International Journal of Theoretical & Applied Sciences, **5**(2): 101-108(2013)

ISSN No. (Print): 0975-1718
ISSN No. (Online): 2249-3247

A Mathematical Model for Blood flow through a Narrow Catheterized Artery

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(Received 05 November, 2013, Accepted 15 December, 2013)

ABSTRACT: The modeling of blood flow through a narrow catheterized artery has been investigated. The flowing blood is considered to be incompressible, Newtonian with variable blood viscosity. The functional dependence of blood viscosity on hematocrit (percentage of red cells) has been duly accounted for in order to improve resemblance to the real solution. It is noticed that the effective viscosity, flow rate and arterial wall shear stress in particular, in the catheterized site, are significantly altered. Flow characteristics assume its lower magnitude in catheterized artery as compared to the un-catheterized artery for any given set of parameter. Numerical results reveal that the effective viscosity in minimal magnitude and consequently the flow rate assumes its maxima magnitude during the artery catheterization at the catheter size approximately fifty percent to that of artery size.

Keywords: Catheterized artery, Hematocrit impedance, Throat, Shear stress, Stenotic region.

AMS Subject Classification (1991): 760z05, 92c35

I. INTRODUCTION

Circulatory disorders are mostly responsible for over seventy five percent of all deaths and stenosis or arteriosclerosis is one of the frequently occurring diseases. The generic medical term 'Stenosis' means a narrowing of anybody passage, tube or orifice, and is a frequently occurring cardiovascular disease in mammalian arteries. Stenosis or arteriosclerosis is the abnormal and unnatural growth in the arterial wall thickness that develops at various locations of the cardiovascular system under diseased conditions which occasionally results into serious consequences (myocardial infarction, cerebral strokes, angina pectoris, etc.). The disease seems to occur due to the deposits of the cholesterol, fatty substances, cellular waste products, calcium and fibrin in the inner lining of the of an artery.

The study of blood flow through an inserted catheter has been the subject of scientific research for a long time. Its plays an important role in the fundamental understanding, diagnosis and treatment of cardiovascular system. Like most of the problem of nature and life sciences. It is complex one due to the complicated structure of blood, the circulatory system and their constituent materials. The experimental studies and the theoretical treatment of blood flow phenomena are very useful for the diagnosis of a number of cardiovascular diseases and development of pathological patterns in human or animal physiology and for other clinical purposes and practical applications. Mathematical Modeling of blood flow has been subject to modification in order to account for the new evidence uncovered through the improved initial experimental observation (Srivastava and Srivastava, 1983).

The use of catheters is of immense importance and has become a standard tool for diagnosis and treatment in modern medicine. Transducers attached to catheters are of great use in clinical works and the technique is used for measuring blood pressure and other mechanical properties in arteries. A catheter is composed of polyester based thermoplastic polyurethane, medical grade polyvinyl chloride, etc. When a catheter is inserted into the stenosed artery, the further increased impedance or frictional resistance to flow will alter the velocity distribution. Karahalios (1990) discussed some possible effects of a catheter on the arterial wall. Back and Denton (1992) investigated some arterial wall shear stress estimates in coronary angioplasty. Srivastava and Rastogi (2009**)** discussed on **e**ffect of hematocrit on impedance and shear stress during stenosed artery catheterization. Again Srivastava and Rastogi (2010) investigated on blood flow through stenosed catheterized artery, effect of hematocrit and stenosis shape. Verma *et. al.* (2011) worked on effect of slip velocity on blood flow through a catheterized artery.

To treat arteriosclerosis in balloon angioplasty, a catheter with a tiny balloon attached at the end is inserted into the artery.

The catheter is carefully guided to the location at which stenosis occurs and the balloon is then inflated to fracture the fatty deposits and widen the narrowed portion of the artery.

