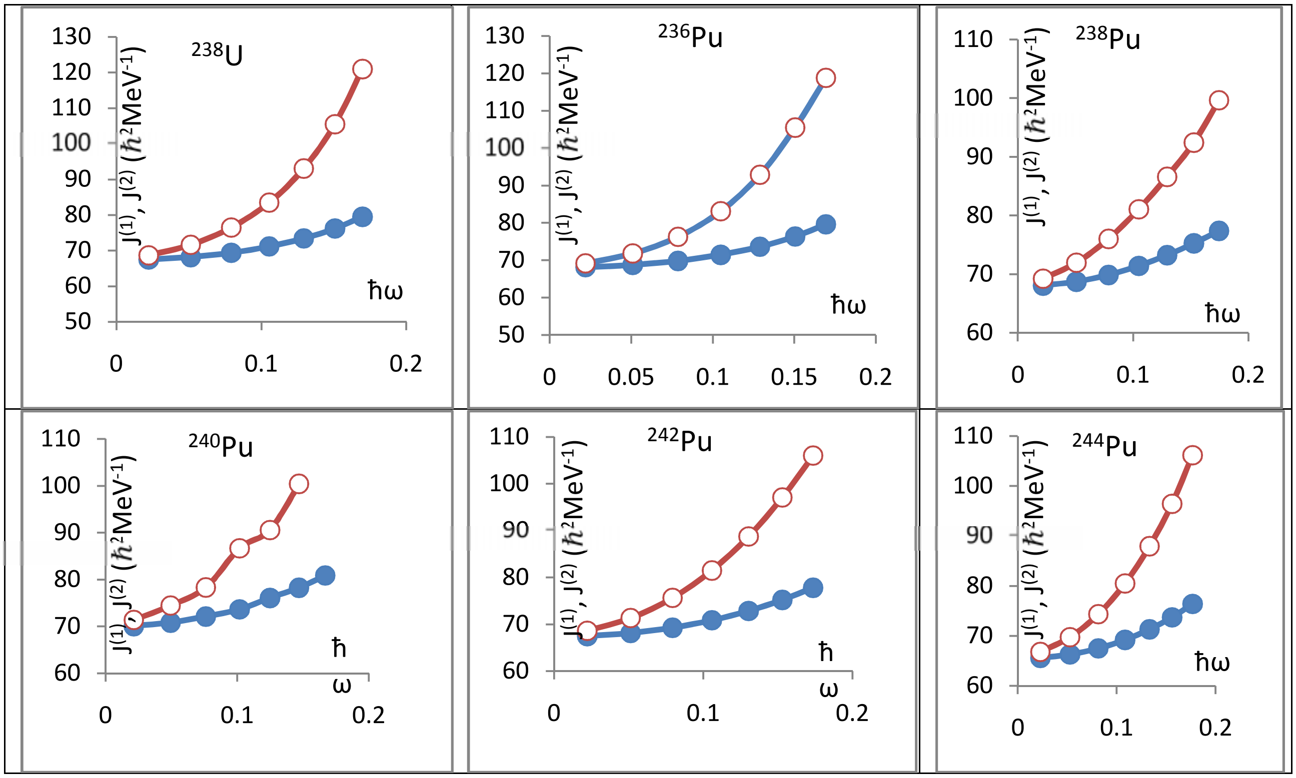

Fig. 1. Kinematic $J^{(1)}$ (closed circle) and dynamic $J^{(2)}$ (open circle) moments of inertia are platted a function of rotational frequency ħω for the ground state bands of even-even Th-U-Pu isotopes.

