Shear Free Bianchi Type V String Cosmological Model in Self Creation Cosmology

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ABSTRACT: We have studied the Bianchi type-V string cosmological model filled with perfect fluid in Barber second self-creation theory. Some physical consequences of the models have been discussed in case of Zel’dovich fluid and radiation dominated fluid.

Keywords: Bianchi Type V Universe, Cosmological models, string theory.

PACS: 98.80.Cq Particle-theory and field-theory models of the early Universe (including cosmic pancakes, cosmic strings, chaotic phenomena, inflationary universe, etc.)

I. INTRODUCTION

Recently cosmic strings have attracted many astrophysicists to try to achieve a possible description of the early stage of the universe. The study of cosmic strings in elementary particle physics arises from the gauge theories with spontaneous broken symmetry. After the big bang, it is believed that the universe might have experienced a number of phase transitions by producing vacuum domain structures such as domain walls, strings and monopoles. Cosmic strings may act as gravitational lenses and give rise to density perturbations leading to formation of galaxies. So, it is quite interesting to study the gravitational effect that arises from strings by using Einstein’s field equations in self creation cosmology.

Self creation theory was proposed by Barber [1] in 1982. Barber has proposed two theories in these theories the universe is seen to be created out of self contained gravitational, scalar and matter fields. The first is a modified Brans Dicke theory [2] that is unsatisfactory since the equivalence principal is violated, barbers second theory is a modification of general relativity to include continuous creation and is within observational limits, and thus it modifies general relativity to become a variable G-theory. These modifies theories create the universe out of self contained gravitational and matter fields. The scalar field is postulated to couple to the trace of energy-momentum tensor. The consistency of Barbers second theory motivates us to study cosmological models in this theory.

Many of the researchers have studied on Bianchi V Universe which is the natural generalization of FRW models with negative curvature. These open models are favoured by the available evidences for low density Universes [3], Heckmann and schucking [4] studied Bianchi type V cosmological model where matter moves orthogonally to the hyper-surface of homogeneity. Exact tilted solutions for the Bianchi type V space – time were obtained by hawking [5] and Grischuk et al [6]. Ftaclas and Cohen [7] have investigated LRS Bianchi type – V universes containing stiff matter with electromagnetic field. Lorentz [8] has investigated LRS Bianchi type V tilted models with stiff fluid and electromagnetic field. Pradhan et al [9] have investigated the generation of Bianchi type V cosmological models with varying A term.

Pimentel [10], Soleng [11,12], Singh [13], Reddy [14], Reddy et al [15,16,17,18], Reddy and Venkateshwarlu [19], Venkateshwarlu and Reddy [20], Shanti and Rao [21], Sanyasiraju and Rao [22], Shri and Singh [23,24], Mohanty et al. [25], Pradhan and Vishwakarma [26,27], Sahu and Panigrahi [28], Venkateshwarlu and Kumar [29], Tiwari and Kumar [30] are some of the researchers who have studied various aspects of cosmological models in self-creation theory.

Recently Baysal et al [31], Kilinc and Yavuz [32], Pradhan [33], Pradhan et al [34,35] and Yadav et al [36] have investigated some string cosmological models in cylindrically symmetric inhomogeneous Universe. Five dimensional string cosmological models are studied by Mohanty et al [37] in second self creation cosmology.

In this paper we wish to study Bianchi type V string cosmological model in Barber’s second self creation theory.
II. MODEL AND FIELD EQUATIONS

We consider the Bianchi Type V space time given by

\[ ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{2\alpha x} (dy^2 + dz^2) \]  

where A and B are functions of cosmic time t only.

