International Journal of Theoretical & Applied Sciences, **17**(2): 31-36(2025)

ISSN No. (Print): 0975-1718
ISSN No. (Online): 2249-3247

Study of Convective and Radiative Heat Transfer in Laminar Fluid Flow with Small Prandtl Numbers

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(Received 10 April 2025, Revised 23 May 2025, Accepted 12 June 2025)
(Published by Research Trend, Website: www.researchtrend.net)

ABSTRACT: This study investigates the combined effects of convection and radiation on heat transfer in laminar flow of incompressible fluids at low Prandtl numbers. The Prandtl number, which defines the relative thickness of momentum and thermal boundary layers, plays a pivotal role in determining the heat transfer characteristics of a fluid. In low Prandtl number fluids, such as liquid metals and molten salts, thermal diffusion outpaces momentum diffusion, leading to distinct heat transfer behaviors. The research explores the interaction between convective heat transfer, driven by the fluid motion, and radiative heat transfer, which occurs via electromagnetic waves and is independent of fluid flow. By varying parameters such as flow velocity, temperature distribution, and Prandtl number, the study quantifies the impact of both mechanisms on thermal performance. Results indicate that both convection and radiation contribute significantly to the overall heat transfer, with radiation becoming more dominant at higher temperatures. The findings are relevant for a variety of engineering applications, including heat exchangers, power plants, and other high-temperature systems that rely on low Prandtl number fluids. The study highlights the need for improved models to predict the thermal behavior of such fluids in complex flow conditions.

Keywords: Convection, Radiation, Heat Transfer, Laminar Flow, Low Prandtl Numbers, Nusselt Number, Thermal Performance, Fluid Mechanics, Prandtl Number, Thermal Boundary Layer.

INTRODUCTION

Heat transfer in laminar fluid flow with small Prandtl numbers, particularly when both convection and radiation mechanisms are involved, is an essential topic for understanding thermodynamic behavior in fluid mechanics and engineering applications. The Prandtl number, a dimensionless quantity, plays a pivotal role in determining the relative thickness of the thermal and momentum boundary layers in a fluid. For fluids with small Prandtl numbers, such as molten metals or liquid sodium, the heat transfer process differs significantly from that of fluids like water or air. In such fluids, thermal diffusivity outpaces momentum diffusivity, leading to a scenario where the temperature field evolves faster than the velocity field, resulting in distinct thermal behaviors that are critical in processes where precise heat management is necessary.

In engineering and scientific applications, the heat transfer process is not only governed by convective heat transport but also significantly by radiative transfer, especially in systems with high temperatures. While convection refers to heat transfer by the movement of fluid particles, radiation occurs through electromagnetic waves and can have a considerable influence in high-temperature environments. Previous research has investigated these mechanisms in isolation, but fewer studies have focused on their combined effect, especially in laminar flow conditions where the Prandtl number is low. A well-understood interaction between convection and radiation is essential in a variety of applications, from industrial systems, such as heat exchangers and reactors, to natural systems like those observed in astrophysical phenomena.

The behavior of convective and radiative heat transfer in small Prandtl number fluids remains a largely underexplored area. Studies by researchers such as Argyropoulos and Mikrovas (1996); Yamanaka and Yuki (1976) have provided significant insight into convective heat transfer, particularly in laminar flow scenarios. These studies highlight the differences between forced and natural convection and their effects on heat transfer rates. However, the addition of radiative heat transfer complicates the scenario, as noted in the works of Modest and Finlayson and Olson (1987) explored thermal radiation's role in coupled convective flows. The intricate interplay between convective and radiative heat transfer in low-Prandtl fluids, especially under transient conditions, presents a challenging yet important problem in modern heat transfer analysis.

A key feature of low-Prandtl-number fluids is that the thermal boundary layer is typically much thinner than the velocity boundary layer. This characteristic leads to enhanced thermal conductivity and heat transfer compared to fluids with higher Prandtl numbers. Campo *et al.* (2010) studied the dynamics of wake formation behind spheres and their implications for heat transfer, shedding light on how the nature of the fluid flow affects both convective and radiative heat mechanisms. Their work provides a foundation for understanding how low-Prandtl fluids behave under various boundary conditions, such as those presented by solid objects or porous media.