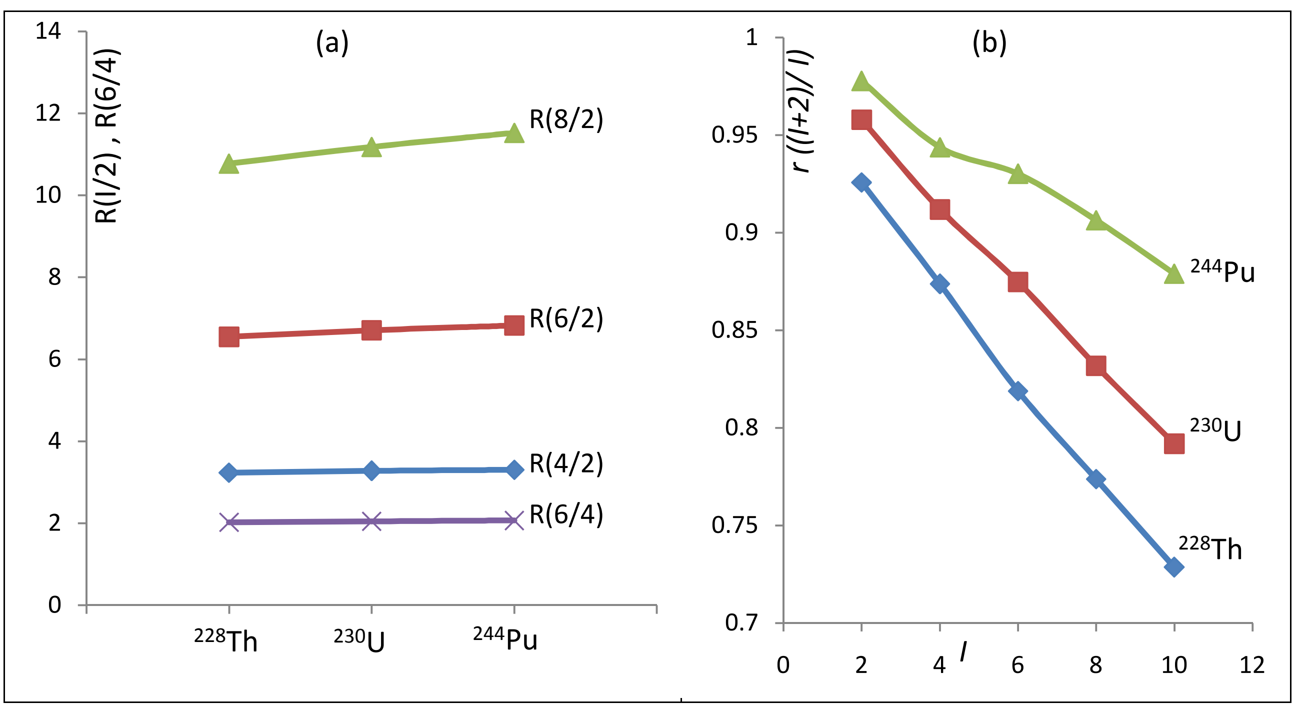

Fig. 2. (a) Systematic of the energy ratios R(I/2) and R(I/4). (b) The r((I+2)/I) energy ratios as a function of spin I for a ground state bands in ^{228}Th , ^{230}U and ^{244}Pu.