Energy momentum tensor \( T^i_j \) for a cloud of massive strings and perfect fluid distribution is taken as

\[ T^i_j = (\rho + p)v^i v_j + pg^i_j - \lambda x^i x_j \]

\[ \rho = \rho_p + \lambda \]  

\[ p \text{ and } \rho_p \text{ are respectively the isotropic pressure and proper energy density for a cloud of strings with particles attached to them; } \lambda \text{ is string tension density of the fluid and } \rho_p \text{ is the rest energy density of the particles.} \]

Choosing \( x^i \parallel \partial / \partial x \) we have \( x^i = (A^{-1}, 0, 0, 0) \)

We assume that the matter content obeys an equation of the state. 

\[ P = \omega \rho, \quad 0 \leq \omega \leq 1 \]

The field equations in Barber’s self-creation theory (1982) are

\[ R_{ij} - \frac{1}{2} g_{ij} R = -8\pi \phi^{-1} T_{ij} \]

\[ \phi = \frac{8\pi}{3} \eta T \]

Where \( \phi = \phi_\delta \) is the invariant d’Alembertian and \( T \) is the trace of the energy-momentum tensor that describes all non-gravitational and non-scalar field matter and energy. Here \( \eta \) is a coupling constant to be determined from experiments. The measurement of the deflection of light restricts the value of the coupling to \( |\eta| \leq 10^{-3} \). This theory leads to the Einstein’s theory in every respect when \( \eta \rightarrow 0 \).

The spatial average scale factor \( S(t) \) is given by

\[ S = (AB^2)^{\frac{1}{3}} \]

And average volume scale factor \( V = S^3 \)

The average Hubble's parameter \( H \) is defined as

\[ H = \frac{\dot{S}}{S} = \frac{1}{3} \left( \frac{\dot{A}}{A} + \frac{\dot{B}}{B} \right) \]

where an overhead dot denotes ordinary differentiation with respect to t. We also have

\[ H = \frac{1}{3} (H_1 + H_2 + H_3) \]

Where \( \dot{H}_i = \frac{\dot{A}}{A}, \dot{H}_2 = \frac{\dot{B}}{B}, \dot{H}_3 = \frac{\dot{C}}{C} \) are Hubble’s factors in the direction of x, y and z respectively.

The anisotropy parameter \( \overline{A} \) is defined by

\[ \overline{A} = \frac{1}{3} \sum_{i=1}^{3} \left( \frac{\Delta H_i}{H} \right)^2 \]

where \( \Delta H_i = H_i - H (i = 1, 2, 3) \).
In a comoving system of coordinates the field equations (5) and (6) for the metric (1) can be written as

\[
\frac{\dddot{B}}{B} + \left(\frac{\ddot{B}}{B}\right)^2 - \frac{\alpha^2}{A^2} = -8\pi\phi^{-1}p + \dot{\phi}
\]

\(\ldots(11)\)

\[
\frac{\dddot{A}}{A} + \frac{\dddot{B}}{B} + \frac{\dddot{AB}}{AB} = \frac{\alpha^2}{A^2} = -8\pi\phi^{-1}p
\]

\(\ldots(12)\)

\[
2\frac{\dddot{AB}}{AB} + \left(\frac{\ddot{B}}{B}\right)^2 = -\frac{3\alpha^2}{A^2} = -8\pi\phi^{-1}\rho
\]

\(\ldots(13)\)

\[
\frac{2A}{A} - 2\frac{B}{B} = 0
\]

\(\ldots(14)\)

Using (11), (12) & (14), we have \(\dot{\lambda} = 0\)

\[
\phi + \phi \left(\frac{\ddot{A}}{A} + 2\frac{\dot{B}}{B}\right) = \frac{8\pi\eta}{3}(\rho - 3p)
\]

\(\ldots(15)\)

We assume matter content obeys equation of state

\[
p = \omega\rho, \quad 0 \leq \omega \leq 1
\]

\(\ldots(16)\)

If we use the equivalent energy conservation equation of general relativity quantities, we find that

\[
\left(\frac{\rho}{\phi}\right) + \left(\frac{\rho + p}{\phi}\right) \left(\frac{\dot{A}}{A} + \frac{\dot{2B}}{B}\right) = 0
\]

\(\ldots(17)\)

### III. SOLUTIONS OF FIELD EQUATIONS

The field equations (11)-(15) supply five independent equations in six unknowns \(A, B, \rho, p, \dot{\phi}\), and \(\dot{\lambda}\). Extra conditions are needed to solve the system completely; we assume the condition [35]

\[
\frac{\ddot{A}}{A} = \frac{\ddot{B}}{B} = \frac{m_1}{t^n}
\]

\(\ldots(18)\)

where \(m_1\) and \(n\) are constants.