II. MATHEMATICAL FORMULATION

Consider the axi- symmetric flow of blood through a narrow catheterized artery with a stenosis. The artery is assumed to be a rigid circular tube of radius R_0 and the catheter as a coaxial rigid tube of radius R_C. The artery length is considered large enough as compared to its radius so that the entrance, end and special wall effects can be neglected. The geometry of stenosis, assumed to be manifested in the arterial segment is described by:

$$\begin{aligned}\frac{R(Z)}{R_0} &= 1-\frac{2\delta}{R_0 L_0}(z-d):\ d\le z\le d+\frac{L_0}{2}\\ &= 1-\frac{\delta}{2R_0}\left[1+\cos\left(\frac{2\pi}{L_0}\right)\left(z-d-\frac{L_0}{2}\right)\right]: d\le z\le d+\frac{L_0}{2} \quad \ldots (1)\\ &= 1 \qquad \textit{otherwise}\end{aligned}$$

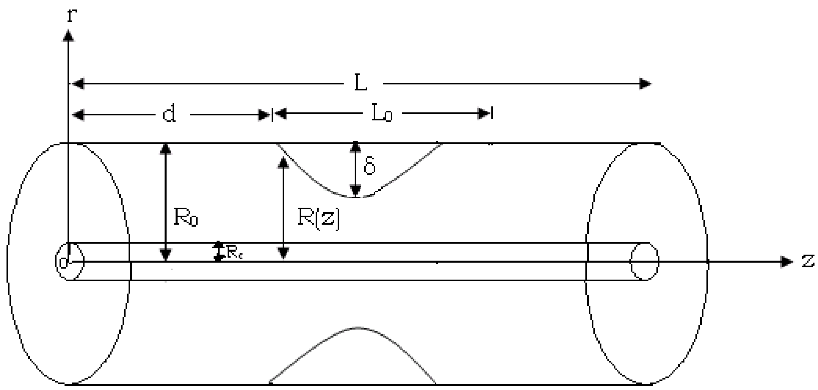

Fig. 1. Flow geometry of stenosis in a catheterized artery.

Where $R(z)$ is the radius of the tube with stenosis and R_C is the artery of the catheter, L_0 is the length of stenosis, d is the location of stenosis, and δ is the maximum height of stenosis located at $d\le z\le d+\frac{L_0}{2}$ Such that $\frac{\delta}{R_0}<<1$. It has been reported that the radial velocity is the negligibly small and can be neglected for a low Reynolds number in a tube with mild stenosis δ.

III. NUMERICAL APPROACH

The governing equation for the model is given by:

$$-\frac{dp}{dz}+\frac{\mu_C}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \mu_C}{\partial r}\right)=0 \quad \ldots (2)$$

$$\frac{dp}{dr}=0 \quad \ldots (3)$$

Where r is radial co-ordinate, u is velocity in the axial direction, p is hydraulic pressure, μ is viscosity of blood.

Blood is visco-elastic flied and its rheological properties, viscosity and elasticity depend on the rate of flow and shear stress. The blood flow in the artery is due to a constant pressure gradient along the artery axis. The artery length is assumed to be large enough as compared to its radius. So that the entrance ends the special wall effect can be neglected.

In addition, the whole blood is Newtonian fluid , at attempt to analyze the system in an exact manner is very difficult due to the complicated structure of blood and the circularly system.

Boundary conditions are:

No slip conditions are assumed at the artery and catheter wall.

$$U = 0 \text{ at } r = R(z) \text{ (artery wall)} \quad \ldots (4)$$

$$U = 0 \text{ at } r = R_c \text{ (catheter wall)} \quad \ldots (5)$$

The solution of equation (2) and (3) subject to the boundary condition (4) and (5) is given by:

$$u = -\frac{R_0^{\,2}}{4\mu}\frac{dp}{dz}\left[\left(\xi^2 - \left(\frac{r}{R_0}\right)^2 + \frac{\xi^2-\varepsilon^2}{\log\left(\frac{R}{R_C}\right)}\log\left(\frac{r}{R}\right)\right)\right] \qquad \ldots (6)$$

where non dimensional catheter radius $\xi = \frac{R}{R_0}$ *and* $\varepsilon = \frac{R}{R_0}$

