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Coupled Fixed Point Theorem in Menger Space

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ABSTRACT: The purpose of this paper is to introduce the new concept of occasionally weakly compatible mappings for coupled maps and prove a coupled fixed point theorem under more general t-norm (H-type norm) in Menger space. Finally, we also given an application.

Keywords. Menger Spaces, Occasionally Weakly Compatible Maps.
AMS Subject Classification (2000). Primary 47H10, Secondary 54H25.

I. INTRODUCTION

Menger [10] introduced the notion of a probabilistic metric space (PM-space) which was, in fact, a generalization of metric space in the year 1942. The idea behind this is to associate a distribution function with a pair of points, say (p,q), denoted by $F_{p,q}(t)$ where $t > 0$ and interpret this function as the probability that distance between p and q is less than t, whereas in the metric space, the distance function is a single positive number. Sehgal [13] initiated the study of fixed points in probabilistic metric spaces. The study of these spaces was expanded rapidly with the pioneering works of Schweizer and Sklar [12]. In 1991, Mishra [11] introduced the notion of compatible mappings in the setting of probabilistic metric space. In 1996, Jungck [8] introduced the notion of weakly compatible. Further, Singh and Jain [14] proved some results for weakly compatible in Menger spaces. Cho, Murthy and Stojakovik [2] proposed the concept of compatible maps of type (A) in Menger space and gave some fixed point theorems. Recently, using the concept of compatible mappings of type (A), semi-compatibility and occasionally weak compatibility in Menger space, Jain et. al. [5, 6, 7] proved some interesting fixed point theorems in Menger space. Fang [3] defined ϕ-contractive conditions and proved some fixed point theorems under ϕ-contractions for compatible and weakly compatible maps in Menger PM-spaces using t-norm of H-type, introduced by Hadžić et. al. [4]. Recently, Bhaskar and Lakshmikantham [1], Lakshmikantham and Ćirić [9] gave some coupled fixed point theorems in partially ordered metric spaces. Now, we introduce the new concept of occasionally weakly compatible mappings for coupled maps and prove a coupled fixed point theorem under more general t-norm (H-type norm) in Menger space.

II. PRELIMINARIES

Definition 2.1. [15] A mapping $F : [0, \infty) \to [0,1]$ is called a distribution function if it is non-decreasing and left-continuous with $\inf_{x\in R} F(x) = 0$. If in addition $F(0) = 0$, then F is called a distance distribution function.
A distance distribution function F satisfying $\lim_{t\to\infty}F(t) = 1$ is called a Menger distance distribution function.
The set of all Menger distance distribution functions is denoted by D^+. This space D^+ is partially ordered by the usual pointwise ordering of functions, that is, $F \le G$ if and only if $F(t) \le G(t)$ for all $t \in [0, \infty)$. The maximal element for D^+ in this order is the distance distribution function ε_0 given by

$$\varepsilon_0(t) = \begin{cases} 0 & , \quad t = 0 \\ 1 & , \quad t > 0. \end{cases}$$

Definition 2.2. [15] A triangular norm (shortly, t-norm) is a binary operation Δ on [0,1] satisfying the following conditions:
(1) Δ is associative and commutative;
(2) Δ is continuous;
(3) $\Delta(a, 1) = a$ for all $a \in [0, 1]$;
(4) $\Delta(a, b) \le \Delta(c, d)$ whenever $a \le c$ and $b \le d$ for all a, b, c, $d \in [0,1]$.
Two typical examples of the continuous t-norm are $\Delta_p(a,b) = ab$, $\Delta_M(a,b) = \min\{a, b\}$ for all $a, b \in [0,1]$.
Now, the t-norm is recursively defined by $\Delta^1 = \Delta$ and

$$\Delta^n(x_1, \ldots, x_{n+1}) = \Delta(\Delta^{n-1}(x_1, \ldots, x_n), x_{n+1})$$

for all $n \ge 2$ and $x_i \in [0,1]$, $i = 1, 2, \ldots, n + 1$.