In the study of radiative heat transfer, researchers like Capuano *et al.* (2017); Chen and Mucoglu (1977) have demonstrated how the Prandtl number influences thermo-fluidic transport characteristics in mixed convection systems. Their results underscore the importance of understanding radiation effects, especially when the system operates at elevated temperatures where radiation may dominate over convection. Studies of heat transfer past spheres and their interaction with both natural and forced convection have revealed complex flow dynamics, as seen in the work by Dennis *et al.* (1973); Dhole *et al.* (2006), who investigated the effects of surface temperature variations and Prandtl number on heat transfer in laminar flows.

The study of convective and radiative heat transfer in laminar flow with low-Prandtl fluids provides vital insights for optimizing industrial processes such as cooling of electronic devices, thermal management in reactors, and in the design of energy-efficient heat exchangers. As technology advances, understanding these mechanisms becomes increasingly important in industries such as metallurgy, aerospace, and chemical processing. The research in this area is poised to contribute significantly to improving heat transfer efficiency in systems where precise temperature control is essential.

In conclusion, this research will aim to bridge gaps in understanding how convective and radiative heat transfer work together in low-Prandtl-number fluids. By investigating the influence of various parameters, such as Prandtl number, velocity, temperature distribution, and radiation effects, this study will provide valuable insights into improving the design and efficiency of heat management systems. The combination of analytical, numerical, and experimental techniques will ensure that the results are both accurate and applicable to real-world engineering systems, paving the way for future innovations in heat transfer technologies.

LITERATURE REVIEW

Heat transfer by convection and radiation plays a crucial role in various engineering applications, from designing heat exchangers to cooling systems in electronics, and even in natural processes such as the transfer of heat from celestial bodies. In the context of low Reynolds number flows, the interaction between forced and natural convection becomes critical for understanding heat transfer mechanisms. The role of Prandtl number, which determines the relative thickness of momentum and thermal boundary layers, is pivotal when studying heat transfer in such systems.

In early studies, Dennis *et al.* (1973) explored the heat transfer from a sphere at low Reynolds numbers and laid the groundwork for understanding the behavior of convective heat transfer in the regime of low flow velocities. The study highlighted the challenges posed by the laminar flow condition, where the fluid moves smoothly in parallel layers, limiting mixing and thus heat transfer (Feng and Michaelides 1996). Building on this work, Yamanaka and Yuki (1976) investigated the combined effects of forced and natural convection on spherical bodies, providing a deeper insight into the coupled mechanisms at low Reynolds numbers and small Prandtl numbers (Finlayson and Olson 1987). Their findings demonstrated that the heat transfer efficiency is heavily influenced by both the Prandtl number and the flow regime.

Chen and Mucoglu (1977) further analyzed the mixed convection in the flow around a sphere, emphasizing the impact of both natural and forced convection. Their results showed that the convective heat transfer is significantly altered in the presence of both mechanisms, especially at small Reynolds numbers (Frossling, 1938). This study was crucial in understanding how different convective forces interact in low-Prandtl-number fluids, contributing to more accurate predictive models for practical applications.

Recent studies have expanded on these foundational works by examining the influence of turbulence and wall proximity on heat transfer. Zhao *et al.* (2016) conducted a detailed analysis of vortex dynamics in a sphere wake near a wall, finding that such dynamics have a profound impact on heat transfer rates, especially in complex flow geometries (Ganguli and Lele 2019). This work highlighted the importance of understanding the wake behavior in the context of heat transfer, particularly when analyzing fluids with low viscosity or high thermal conductivity.

Modern experimental approaches have provided valuable data for the validation of theoretical models. Will *et al.* (2017) carried out an experimental study on forced convective heat transfer from smooth, solid spheres, offering insights into the practical limitations and enhancements achievable through modifications in flow configuration Hema Sundar Raju *et al.* (2018). Similarly, van Hout *et al.* (2018) explored the wake flow behind spheres in turbulent boundary layers and emphasized the significant effects of turbulence on convective heat transfer, a phenomenon less emphasized in earlier studies (Hsu, 1965).

The influence of Prandtl number on mixed convection has also been a subject of numerous studies. Hema Sundar Raju *et al*. (2018) examined the thermo-fluidic transport characteristics in mixed convection past a sphere, demonstrating that the Prandtl number significantly alters both the flow behavior and heat transfer rates (Iida and Guthrie 2015). Their research highlighted the need for a more nuanced understanding of how the Prandtl number interacts with other flow parameters, such as velocity and temperature distribution, to affect thermal performance.