The non dimensional volumetric flow rate is given by:

$$Q = \int_{R_C}^{R} 2\pi\, r\, u\, du = -\frac{\pi}{8\mu} R_0^{\,4}\left\{\xi^2-\varepsilon^2\right\}\frac{dp}{dz}\left[\xi^2+\varepsilon^2-\frac{\left(\xi^2-\varepsilon^2\right)}{\log\left\{\frac{\xi}{\varepsilon}\right\}}\right] \qquad \ldots (7)$$

From equation (7) one now obtains with the help of equation (4) and (5), we obtain as.

$$\frac{dp}{dz} = \frac{8\mu Q}{\pi R_0^{\,4}}\phi(z) \quad with \quad \phi(z) = \frac{1}{C(z)} \qquad where \quad C(z) = \left\{\xi^2-\varepsilon^2\right\}\left[\xi^2+\varepsilon^2-\frac{\left(\xi^2-\varepsilon^2\right)}{\log\left\{\frac{\xi}{\varepsilon}\right\}}\right]$$

The pressure drop across the stenosis in the artery of length, L is obtained as:

$$\Delta\, p = \int_0^L\left(-\frac{dp}{dz}\right)dz = \frac{8\mu Q}{\pi R_0^{\,4}}\psi \quad \text{where} \quad \psi = \int_0^d\left[\phi(z)\right]_{\xi=1}dz + \int_d^{d+L_0}\left[\phi(z)\right]_{\xi=1}dz + \int_{d+L_0}^{L}\left[\phi(z)\right]_{\xi=1}dz \qquad \ldots (8)$$

The first and third integration in the expression for ψ obtained above are straight forward. The expression for the impedance λ, the wall shear stress distribution in the stenotic region τ_W, shear stress at the stenosis throat τ_S, in their non-dimensional form as:

$$\lambda = \frac{1-\frac{L_0}{L}}{\eta} + \frac{L_0}{2\pi L}\int_0^{2\pi}\frac{da}{\left(b^2-\varepsilon^2\right)\left[b^2+\varepsilon^2-\frac{\left(b^2-\varepsilon^2\right)}{\log\frac{b}{\varepsilon}}\right]} \qquad \ldots (9)$$

$$w = \frac{\xi}{\left\{\xi^2-\varepsilon^2\right\}\left[\xi^2+\varepsilon^2-\left\{\frac{\xi^2-\varepsilon^2}{\log\left(\frac{\xi}{\varepsilon}\right)}\right\}\right]} \qquad \ldots (10)$$

$$\tau_S = \frac{\theta}{\left\{\theta^2-\varepsilon^2\right\}\left[\theta^2+\varepsilon^2-\left\{\frac{\theta^2-\varepsilon^2}{\log\frac{\theta}{\varepsilon}}\right\}\right]} \qquad \ldots (11)$$

where

$$a = 1 - \frac{\delta}{2R},\ \theta = 1 - \frac{\delta}{2R_0},\ b = b(\alpha) = a + \theta\cos\alpha,\ \alpha = \pi - \left(\frac{2\pi}{L_0}\right)\left(z - d - \frac{L_0}{2}\right)$$

$$\eta = (1-\xi^2)\left\{1 + \xi^2 + \frac{(1-\xi^2)}{\log\varepsilon}\right\},\ \lambda = \frac{\bar{\lambda}}{\lambda_0},\ (\tau_W, \tau_S) = \frac{\bar{\tau}_W.\bar{\tau}_S}{\tau_0},\ \lambda_0 = \frac{8\mu L}{\pi R_0{}^4},\ \tau_0 = \frac{4\mu Q}{\pi R_0{}^3}$$

are the flow resistance and shear stress, respectively for a Newtonian fluid in a normal artery and $\bar{\lambda}$, $\bar{\tau}_w$, $\bar{\tau}_s$ are the impedance, wall shear stress and shear stress at stenosis throat respectively in their dimensional form obtained from the definitions:

$$\bar{\lambda} = \frac{\Delta p}{Q},\ \bar{\tau}_w = \frac{-R}{2}\left(\frac{dp}{dz}\right), \bar{\tau}_s = (\bar{\tau}_w)_{\xi=b}$$

IV. RESULT AND DISCUSSION

The present model has been investigated to analyse the effect of the stenosis height, shape, catheter radius and slip velocity on axial velocity, shear stress and effective viscosity. The numerical evaluations of the analytical results obtained in equations (8)-(11). The various parameter values are selected from Young (1968) and Srivastava *et. al*, (2009, 2010) as:

Lo = L = 1(cm); ε (non- dimensional catheter radius = 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, δ/R (non-dimensional stenosis height) = 0, 0.05, 0.10, 0.15, 0.20. (It is to note here that the present study corresponds to the flow in un-catheterized and normal (no stenosis) artery for parameter values $\varepsilon = 0$ and $\delta/R = 0$, respectively.