A t-norm Δ is said to be of Hadžić-type if the family $\{\Delta^n\}$ is equicontinuous at x = 1, that is, for any $\varepsilon \in (0,1)$, there exists $\delta \in (0,1)$ such that

$$a > 1 - \delta \Rightarrow \Delta^n(a) > 1 - \varepsilon$$

for all $n \geq 1$.

Δ_M is a trivial example of a t-norm of Hadžić-type [11].

Definition 2.3. [15] A Menger probabilistic metric space (briefly, a Menger PM-space) is a triple $(X, \mathcal{F}, \Delta)$, where X is a nonempty set, Δ is a continuous t-norm and $\mathcal{F}$ is a mapping from $X \times X \to D^+$ ($F_{x,y}$ denotes the value of $\mathcal{F}$ at the pair (x,y)) satisfying the following conditions:

(PM-1) $F_{x,y}(t) = 1$ for all $x, y \in X$ and $t > 0$ if and only if x = y;

(PM-2) $F_{x,y}(t) = F_{y,x}(t)$ for all $x, y \in X$ and $t > 0$;

(PM-3) $F_{x,z}(t + s) \geq \Delta(F_{x,y}(t), F_{y,z}(s))$ for all $x, y, z \in X$ and $t, s \geq 0$.

Definition 2.4. [15] Let $(X, \mathcal{F}, \Delta)$ be a Menger PM-space.

(1). A sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$ (write $x_n \to x$) if, for any $t > 0$ and $0 < \varepsilon < 1$, there exists a positive integer N such that

$F_{x_n, x}(t) > 1 - \varepsilon$

whenever $n \geq N$;

(2). A sequence $\{x_n\}$ in X is called a Cauchy sequence if, for any $t > 0$ and $0 < \varepsilon < 1$, there exists a positive integer N such that

$F_{x_n, x_m}(t) > 1 - \varepsilon$ whenever $m, n \geq N$.

(3). A Menger PM-space $(X, \mathcal{F}, \Delta)$ is said to be complete if every Cauchy sequence in X is convergent to a point in X.

Definition 2.5. [1] Let X be a non-empty set and $T : X\times X \to X$ be a mapping. An element $(x, y) \in X\times X$ is said to be a coupled fixed point of T if

$T(x, y) = x, \quad T(y, x) = y.$

Definition 2.6. [9] Let X be a non-empty set and $T : X\times X \to X$, $h : X \to X$ be two mappings.

(1) An element $(x, y) \in X\times X$ is said to be a coupled coincidence point of h and T if

$T(x, y) = h(x), \quad T(y, x) = h(y);$

(2) An element $(x, y) \in X \times X$ is said to be a coupled common fixed point of h and T if

$T(x, y) = h(x) = x, \quad T(y, x) = h(y) = y.$

Definition 2.7. [15] Let $(X, \mathcal{F}, \Delta)$ be a Menger PM-space and $T : X \times X \to X$, $h : X \to X$ be two mappings. The mappings T and h are said to be weakly compatible (or w-compatible) if they commute at their coupled coincidence points, i.e., if (x, y) is a coupled coincidence point of T and h, then g(F(x, y)) = F(gx, gy).

Definition 2.8. Let $(X, \mathcal{F}, \Delta)$ be a Menger PM-space and $T : X \times X \to X$, $h : X \to X$ be two mappings. The mappings T and h are said to be occasionally weakly compatible if there is a point $x \in X$ which is a coupled coincidence point of f and g at which f and g commute.

Definition 2.9. [12] Define $\Phi = \{ \phi : R^+ \to R^+\}$, where $R^+ = [0, +\infty)$ and each $\phi \in \Phi$ satisfies the following conditions :

(ϕ-1) ϕ is non-decreasing;

(ϕ-2) ϕ is upper semi-continuous from the right;

(ϕ-3) $\sum_{n=0}^{\infty} \phi^n(t) < \infty$ for all $t > 0$, where $\phi^{n+1}(t) = \phi(\phi^n(t))$, $n \in N$.