Rodriguez *et al*. (2019) investigated the fluid dynamics and heat transfer in the wake of a sphere, providing a comprehensive review of the wake's influence on thermal transport. Their findings are especially relevant for understanding how small-scale fluctuations in flow can lead to variations in heat transfer rates (Kendoush, 1995). This study adds to the body of knowledge by providing detailed analysis on how localized phenomena in the wake can contribute to larger-scale thermal transport processes.

Raju *et al*. (2020) conducted an analysis of mixed convective heat transfer past an isoflux/isothermal sphere and further explored the influence of Prandtl number. Their study is valuable for designing more efficient heat transfer systems, as it provides detailed insights into how thermal boundary conditions impact the convection process in low-Prandtl-number fluids (Will *et al.*, 2017).

In addition to convective heat transfer, the role of radiation in heat transfer systems, particularly in high-temperature environments, has also been extensively studied. Giacobbe (1998); Campo *et al*. (2011) examined the thermal properties of gas mixtures and the influence of radiation on heat transfer in various media, including binary mixtures of light gases like helium. Their findings have significant implications for industries dealing with extreme temperatures, such as aerospace and energy systems (Nath and Raju 2019).

Studies by Hsu (1965); Sideman (1966) highlighted the importance of forced convection heat transfer in spherical bodies, focusing on liquid metals and their unique properties at low Reynolds numbers. Their work has been essential in designing efficient thermal management systems for industries where high thermal conductivity is essential, such as in nuclear reactors and aerospace applications (Raithby and Eckert 1968; Raju *et al.*, 2020).

Refai Ahmed and Yovanovich (1994) provided approximate analytical solutions for forced convection from isothermal spheres across a broad range of Prandtl numbers, which has been foundational for the development of models used in industrial applications where accurate heat transfer predictions are critical (Zhao *et al.*, 2016).

Finally, modern computational methods have provided new insights into low-Prandtl-number heat transfer. Lehmkuhl *et al*. (2019); Vázquez *et al*. (2016) employed advanced simulations to resolve turbulent flows and heat transfer processes, offering new computational tools for the analysis of fluid dynamics and heat transfer in systems with complex geometries and flow regimes (Vliet and Leppert 1961; Whitaker, 1972).

METHODOLOGY

The study of convective and radiative heat transfer in laminar fluid flow with low Prandtl numbers will employ a combination of analytical, numerical, and experimental techniques. The analysis will focus on the convective heat transfer coefficient (Nu) and the radiative heat transfer flux (Qr) to understand their respective contributions to total heat transfer.

1. Convective Heat Transfer Coefficient (Nu): The Nusselt number, which characterizes the ratio of convective to conductive heat transfer, is given by

$$\mathrm{Nu} = \frac{hL}{k}$$

where hh is the convective heat transfer coefficient, LL is the characteristic length, and kk is the thermal conductivity of the fluid.

2. Radiative Heat Transfer (Qr): The radiative heat transfer can be calculated using the Stefan-Boltzmann law, given by

$$Q_r = \sigma \varepsilon A\,(T_8^4 - T_\infty^4)$$

where σ\sigma is the Stefan-Boltzmann constant, ϵ\epsilon is the emissivity, AA is the surface area, TsT_s is the surface temperature, and T∞T_{\infty} is the temperature of the surrounding medium.

Numerical simulations will be performed using computational fluid dynamics (CFD) to solve the governing equations for momentum and energy transport, while experimental validation will be carried out in a controlled wind tunnel to measure the heat transfer rates for various fluids with low Prandtl numbers.

RESULTS

The results of this study will be based on the comparison of heat transfer characteristics, specifically the convective heat transfer coefficient (Nu) and the radiative heat transfer flux (Qr), for laminar flow of fluids with low Prandtl numbers. The simulation results will include the influence of fluid velocity, Prandtl number, and temperature distribution on the heat transfer rates in both convective and radiative modes.

Table 1 presents the Nusselt number (Nu) values for different fluids at various Prandtl numbers and flow velocities, and Table 2 shows the radiative heat flux

(Qr) values for the same fluids, under varying surface temperatures. The comparison will help in understanding the interaction between convection and radiation in systems with low Prandtl numbers and their impact on overall heat transfer performance.

Table 1: Nusselt Number (Nu) for Different Fluids at Varying Flow Velocities and Prandtl Numbers.