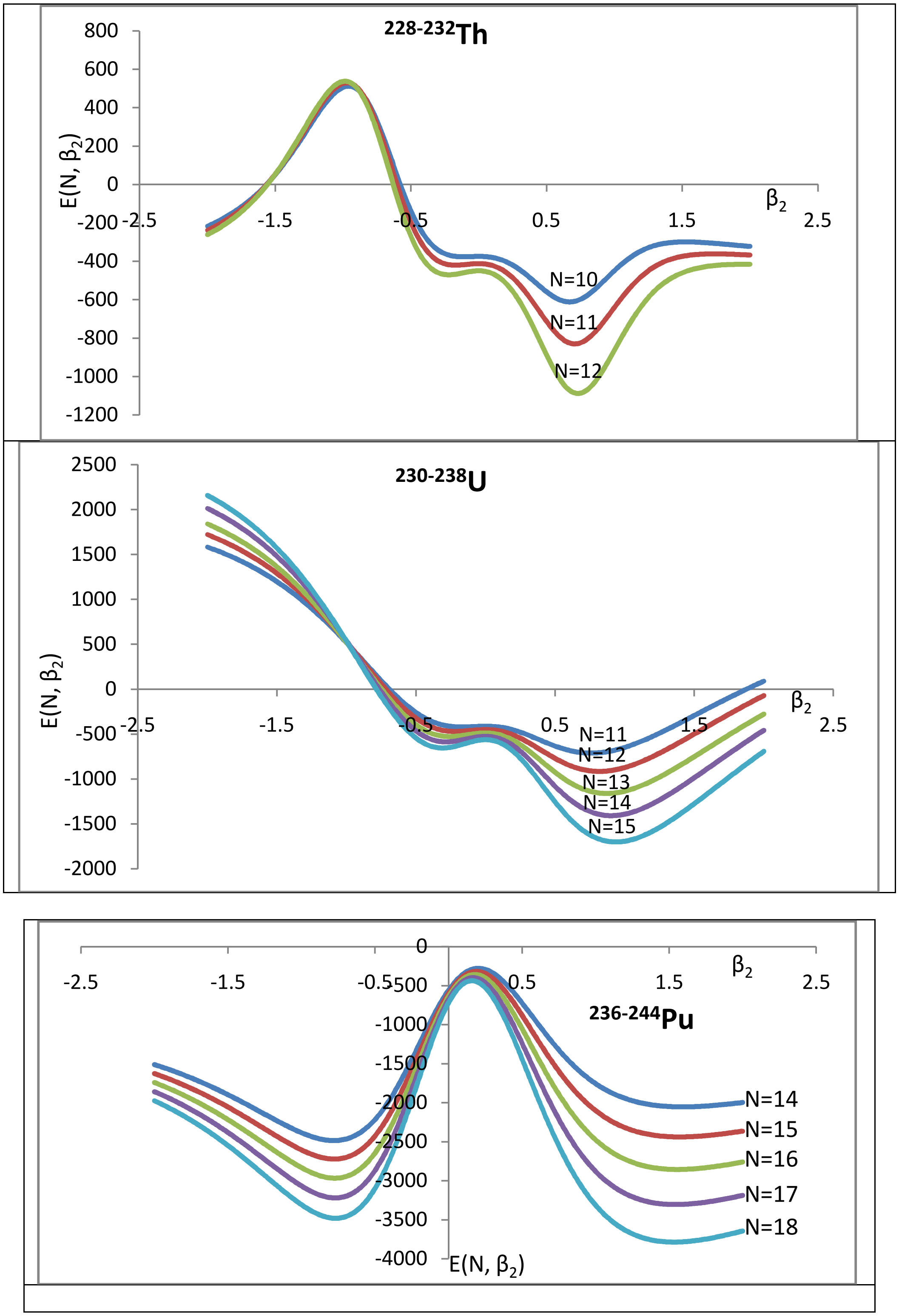

Fig. 3. Sketches of the calculated PES's , $E(N,\beta_2)$, as a function of the quadrupole deformation parameter β_2 for Th / U / Pu isotopic chains. The total number of valence pairs of nucleons are indicated in the graph.