Clearly, if $\phi \in \Phi$, then $\phi(t) < t$ for all $t > 0$.

III. MAIN RESULT

Theorem 3.1. Let $(X, \mathcal{F}, *)$ be Menger PM-Space, * being continuous t-norm of H-type. Let $f : X \times X \to X$ and $g : X \to X$ be two mappings and there exists $\phi \in \Phi$ such that followings hold:

(3.1) $F_{f(x,y),f(u,v)}(\phi(t)) \geq (F_{gx,gu}(t)*F_{gy,gv}(t))$, for all x, y, u, v in X and $t > 0$;

(3.2) Suppose that $f(X \times X) \subseteq g(X)$;

(3.3) pair (f, g) is occasionally weakly compatible;

(3.4) range space of one of the maps f or g is complete.

Then f and g have a coupled coincidence point. Moreover, there exists a unique point x in X such that f(x,y) = g(x).

Proof. Let x_0, y_0 be two arbitrary points in X . Since $f(X\times X) \subseteq g(X)$, we can choose x_1, y_1 in X such that $g(x_1) = f(x_0,y_0)$, $g(y_1) = f(y_0,x_0)$. Continuing in this way we can construct two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$g(x_{n+1}) = f(x_n, y_n)$ and $g(y_{n+1}) = f(y_n, x_n)$ for all $n \geq 0$.

Step 1. Firstly we show that $\{gx_n\}$ and $\{gy_n\}$ are Cauchy sequences.

Since * is a t-norm of H-type, for any $\varepsilon > 0$, there exists $\delta > 0$ such that

(3.5) $$\underbrace{(1-\delta)*(1-\delta)*...*(1-\delta)}_{p} \geq (1-\varepsilon),$$ for all $p \in N$.

Since $\lim_{t\to\infty} F_{x,y}(t) = 1$, for all x, y in X, there exists $t_0 > 0$ such that

$F_{gx_0, gx_1}(t_0) \geq (1 - \delta)$ and

$F_{gy_0, gy_1}(t_0) \geq 1 - \delta$.

Since $\phi \in \Phi$ and using condition (ϕ-3), we have

$\sum_{n=1}^{\infty} \phi^n(t_0) < \infty$. Then for any $t > 0$, there exists $n_0 \in N$ such that

(3.6) $$t > \sum_{k=n_0}^{\infty} \phi^k(t_0).$$

From (3.1), we have

$F_{gx_1,gx_2}(\phi(t_0)) = F_{f(x_0,y_0),f(x_1,y_1)}(\phi(t_0))$

$\geq F_{gx_0,gx_1}(t_0)*F_{gy_0,gy_1}(t_0)$

$F_{gy_1,gy_2}(\phi(t_0)) = F_{f(y_0,x_0),f(y_1,x_1)}(\phi(t_0))$

$\geq F_{gy_0,gy_1}(t_0) * F_{gx_0,gx_1}(t_0).$

Similarly, we can also get

$F_{gx_2,gx_3}(\phi^2(t_0)) = F_{f(x_1,y_1),f(x_2,y_2)}(\phi^2(t_0))$

$\geq F_{gx_1,gx_2}(\phi(t_0))*F_{gy_1,gy_2}(\phi(t_0))$

$F_{gy_2,gy_3}(\phi^2(t_0)) = F_{f(y_1,x_1), f(y_2,x_2)}(\phi^2(t_0))$

$\geq [F_{gy_0, gy_1}(t_0)]^2*[F_{gx_0,gx_1}(t_0)]^2.$

Continuing in this way, we can get

$F_{gx_n, gx_{n+1}}(\phi^n(t_0)) \geq [F_{gx_0, gx_1}(t_0)]^{2^{n-1}}*[F_{gy_0, gy_1}(t_0)]^{2^{n-1}}$