Fig. 2 indicates that the variation of impedance, λ and catheter size ε.

It is increases with the catheter size, ε for any given stenosis height, δ/R_0 . One notice that for any given stenosis height, a significant increase in the magnitude of the flow resistance, λ occurs for any small increase in the catheter size, ε .

Numerical results reveal that for any given set of other parameters, the impedance, λ decreases with increasing the tube length, L which interns implies that the flow resistance, λ increases with the stenosis length, L_0 . The flow resistance, λ steeply increases with the catheter size, ε (≤ 0.3) but increases rapidly with increasing the catheter size, ε and depending on the height of the stenosis, attains a very high asymptotic magnitude with increasing the catheter size, ε (Fig. 3).

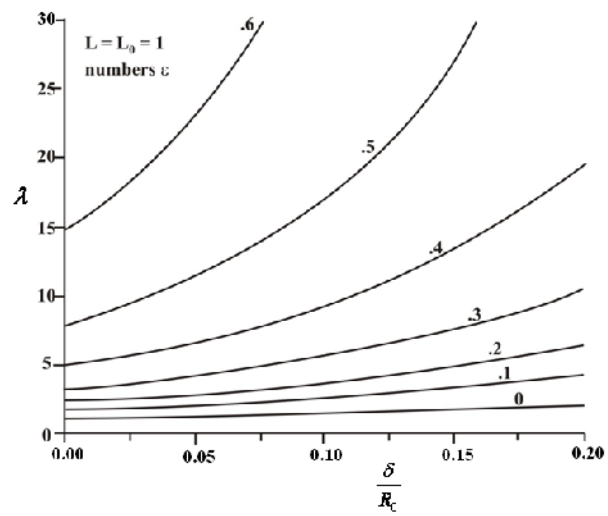

Fig. 2. Impedance, λ versus εfor different $\frac{\delta}{R_0}$.