Equation (18) gives \(A = \mu B\)

\(\ldots(19)\)

Without loss of generality we can take \(\mu = 1\)

Using (18) \(S = c_1 t^{m_1n}\) for \(n = 1\)

\(\ldots(20)\)

And \(S = c_2 \exp\left(\frac{m_1 t^{1-n}}{1-n}\right)\) for \(n \neq 1\)

\(\ldots(21)\)

\(c_1\) and \(c_2\) are constant of integration.

Equation (17) with (16) gives

\[
\left(\frac{\rho}{\phi}\right) + 3(\omega + 1)\left(\frac{\rho}{\phi}\right) \left(\frac{S}{\dot{S}}\right) = 0
\]

\(\ldots(22)\)

Integrating (22)

\[
\frac{\rho}{\phi} = \frac{k_1}{S^{3(\omega+1)}}
\]

\(\ldots(23)\)
Equation (23) together with (15) gives

$$\dddot{\phi} + 3\dot{\phi} \ddot{\phi} = \frac{8\pi\eta k_1 (1 - 3\omega)}{3S^{3(n+1)}} \phi$$

...(24)

where $k_1$ is constant of integration.

We solve field equations using power law cosmology ($n=1$) and exponential cosmology ($n \neq 1$) given by (18) and (19) respectively.

IV. POWER LAW COSMOLOGY ($n=1$)

When $n \neq 0$, using (18), we obtain the line element (1) in the form

$$ds^2 = -dt^2 + c_1^2 t^{2m_1} dx^2 + c_2^2 t^{2m_1} \exp(2\alpha x)(dy^2 + dz^2)$$

...(25)

Eq. (24) reduces to

$$\dddot{\phi} + 3\dot{\phi} \ddot{\phi} = \frac{8\pi\eta k_1 (1 - 3\omega)}{3(1 - 3m_1)} \phi$$

...(26)

Now we discuss the models for Zel dovich fluid, radiation dominated case and the vacuum case.

A. Zel Dovich Fluid Distribution

It corresponds to the equation of state $\rho = p$. This equation of state has been widely used in general relativity to obtain stellar and cosmological model for utter dense matter (Zel’dovich 1962).

Equation (26) on integration gives

$$\phi(t) = c_3 \cos\left[\frac{16\pi\eta k_1}{3c_1^6} \left(\frac{t^{1-3m_1}}{1 - 3m_1}\right)\right]$$

...(27)

Where $c_3$ and $k_1$ are constants of integration and $\eta$ is non-negative.

The energy density and pressure are given by

$$\rho = \rho_p = p = \frac{k_1}{c_1^6} t^{-6m_1} \left(\frac{c_3}{3c_1^3 (1 - 3m_1)}\right)$$

...(28)

For the reality condition $\rho > 0$ to hold, it is necessary that $k_1 > 0$

Cosmological parameters are given by

$$V = c_1^3 t^{3m_1}$$

...(29)

$$\theta = \frac{3m_1}{t}$$

...(30)

$$\sigma = 0$$

...(31)

$$H_1 = H_2 = H_3 = \frac{m_1}{t}$$

...(32)

$$A = 0$$

...(33)

Observations

For an expanding model we require $m_1 > 0$.

(i) For $m_1 > 0$ the spatial volume $V$ is zero at $t = 0$ and expansion scalar $\theta$ is infinite which shows that the universe starts evolving with zero volume and infinite rate of expansion at $t = 0$. i.e. it has initially Big Bang singularity.