$F_{gy_n, gy_{n+1}}(\phi^n(t_0)) \geq [F_{gy_0, gy_1}(t_0)]^{2^{n-1}} * [F_{gx_0, gx_1}(t_0)]^{2^{n-1}}.$

So, from (3.5) and (3.6), for $m > n \geq n_0$, we have

$$F_{gx_n,gx_m}(t) \geq F_{gx_n, gx_m}\left(\sum_{k=n_0}^{\infty}\phi^k(t_0)\right)$$

$$\geq F_{gx_n, gx_m}\left(\sum_{k=n}^{m-1}\phi^k(t_0)\right)$$

$$\geq F_{gx_n,gx_{n+1}}(\phi^n(t_0)) * F_{gx_{n+1},gx_{n+2}}(\phi^{n+1}(t_0)) * \ldots *F_{gx_{m-1}, gx_m}(\phi^{m-1}(t_0))$$

$$\geq [\{[F_{gx_0,gx_1}(t_0)]^{2^{n-1}} * [F_{gy_0, gy_1}(t_0)]^{2^{n-1}}\}*\{[F_{gx_0, gx_1}(t_0)]^{2^n} * [F_{gy_0, gy_1}(t_0)]^{2^n}\}*\ldots *\{[F_{gx_0, gx_1}(t_0)]^{2^{m-2}} *[F_{gy_0, gy_1}(t_0)]^{2^{m-2}}\}]$$

$$= [F_{gx_0, gx_1}(t_0)]^{2^{n-1}(2^{m-n}-1)} * [F_{gy_0, gy_1}(t_0)]^{2^{n-1}(2^{m-n}-1)}$$

$$\geq \underbrace{(1-\delta)*(1-\delta)*\ldots*(1-\delta)}_{2^n(2^{m-n}-1)} \geq (1-\varepsilon)$$

which implies that

$F_{gx_n, gx_m}(t) \geq (1 - \varepsilon)$, for all $m, n \in N$ with $m > n \geq n_0$ and $t > 0$.

So, $\{gx_n\}$ is a Cauchy sequence. Similarly, we can get that $\{gy_n\}$ is also a Cauchy sequence.

Step 2. Now we show that f and g have a coupled coincidence point.

Without loss of generality, we assume that g(X) is complete, then there exists points x, y in g(X) so that $\lim_{n\to\infty} g(x_{n+1}) = x$, $\lim_{n\to\infty} g(y_{n+1}) = y$.

Again $x, y \in g(X)$ implies the existence of p, q in X so that $g(p) = x$, $g(q) = y$ and hence $\lim_{n\to\infty} g(x_{n+1}) = \lim_{n\to\infty} f(x_n, y_n) = g(p) = x$,

$$\lim_{n\to\infty} g(y_{n+1}) = \lim_{n\to\infty} f(y_n, x_n) = g(q) = y.$$

From (3.1),

$$F_{f(x_n, y_n), f(p, q)}(\phi(t)) \geq F_{gx_n, g(p)}(t) * F_{gy_n, g(q)}(t).$$

Taking limit as $n \to \infty$, we get

$F_{g(p),f(p,q)}(\phi(t)) = 1$ that is, $f(p,q) = g(p) = x$.

Similarly, $f(q, p) = g(q) = y$.

But f and g are occasionally weakly compatible, so that $f(p,q) = g(p) = x$ and $f(q,p) = g(q) = y$ implies $gf(p, q) = f(g(p), g(q))$ and $gf(q, p) = f(g(q), g(p))$, that is $g(x) = f(x,y)$ and $g(y) = f(y,x)$.

Hence f and g have a coupled coincidence point.

Step 3. Now we show that $g(x) = y$ and $g(y) = x$.

Since * is a t-norm of H -type, for any $\varepsilon > 0$, there exists $\delta > 0$ such that

$$\underbrace{(1-\delta)*(1-\delta)*\ldots*(1-\delta)}_{p} \geq (1-\varepsilon),$$

for all $p \in N$.

