International Journal of Theoretical & Applied Sciences, 8(1): 60-66(2016)

ISSN No. (Print): 0975-1718
ISSN No. (Online): 2249-3247

Temperature Effect in a Thick Circular Plate Due to Asymmetric Heat Supply

Navneet Kumar
Research Scholar, Department of Mathematics,
RTM Nagpur University, Nagpur (Maharashtra State), India.

(Corresponding author: Navneet Kumar)
(Received 11 April, 2016 Accepted 02 May, 2016)
(Published by Research Trend, Website: www.researchtrend.net)

ABSTRACT: This paper presents the study of transient thermal stresses in a thick circular plate subjected to asymmetric temperature field on the Axial direction of the cylindrical co-ordinate system. Initially the plate is kept at zero temperature and an Arbirary heat flux is prescribed over the upper surface while lower surface maintain at zero degree with its circular boundary insulated. The displacement potential and thermal stresses are obtained by using integral transforms techniques.

Keywords: Quasi-static, Thick Plate, Asymmetric Heat Supply, Thermoelastic Problem, Thermal Stresses.

I. INTRODUCTION

During the second half of the Twentieth century, non isothermal problem of the theory of elasticity became increasingly important this is due to their wide application in diverse fields. The high relocation of modern aircraft give rise to aerodynamic heating, which produces intense thermal stresses that reduce the strength of the aircraft structure.

The initial boundary value problem of heat conduction is of considerable technological importance. Graya and Kozlowski (1982), investigated one dimensional transient thermoelastic problem and derived the heating temperature and heat flux on the surface of isotropic infinite slab [1]. Noda (1983) studied transient thermal stress problem in a finite circular transversely isotropic solid cylinder subjected to an asymmetrical temperature distribution on a cylindrical surface [2]. The stress fields are found by use of potential functions method. Deshmukh (1996) solved an inverse problem of thermoelasticity in a thin circular plate by determining the temperature on the curved surface of the plate, displacement and thermal stresses in the plate by using quasis-static approach by employing integral transform techniques [3].

Khobragade and Kumar (2011) determined coupled thermal stresses in a axisymmetric hollow cylinder [4]. Recently, Kumar and Khobragade (2012) found thermal stresses in a thin object by using Integral transform methods for inverse problem of heat conduction with known boundary [5].

This paper deals with the study of transient thermal stresses in a thick circular plate subjected to a arbitrary heat conduction at upper surface while the lower surface of the circular plate kept at zero temperature and its circular boundary is insulated. Initially thick circular plate kept at zero temperature.The governing heat condition equation is solved by using integral transform technique.

This paper contains, new and novel contribution of thermal stresses in quasi-static thick plate under steady state. The result presented here will be more useful in engineering problem particularly in the determination of the state of strain in thick circular plate constituting foundations of containers for hot gases or liquids, in the foundations for furnaces etc.

II. FORMULATION OF PROBLEMS

Let usconsider a quasi static thick circular plate of radius a and thickness $2h$ occupying the region $0 \le r \le a$, $h \le z \le h$ as shown in Fig. 1. Also it is assumed that plate be subjected to asymmetric temperature field on the Axial direction of the cylindrical co-ordinate system. Initially the plate is kept at zero temperature the Arbirary heat flux is prescribed over the upper surface ($z = h$) and lower surface ($z = -h$) maintain at zero degree while the fixed circular edge ($r = a$) thermally insulated. Assume the upper and lower surface of the thick circular plate are traction free under these prescribed condition transient thermal stresses are required to be determined.