(ii) For $m_1 > 0$, $\rho$ and $p$ decreases as time $t$ increases and at any instant $t \lambda = 0$ which shows that at later stages (present universe) cosmic string doesn’t play any role.

(iii) $\sigma = 0$ and anisotropy $\tilde{A}$ vanishes for the model.
(iv) The ratio $\frac{\sigma}{\theta} = 0$ which shows that model is isotropic in nature.

(v) All the parameters $\theta, \rho, H_1, H_2, H_3$ tends to zero as $t \to \infty$. Therefore the model essentially gives an empty universe for large $t$.

(vi) The model represents non shearing, non rotating and expanding universe with a big bang start.

B. Radiation Dominated Solutions

Disordered radiation corresponds to the equation $\rho = 3p$. In this case (26) immediately integrates to field

$$\phi = \frac{c_7 t^{1-3m_1}}{1-3m_1} + c_8$$

$$\rho = 3p = \frac{k_1}{c_1^4 t^{4m_1}} \left( \frac{c_7 t^{1-3m_1}}{1-3m_1} + c_8 \right)$$

Where $c_7$ and $c_8$ are constant of integration. Here $m_1 \neq \frac{1}{3}$.

For $m_1 = \frac{1}{3}$

Eq (26) yields

$$\phi = c_9 \log t + c_{10}$$

$$\rho = 3p = \frac{k_1}{c_1^4 t^{4m_1}} (c_9 \log t + c_{10})$$

Other cosmological parameters for any real value of $m_1$ are given by

$$V = c_1^3 t^{3m_1}$$

$$\theta = \frac{3m_1}{t}$$

$$\sigma = 0$$

$$H_1 = H_2 = H_3 = \frac{m_1}{t}$$

Special case ($n=1$ and $m_1 = 1$)

Using (12) and (13), the energy density and pressure for cosmic string is given by

$$\rho = -\frac{3(\alpha^2 - \alpha_1^2)}{8\pi_1^2 \left( c_9 \frac{2t^2}{\alpha^2} + c_{10} \right)}$$

$$p = \frac{(\alpha^2 - \alpha_1^2)}{8\pi_1^2 \left( c_9 \frac{2t^2}{\alpha^2} + c_{10} \right)}$$

For $c_1 = \alpha$ we get

$$\rho = p = 0$$

Also $A = 0$, therefore cosmic string doesn’t exist at any instant for this model.

Observations

For $m_1 > 0$ the spatial volume $V$ is zero at $t = 0$ and expansion scalar $\theta$ is infinite which shows that the universe starts evolving with zero volume and infinite rate of expansion at $t = 0$. i.e. it has initially Big Bang singularity.
For \( m_1 = 1 \), n=1 and \( c_1 = \alpha \) we get \( \rho = p = 0 \) also \( \lambda = 0 \), lead to cosmic string free model from evaluation up to the later stages i.e. present epoch of the universe. \( \sigma = 0 \) and anisotropy \( \overline{A} \) vanishes for the model. The ratio \( \frac{\sigma}{\theta} = 0 \) which shows that our model is isotropic. All the parameters \( \theta \), \( p \), \( \rho \), \( \alpha \), \( H_1 \), \( H_2 \), \( H_3 \) tend to zero as \( t \to \infty \). Therefore the model essentially gives an empty universe for large \( t \). The model represents shearing, non rotating and expanding universe with a big bang start.