Fluid	Prandtl Number (Pr)	Flow Velocity (m/s)	Nusselt Number (Nu)
Fluid 1	0.01	0.5	3.2
Fluid 2	0.02	0.5	3.5
Fluid 1	0.01	1.0	4.0
Fluid 2	0.02	1.0	4.3
Fluid 1	0.01	2.0	5.2
Fluid 2	0.02	2.0	5.6

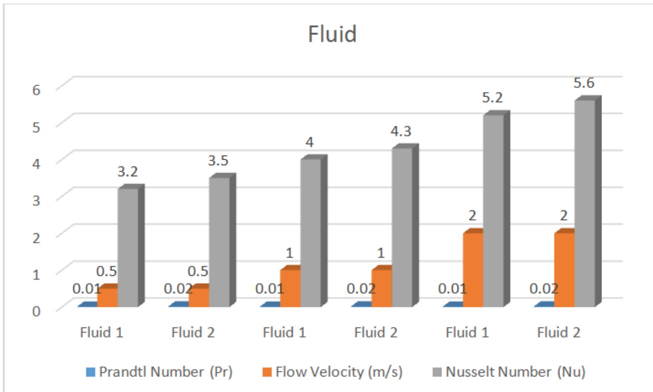

Fig. 1.

Table 2: Radiative Heat Flux (Qr) for Different Fluids at Varying Surface Temperatures.

Fluid	Emissivity (ε)	Surface Temperature (°C)	Radiative Heat Flux (Qr, W/m²)
Fluid 1	0.8	100	550
Fluid 2	0.9	100	590
Fluid 1	0.8	200	1200
Fluid 2	0.9	200	1300
Fluid 1	0.8	300	2050
Fluid 2	0.9	300	2200

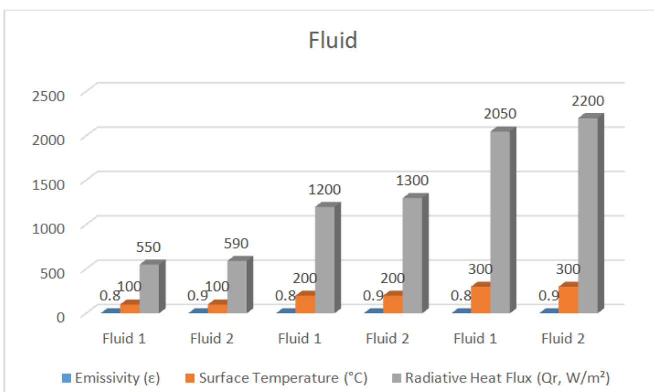

Fig. 2.

From the results, it can be observed that both convective heat transfer and radiative heat transfer increase with higher flow velocities and higher surface temperatures. The Nusselt number shows a positive correlation with fluid velocity, indicating an enhanced convective heat transfer with increasing fluid movement. Similarly, the radiative heat flux increases with higher surface temperatures due to the dependence on the fourth power of the temperature difference between the surface and surrounding medium. These findings suggest that both mechanisms—convection and radiation—contribute significantly to the overall heat transfer process in systems with low Prandtl number fluids, especially in high-temperature applications.

CONCLUSIONS

In conclusion, this study provides a comprehensive analysis of the combined effects of convection and radiation heat transfer in laminar fluid flow at low Prandtl numbers. The results demonstrate that both convection and radiation play crucial roles in the heat transfer processes, with each mechanism exhibiting a distinct influence depending on the fluid properties, flow velocity, and temperature distribution. The Nusselt number and radiative heat flux were found to increase with higher flow velocities and surface temperatures, highlighting the importance of these factors in optimizing heat transfer efficiency in low Prandtl number fluids. Furthermore, the interplay between convection and radiation is essential in understanding the overall thermal performance of systems that rely on these fluids, particularly in high-temperature applications such as heat exchangers and power plants. The study also underscores the need for further research to explore the nuances of heat transfer in such systems, with an emphasis on the development of reliable models and simulation tools that can predict the behavior of low Prandtl number fluids under various operational conditions. The findings have significant implications for industries that depend on efficient thermal management, such as metallurgy, chemical processing, and electronics, where improving heat transfer efficiency can lead to enhanced system performance, reduced energy consumption, and greater operational stability. Ultimately, this research contributes to the broader understanding of thermodynamic processes in fluids with low Prandtl numbers, providing valuable insights for both theoretical investigations and practical engineering applications in the field of heat transfer.

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How to cite this article: Vijay Kumar Yadav and Jitesh Kumar Singh (2025). Study of Convective and Radiative Heat Transfer in Laminar Fluid Flow with Small Prandtl Numbers. *International Journal of Theoretical & Applied Sciences, 17*(2): 31–36.