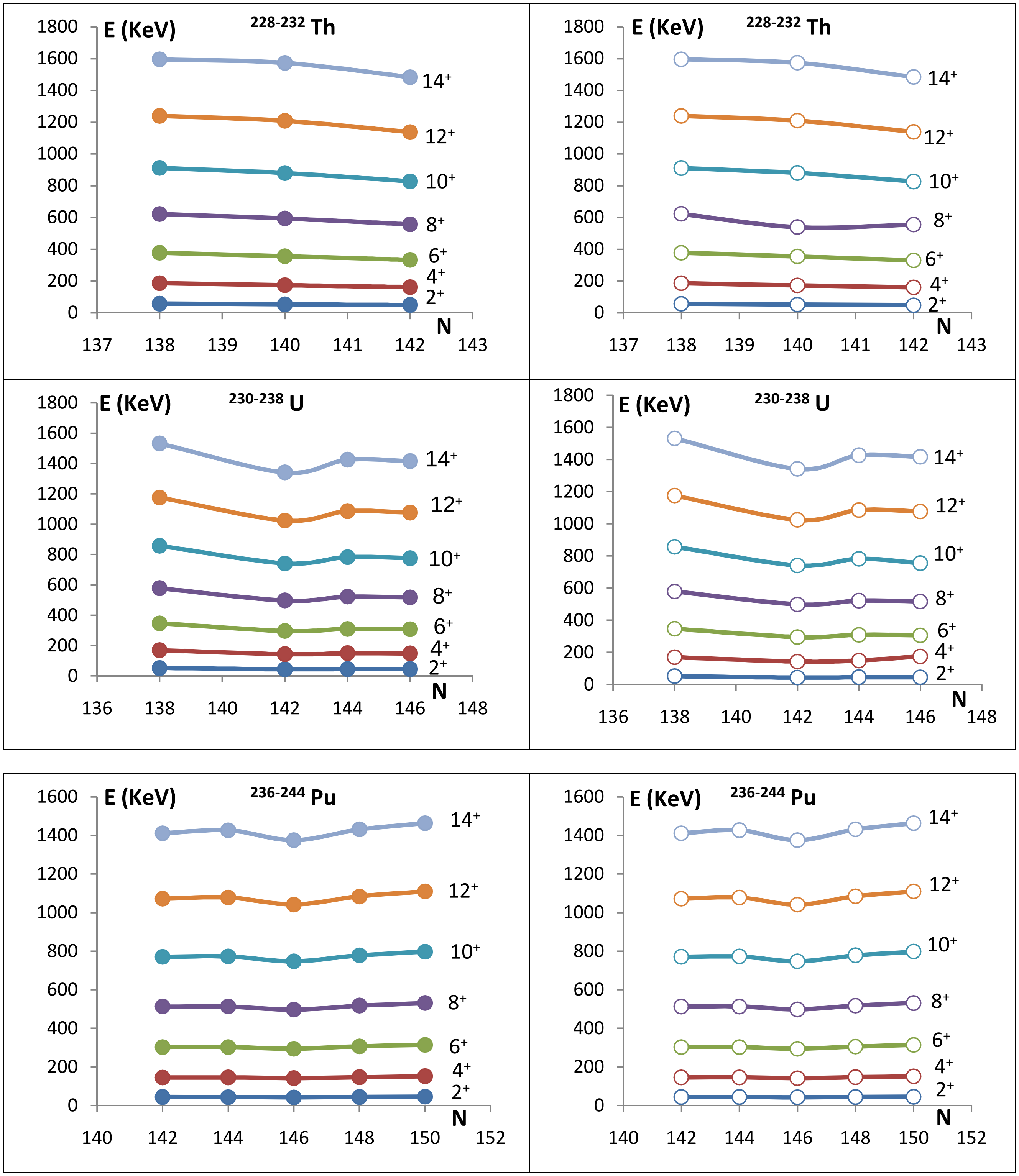

Fig. 4. Comparison of experimental level energies (closed circles) and our calculations (open circles) for the ground rotational bands in Th- U – Pu isotopic series [1].

VI. CONCLUSION

The ground state bands in Th-U-Pu nuclei have been studied in the framework of simple Harris expansion containing four terms. The optimized four parameters have been deduced by using a computer simulated search program in order to obtain a minimum root mean square division between the calculated and experimental excitation energies. The kinematic and dynamitic moments of inertia are examined. Pair of identical bands is investigated for ^{236}U and ^{238}U. The energy ratios R((I+2)/2) and the quantities r((I+2)/I) are useful tools for the characterization of collective bands. The isotopes chosen are all well deformed rotators with energy ratios R(4/2) close to 3.3. The PES's are calculated and show rotational behavior to all studied nuclei where they are mainly prolate deformed nuclei.

REFERENCES

[1]. National Nuclear Data Center NNDC Brookhaven National Laboratory[Cited on July 2012] http.//www. nndc.bnl.gov/chart/

[2]. Shneidman T.M. et.al, *Phys. Rev*. C, 2003, v.**67**, 014313.

[3]. Bulk B, Merchant A.C. and Perez S. M. , *Phys. Rev*. C,2003,v.**68**,024313.

[4]. Ganev V.P., Garistov V.P. and Georgieva A.I., *Phys. Rev*. C, 2004, v.**69**, 014305.

[5]. Balasubramaniam M. , Kumarasamy S. and Arunachalam N., *Phys. Rev.* C, 2004, v.**70**,017301.

[6]. Buck B. et al, *J. Phys*. G. 2005,v.**31**,1499.

[7]. Ghanmugam G., Sudhakar S. and Niranjani S., *Phys. Rev. C*, 2005, v.**72**, P034310.

[8]. Lenis D. and Bonatsos D, *Phys. Lett. B*, 2006, v.633,474.

[9]. Xu F.R. and Pei J.C., *Phys. Lett. B*, 2006, v.**642**,322.

[10]. Butler P.A. and Nazarewicz W. 1996, *Rev. Mod. Phys.*, v.**68**,349.

[11]. Hackman G. et al, *Physical Review* C,v.**57,** R1056-R1059.

[12]. Tsvelkov A., Kvasil J. and Nazmitdinov R.G., 2002, *J. Phys. G. Nucl. Part. Phys*., v.**28**,2187-2206.

[13]. Delaroche J.P. et al, *Nuclear Physics*, 2006,v.**771**A,103-168.

[14]. Buck B., Merchant A.C. and Perez S.M., *J. Phys. G. Nucl. Phys.* 2008,v. **35**, 085101.

[15]. Aberg S., Flocard H. and Nazarewicz W.1990, *Ann. Rev. Nucl. Part. Sci.* v.**40**,439.

[16]. Bonche P. *et al*, 1986, *Phys. Lett. B*. v.**175**,387.

[17]. Egido J. L. and Robledo L.M. 1990, *Nucl. Phys.* A v.**518**, 475.

[18]. Garrote E., Egido J. L. and Robledo L.M., 1997, *Phys. Lett. B* v.**410,** 86.

[19]. Garrote E., Egido J. L. and Robledo L.M., 1998, P*hys. Rev. Lett.*, **v.80**, 4398.

[20]. Harris S.M., *Phys. Rev.* **138**(1965) 509.

[21]. Bohr A. and Mottelson B., Nuclear Structure, Benjamin, New York, 1975, vol. **II**.

[22]. Mallmann C., *Phys. Rev. Lett.*, **2**(1969) 507.

[23]. Bonatsos D. and Skouras L. D., *The Americal Physical Socity*, **43**(1991) R 952.