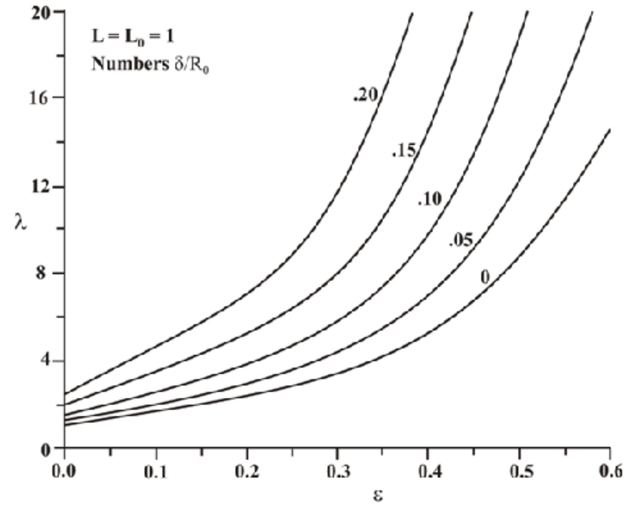

Fig. 3. Impedance, λ versus ε for different $\frac{\delta}{R_0}$

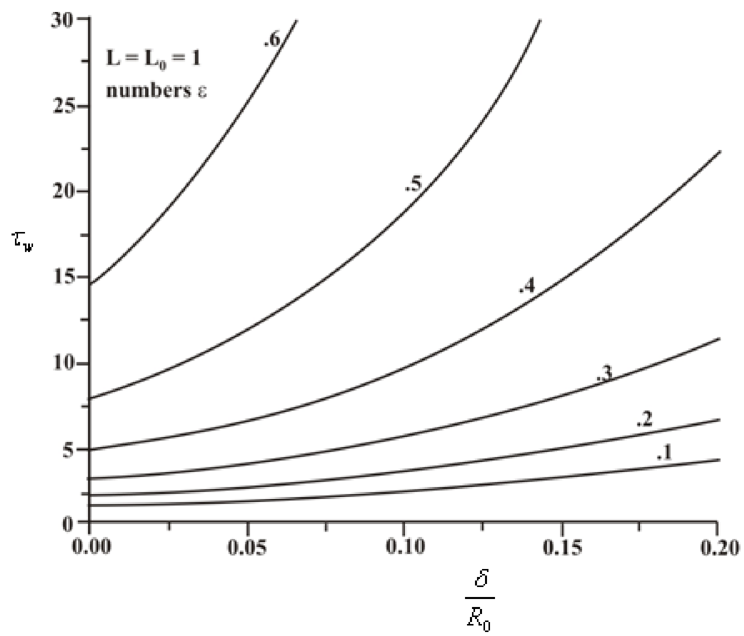

Fig. 4. Shear stress at stenosis throat, τ_w versus $\frac{\delta}{R_0}$ for different ε.

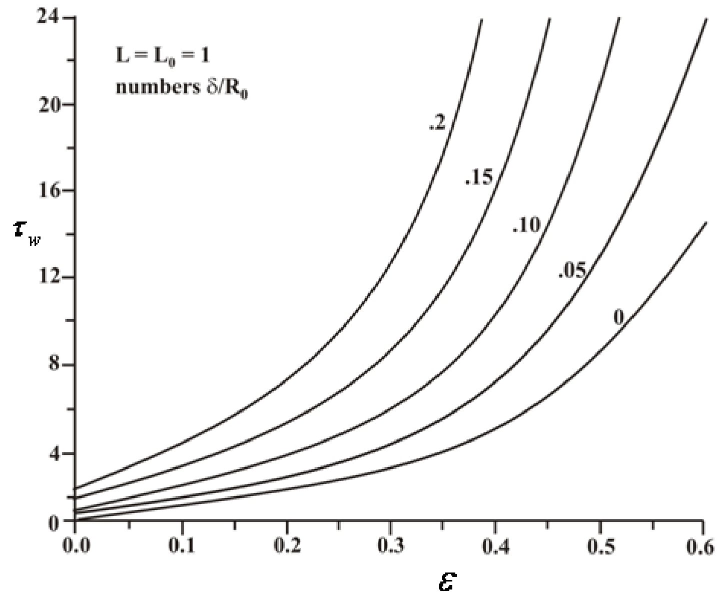

Fig. 5. Shear stress at stenosis throat, τ_w versus ε for different $\frac{\delta}{R_0}$.

The high asymptotic magnitude of λ occurs for $\delta/R = 0.1(19\%\ stenosis)$, 0.15 (28% stenosis) and 0.2 (38% stenosis) at catheter size, $\varepsilon = 5.5,\ 4.5\ \ and\ 4.0$ respectively. One further notice that τ_W is increases with increasing stenosis height, δ/R_0 (Fig. 4). The wall shear stress in the stenotic region, τ_W increases rapidly with the catheter size, ε (Fig. 5).

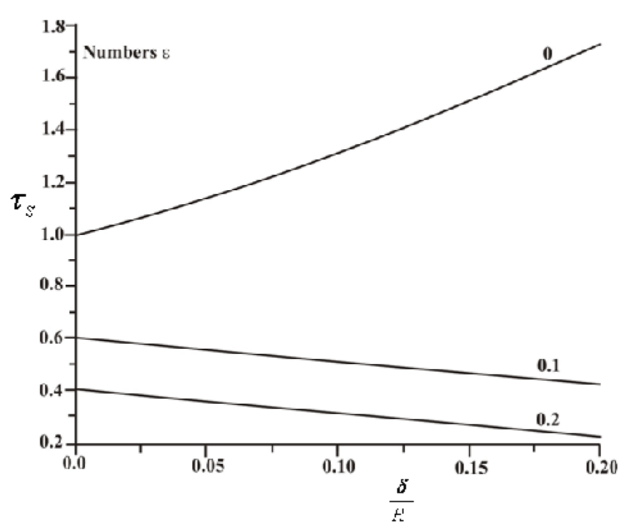

Fig. 6. Variation of shear stress at stenosis throat, τ_s with $\frac{\delta}{R_0}$ for different ε.