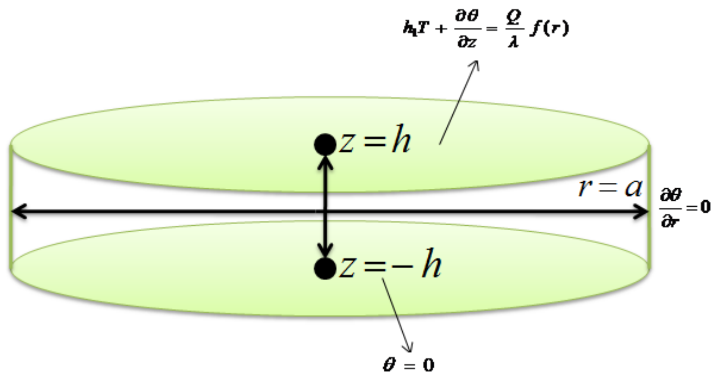

Fig. 1. Configuration of the thick circular plate.

A. Heat Conduction

The temperature of the plate at *t* satisfies the heat conduction equation [6]

$$\frac{\partial^2\theta}{\partial r^2}+\frac{1}{r}\frac{\partial\theta}{\partial r}+\frac{\partial^2\theta}{\partial z^2}=\frac{1}{k}\frac{\partial\theta}{\partial t} \tag{1}$$

with Boundary Conditions

$$h_1T+\frac{\partial\theta}{\partial z}=\frac{Q}{\lambda}f(r) \qquad for\ z=h,\ 0\le r\le a \tag{2}$$

$$\theta=0 \qquad for\ z=-h,\ 0\le r\le a \tag{3}$$

$$\frac{\partial\theta}{\partial r}=0 \qquad at\ \ r=a\ ,\ -h\le z\le h \tag{4}$$

and the initial condition

$$\theta=0 \qquad at \qquad t=0 \tag{5}$$

Where *k* thermal diffusivity of the material of the plate.

B. Thermal Stresses and Displacement Potential

The differential equation governing the displacement potential function ϕ(r,z,t) is given

$$\frac{\partial^2\phi}{\partial r^2}+\frac{1}{r}\frac{\partial\phi}{\partial r}+\frac{\partial^2\phi}{\partial z^2}=k\tau \tag{6}$$

Where k is restraint coefficient and temperature change $\tau=\theta-\theta_i$ is the initial temperature displacement function ϕ is known as Goodier's thermoelastic displacement Potential.

The displacement function in the cylindrical co-ordinate system are represented by Michell's function

Michell's function M must satisfy $\nabla^2\nabla^2M=0$ (7)

where

$$\nabla^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2} \tag{8}$$

$$U_r=\frac{\partial\phi}{\partial r}+\frac{\partial^2M}{\partial r\partial z} \tag{9}$$

$$U_z=\frac{\partial\phi}{\partial z}+2(1-v)\nabla^2M-\frac{\partial^2M}{\partial z^2} \tag{10}$$

the component of the stresses are represented by the thermoelastic displacement potential ϕ and Michell's function M

$$\sigma_{rr}=2G\left[\frac{\partial^2\phi}{\partial r^2}-k\tau+\frac{\partial}{\partial z}\left(v\nabla^2M-\frac{\partial^2M}{\partial r^2}\right)\right] \tag{11}$$

$$\sigma_{\theta\theta} = 2G\left[\frac{1}{r}\frac{\partial \phi}{\partial r} - k\tau + \frac{\partial}{\partial z}\left(v\nabla^2 M - \frac{1}{r}\frac{\partial M}{\partial r}\right)\right] \tag{12}$$

$$\sigma_{zz} = 2G\left[\frac{\partial^2 \phi}{\partial z^2} - k\tau + \frac{\partial}{\partial z}\left((2-v)\nabla^2 M - \frac{\partial^2 M}{\partial z^2}\right)\right] \tag{13}$$

and

$$\sigma_{rz} = 2G\left[\frac{\partial^2 \phi}{\partial rdz} + \frac{\partial}{\partial z}\left((1-v)\nabla^2 M - \frac{\partial^2 M}{\partial z^2}\right)\right] \tag{14}$$

G and *v* are shear modulus and Poisson's ratio respectively for traction free surface stresses function.

$$\left.\begin{aligned} \sigma_{zz} = \sigma_{rz} = 0 \qquad & at \quad z = h \\ \sigma_{rr} = \sigma_{rz} = 0 \qquad & at \quad r = a \end{aligned}\right\} \tag{15}$$

The equation (1) to (15) constitute Mathematical formulation of the problem.