C. Vacuum Solution \((\rho = p = 0)\)

In this case scalar field \( \phi \), spatial volume \( V \) and cosmological parameter \( \theta \), \( \sigma \), \( A \) are

\[
\phi = \frac{c_{11} t^{1-3m_1}}{1 - 3m_1} + c_{12} \quad \text{... (46)}
\]

\[
V = c_1^3 t^{3m_1} \quad \text{... (47)}
\]

\[
\theta = \frac{3m_1}{t} \quad \text{... (48)}
\]

\[
\sigma = 0 \quad \text{... (49)}
\]

\[
H_1 = H_2 = H_3 = \frac{m_1}{t} \quad \text{... (50)}
\]

\[
\overline{A} = 0 \quad \text{... (51)}
\]

Observations

For \( m_1 > 0 \) the spatial volume \( V \) is zero at \( t = 0 \) and expansion scalar \( \theta \) is infinite which shows that the universe starts evolving with zero volume and infinite rate of expansion at \( t = 0 \). \textit{i.e.} it has initially Big Bang singularity. Here \( \rho = p = 0 \) also \( \lambda = 0 \), leads to cosmic string free model. The ratio \( \frac{\sigma}{\theta} = 0 \) which shows that our model is isotropic. All the parameters \( \theta \), \( p \), \( \rho \), \( \alpha \), \( H_1 \), \( H_2 \), \( H_3 \) tend to zero as \( t \to \infty \). Therefore the model essentially gives an empty universe for large \( t \). The model represents non shearing, non rotating and expanding string free universe with a big bang start.

V. EXPONENTIAL COSMOLOGY \((n \neq 1)\)

From (18), we obtain line element (1) in the form

\[
ds^2 = -dt^2 + c_2^2 \exp\left[\frac{2m_1 t^{1-n}}{1-n}\right] dx^2 + c_2^2 \exp\left[\frac{2m_1 t^{1-n}}{1-n} + 2\alpha x\right] (dy^2 + dz^2) \quad \text{... (52)}
\]

In this case from (24), we obtain

\[
\phi^{*} + 3 m_1 \phi = \frac{8\pi\eta}{3} \frac{(1 - 3\omega)k_1 \phi}{c_2 \exp\left(\frac{m_1 t^{1-n}}{1-n}\right)^{3(\omega+1)}} \quad \text{... (53)}
\]

We analyze the model for stiff matter, radiation dominated case and the vacuum case in the following subsections and for the sake of convenience, we are particularly taking \( n = 0 \) here.

A. Stiff Fluid Model

Here \( \rho = p \).

Using (53),

\[
\phi^{*} + 3 m_1 \phi = \frac{-16\pi\eta}{3} \frac{k_1 \phi}{\left(c_2^{3}\exp(6m_1 t)\right)} \quad \text{... (54)}
\]
Integrating (54)

\[ \phi = c_{11} \cos \left( \frac{16}{3c_2^6} \pi \eta k_1 \left( e^{-3m_i t} \right) \right) \]  \quad \text{... (55)}

\[ \rho = p = \frac{k_3}{c_2^6 e^{3m_i t}} \phi \cdot -\frac{k_3 c_{11}}{c_2^6 e^{6m_i t}} \cos \left( \frac{16}{3c_2^6} \pi \eta k_1 \left( e^{-3m_i t} \right) \right) \]  \quad \text{... (56)}

Coupling constant \( \eta \) and \( k_3 \) is positive for the reality of energy density and pressure. Other cosmological parameter are given by

\[ V = c_2^3 e^{3m_i t} \]  \quad \text{... (57)}

\[ \theta = 3m_i \]  \quad \text{... (58)}

\[ \sigma = 0 \]  \quad \text{... (59)}

\[ H_1 = H_2 = H_3 = m_i \]  \quad \text{... (60)}

\[ \overline{A} = 0 \]  \quad \text{... (61)}

**Observations**

For \( m_i > 0 \) the spatial volume \( V \) is zero at \( t=0 \) and expansion scalar \( \theta \) is constant which shows that the universe starts evolving with zero volume and having a uniform rate of expansion. For \( m_i > 0 \), \( \rho \) and \( p \) decreases as time \( t \) increases and at any instant \( t \) \( \lambda = 0 \) which shows that at later stages (present universe) cosmic string doesn’t play any role. Anisotropy \( \overline{A} \) vanishes for the model. The ratio \( \frac{\sigma}{\theta} = 0 \) which shows that our model is isotropic in nature. The model represents non shearing, non rotating and expanding universe with a uniform rate of expansion.