Since $\lim_{t\to\infty} F_{x, y}(t) = 1$, for all x, y in X, there exists $t_0 > 0$ such that

$F_{gx,y}(t_0) \geq (1 - \delta)$ and $F_{gy,x}(t_0) \geq (1 - \delta)$. Since $\phi \in \Phi$ and using condition (ϕ -3), we have

$$\sum_{n=1}^{\infty}\phi^n(t_0) < \infty.$$

Then for any $t > 0$, there exists $n_0 \in N$ such that

$$t > \sum_{k=n_0}^{\infty}\phi^k(t_0).$$

Using (3.1), we have

$F_{gx, gy_{n+1}}(\phi(t_0)) = F_{f(x,y),f(y_n,x_n)}(\phi(t_0))$

$\geq F_{gx, gy_n}(t_0) * F_{gy, gx_n}(t_0).$

Letting $n \to \infty$, we get

$F_{gx, y}(\phi(t_0)) \geq F_{gx, y}(t_0) * F_{gy, x}(t_0).$

By this way, we can get for all $n \in N$,

$F_{gx, y}(\phi^n(t_0)) \geq F_{gx, y}(\phi^{n-1}(t_0)) * F_{gy, x}(\phi^{n-1}(t_0))$

$\geq [F_{gx, y}(t_0)]^{2^{n-1}}*[F_{gy, x}(t_0)]^{2^{n-1}}$

thus, we have

$$F_{gx, y}(t) \geq F_{gx, y}\left(\sum_{k=n_0}^{\infty}\phi^k(t_0)\right)$$

$$\geq F_{gx, y}\left(\phi^{n_0}(t_0)\right)$$

$$\geq [F_{gx, y}(t_0)]^{2^{n_0-1}} * [F_{gy, x}(t_0)]^{2^{n_0-1}}$$

$$\geq \underbrace{(1-\delta)*(1-\delta)*\ldots*(1-\delta)}_{2^{n_0}} \geq (1-\varepsilon).$$

So, for any $\varepsilon > 0$, we have $F_{gx,y}(t) \geq (1 - \varepsilon)$, for all $t > 0$.

This implies $g(x) = y$. Similarly, $g(y) = x$.

Step 4. Next we shall show that $x = y$.

Since * is a t-norm of H-type, for any $\varepsilon > 0$, there exists $\delta > 0$ such that

$$\underbrace{(1-\delta)*(1-\delta)*...*(1-\delta)}_{p} \geq (1-\varepsilon),$$

for all $p \in N$.

Since $\lim_{t\to\infty} F_{x,y}(t) = 1$, for all x, y in X , there exists $t_0 > 0$ such that

$$F_{x,y}(t_0) \geq (1-\delta).$$

Since $\phi \in \Phi$ and using condition (ϕ -3), we have $\sum_{n=1}^{\infty} \phi^n(t_0) < \infty.$

Then for any $t > 0$, there exists $n_0 \in N$ such that

$$t > \sum_{k=n_0}^{\infty} \phi^k(t_0).$$

Using (3.1), we have

$$F_{gx_{n+1}, gy_{n+1}}(\phi(t_0)) = F_{f(x_n,y_n),f(y_n,x_n)}(\phi(t_0)) \geq F_{gx_n, gy_n}(t_0) * F_{gy_n, gx_n}(t_0).$$

Letting $n \to \infty$, we get

$$F_{x,y}(\phi(t_0)) \geq F_{x,y}(t_0) * F_{y,x}(t_0).$$

By this way, we can get for all $n \in N$, $F_{x,y}(t) \geq$

$$F_{x,y}\left(\sum_{k=n_0}^{\infty} \phi^k(t_0)\right) \geq F_{x,y}\left(\phi^{n_0}(t_0)\right) = [F_{x,y}(t_0)]^{2^{n_0-1}} * [F_{y,x}(t_0)]^{2^{n_0-1}} \geq \underbrace{(1-\delta)*(1-\delta)*...*(1-\delta)}_{2^{n_0}} \geq (1-\varepsilon)$$

which implies that x = y.