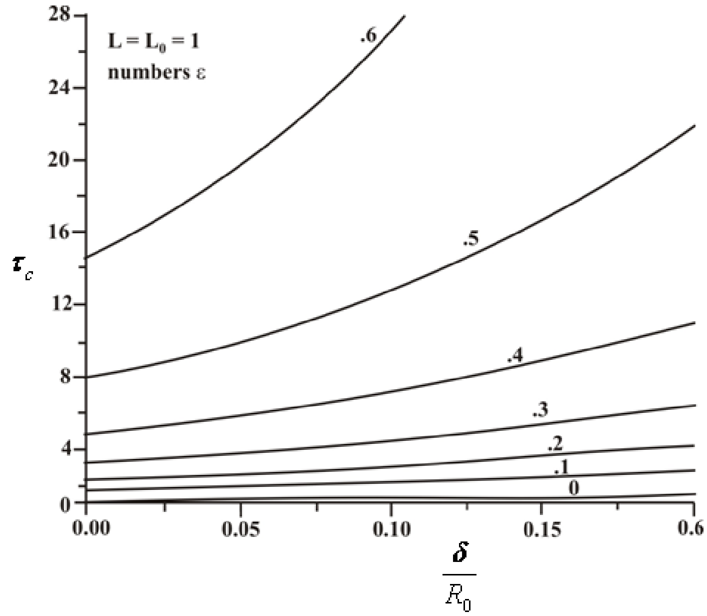

Fig. 7. Shear stress at stenosis critical height, τ_c versus $\frac{\delta}{R_0}$ for different ε.

The shear stress at the stenosis throat τ_s increases with the catheter size, ε as well as with the stenosis height, δ/R (Fig. 6). At stenosis critical height, the shear stress τ_c too increases with the catheter size, ε and stenosis height δ/R (Fig. 7).

The flow characteristic, τ_s assumes higher magnitude for higher stenosis height for small catheter size, ε (Fig. 8). One notices that τ_s achieves an asymptotic magnitude when the catheter size becomes approximately fifty percent of the artery size.

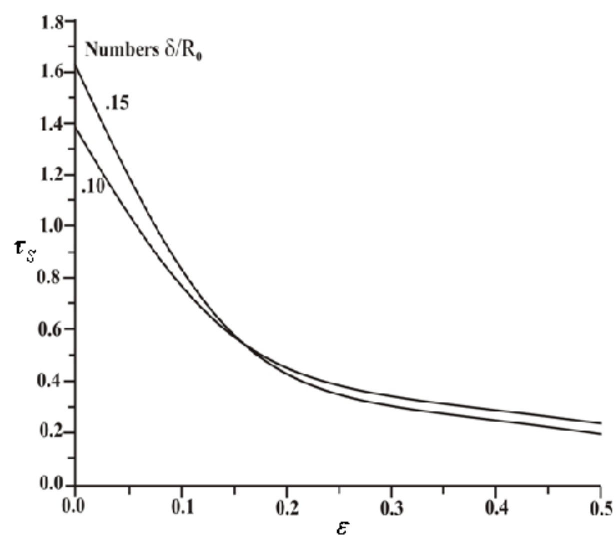

Fig. 8. Variation of shear stress at stenosis throat, τ_s for different $\frac{\delta}{R_0}$.

V. CONCLUSION

In the present work, the increased impedance and shear stress during a narrow catheterized has been analyzed assuming that the flowing blood is represented by a Newtonian fluid. The impedance increases with increasing catheter size and depends on stenosis height. A numerical result reveals that the shear stress at the stenosis throats and at the stenosis critical height possesses the characteristics similar to the flow resistance with respect to any parameter. Thus, the magnitude of the stenosis critical height is much smaller than its corresponding value at the stenosis throats.

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