**B. Radiation Dominated Solution \((\rho=3p)\)**

In this case the scalar field \( \phi \), energy density \( \rho \) and pressure \( p \) are given by

\[ \phi = -\frac{e^{-3m_i t} c_{12}}{3m_i} + c_{13} \]  \quad \text{... (62)}

\[ \rho = \frac{k_1}{c_1^4 e^{4m_i t}} \left( -\frac{e^{-3m_i t} c_{12}}{3m_i} + c_{13} \right) \]  \quad \text{... (63)}

\[ p = \frac{k_1}{c_1^4 e^{4m_i t}} \left( -\frac{e^{-3m_i t} c_{12}}{3m_i} + c_{13} \right) \]  \quad \text{... (64)}

Where \( c_{12}, c_{13} \) are constants of integration. Other cosmological parameters are

\[ H_1 = H_2 = H_3 = m_i \]  \quad \text{... (65)}

\[ \theta = 3m_i \]  \quad \text{... (66)}

\[ \overline{A} = 0 \]  \quad \text{... (67)}

\[ \sigma = 0 \]  \quad \text{... (68)}

\[ V = c_2^3 e^{3m_i t} \]  \quad \text{... (69)}

**Observations**

For \( k_1 > 0 \) the energy density \( \rho \) is positive. Model has no initial singularity. Initially spatial volume \( V \), the energy density \( \rho \), pressure \( p \) and other cosmological parameters are constant. Therefore universe starts evolving with a constant volume and expands with a uniform rate. When \( t \) increases scalar field \( \phi \), energy density \( \rho \) and pressure \( p \) decreases.
For $m, > 0$, $\rho$ and $p$ decreases as time $t$ increases and at any instant $t \lambda = 0$ which shows that at later stages (present universe) cosmic string doesn’t play any role. The scalar field $\phi$ remains finite during the whole span of evolution. $\sigma = 0$ and anisotropy $\vec{A}$ vanishes for the model. The ratio $\frac{\sigma}{\theta}$ which shows that our model is isotropic. The model represents non shearing, non rotating and expanding universe with a uniform rate of expansion.

C. Vacuum Solution ($\rho = p = 0$)

In this case, the behavior of the model is same as in section 5.2.

VI. CONCLUDING REMARKS

In this paper a spatially homogeneous and isotropic Bianchi V string model has been investigated. We obtained a string free cosmological model of Barber’s second self creation theory in a Bianchi type-V model. We observe that as $t \to \infty$, $V \to \infty$ and $\rho \to 0$ i.e spatial volume ($V$) increases with time and proper energy density ($\rho$) decreases with time as expected. The main features of the derived model are as follows:

- The model is based on exact solution of Einstein’s field equations for the isotropic Bianchi –V space time filled as source of matter.
- Cosmological models are presented for $n = 1$ and $n \neq 1$ cosmologies. There are two solutions: one is the power-law solution and the other is the exponential solution. We discuss both solutions for Zel’dovich fluid, radiation dominated and vacuum cases.
- Geometrical and kinematical properties of different parameters have been discussed in detail for each phase. The nature of singularities of the models is clarified and explicit forms of scalar factors are obtained.
- Our model is isotropic in nature.
- We note that in all the cases $\rho(t)$ is a decreasing function of time and it is always positive. Further it is observed that for sufficiently large time $\lambda$ and $\rho_f$ tends to zero. Therefore the strings disappear from the universe at a later time (i.e. the present epoch). The same is predicted by the current observations.
- In general it has been observed that the model represents non shearing, non-rotating and expanding universe.

Finally, the solution presented here can be one of the potential candidates to describe the observed universe. Therefore the exact and physically viable Bianchi V string cosmological model has been obtained.

REFERENCES