Thus, f and g have a common fixed point x in X.

Step 5. Uniqueness.

Suppose z be any point in X such that $z \neq x$ with $g(z) = z = f(z, z)$.

Since * is a t-norm of H-type, for any $\varepsilon > 0$, there exists $\delta > 0$ such that

$$\underbrace{(1-\delta)*(1-\delta)*...*(1-\delta)}_{p} \geq (1-\varepsilon),$$

for all $p \in N$.

Since $\lim_{t\to\infty} F_{x,y}(t) = 1$, for all x, y in X , there exists $t_0 > 0$ such that

$$F_{x,z}(t_0) \geq (1-\delta).$$

Since $\phi \in \Phi$ and using condition (ϕ -3), we have $\sum_{n=1}^{\infty} \phi^n(t_0) < \infty.$

Then for any $t > 0$, there exists $n_0 \in N$ such that

$$t > \sum_{k=n_0}^{\infty} \phi^k(t_0).$$

Using (3.1), we have

$$F_{x,z}(\phi(t_0)) = F_{f(x,x),f(z,z)}(\phi(t_0)) \geq F_{g(x),g(z)}(t_0) * F_{g(x),g(z)}(t_0) = F_{x,z}(t_0) * F_{x,z}(t_0)[F_{x,z}(t_0)]^2 .$$

Thus, we have

$$F_{x,z}(t) \geq F_{x,z}\left(\sum_{k=n_0}^{\infty} \phi^k(t_0)\right) \geq F_{x,z}\left(\phi^{n_0}(t_0)\right) \geq \{[F_{x,z}(t_0)]^{2^{n_0-1}}\}^2 = [F_{x,z}(t_0)]^{2^{n_0}} \geq \underbrace{(1-\delta)*(1-\delta)*...*(1-\delta)}_{2^{n_0}} \geq (1-\varepsilon)$$

,

which implies that x = z.

Hence, f and g have a unique common fixed point in X.

Next we give an application of Theorem 3.1.

IV. AN APPLICATION

Theorem 4.1. Let (X, $\mathcal{F}$, *) be a Menger PM-space, * being continuous t-norm defined by $a*b = \min\{a,b\}$ for all a, b in X. Suppose P and Q be occasionally weakly compatible self maps on X satisfying the following conditions:

(4.1) $P(X) \subseteq Q(X)$,

(4.2) there exists $\phi \in \Phi$ such that

$F_{Px,Py}(\phi(t)) \geq F_{Qx,Qy}(t)$ for all x, y in X and $t > 0$.

If range space of any one of the maps P or Q is complete, then P and Q have a unique common fixed point in X.

Proof. By taking $f(x,y) = P(x)$ and $g(x) = Q(x)$ for all $x, y \in X$ in Theorem 3.1, we get the desired result.

Taking $\phi(t) = kt$, $k \in (0, 1)$, we have the following:

Corollary 4.1. Let (X, $\mathcal{F}$, *) be a Menger PM-space, * being continuous t -norm defined by $a*b = \min\{a,b)$ for all a, b in X . Suppose P and Q be occasionally weakly compatible self maps on X satisfying (4.1) and the following condition:

(4.3) there exists $k \in (0,1)$ such that

$F_{Px,Py}(kt) \geq F_{Qx,Qy}(t)$ for all x, y in X and $t > 0$.

If range space of any one of the maps P or Q is complete, then P and Q have a unique common fixed point in X.

Taking Q = I, the identity map on X, we have the following:

Corollary 4.2. Let (X, $\mathcal{F}$, *) be a Menger PM-space, * being continuous t-norm defined by a*b = min{a, b} for all a, b in X. Suppose ρ and Q be occasionally weakly compatible self maps on X satisfying (4.1) and the following condition:

(4.4) there exists $k \in (0,1)$ such that

$F_{Px,Py}(kt) \geq F_{x,y}(t)$ for all x, y in X and $t > 0$.

If range space of the map P is complete, then P and Q have a unique common fixed point in X.

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