III. SOLUTION OF THE PROBLEM

On applying the Laplace and Hankel transform technique and their inversions to the equation (1) and using the equations (1) to (5) one obtain,

$$\theta = \left(\frac{2Q}{a^2\lambda}\right)\sum_{n=1}^{\infty}\frac{\overline{f}(\alpha_n)J_0(\alpha_n r)}{p_m J_0^2(\alpha_n a)}$$

$$\left[\frac{k\sqrt{\alpha_n^2+\frac{P_m}{k}}.Sinh\left[2h\sqrt{\alpha_n^2+\frac{P_m}{k}}.(z+h)\right]}{2h.Cosh\left[2h\sqrt{\alpha_n^2+\frac{P_m}{k}}\right]\langle 2hh_1+1\rangle+2h\sqrt{\alpha_n^2+\frac{P_m}{k}}.Sinh\left[2h\sqrt{\alpha_n^2+\frac{P_m}{k}}.(z+h)\right]}\right]\left(e^{p_m t}-1\right) \tag{16}$$

Where p is Laplace transform parameter.

And P_m be the root of $\left\{h_1 Sinh\left(2h\sqrt{\alpha_n^2+\frac{P}{k}}\right)+\sqrt{\alpha_n^2+\frac{P}{k}}.Cosh\left[2h\sqrt{\alpha_n^2+\frac{P}{k}}\right]\right\}$

A. Goodiers thermoelastic Displacement Potential ϕ.

Assuming the displacement function ϕ (r,z,t) which satisfy (6)

$$\frac{\partial^2\phi}{\partial r^2}+\frac{1}{r}\frac{\partial\phi}{\partial r}+\frac{\partial^2\phi}{\partial z^2}=k\tau.$$

$$\phi = \left(\frac{2Qk}{a^2\lambda}\right)\sum_{n=1}^{\infty}\overline{f}(\alpha_n)\frac{J_0(\alpha_n r)}{p_m.J_0^2(\alpha_n r)}$$

$$\left[\frac{k\sqrt{\alpha_n^2+\frac{P_m}{k}}.Sinh\left[2h\sqrt{\alpha_n^2+\frac{P_m}{k}}.(z+h)\right]}{2h\,Cosh\left[2h\sqrt{\alpha_n^2+\frac{P_m}{k}}\right]\langle 2hh_1+1\rangle+2h\sqrt{\alpha_n^2+\frac{P_m}{k}}\left(Sin\,h\left[\left(2h\sqrt{\alpha_n^2+\frac{P_m}{k}}\right)\right]\right)}\right]\left(\frac{e^{(p_m t)}-1}{(4h^2-1)\alpha_n^2+\frac{4h^2 p_m}{k}}\right) \tag{17}$$

Now let assume Michell's function M which satisfy the condition (7)

$$M = \left(\frac{2Qk}{a^2\lambda}\right)\sum_{n=1}^{\infty}\left\{\left(\frac{\overline{f}(\alpha_n).J_0(\alpha_n r)}{p_m.J_0^2(\alpha_n a)}\right)\left(H_n\,Sinh\left[\alpha_n(z+h)\right]+R_n\,\alpha_n(z+h)Cosh\left[\alpha_n(z+h)\right]\right)\right\} \tag{18}$$

H_n and R_n are Arbitrary Constant.

IV. DISPLACEMENT AND THERMAL STRESSES DETERMINATION

Now using the equation (16) to (18) in the equation (9) and (10) one obtain the expression for displacement as,

$$U_r = \left(\frac{2Qk}{a^2\lambda}\right)\sum_{n=1}^{\infty}\frac{\overline{f}(\alpha_n)J_1(\alpha_n r)}{p_m.J_0^2(\alpha_n a)}$$

$$\left[\frac{(-\alpha_n)k\sqrt{\alpha_n^2+\frac{P_m}{k}}.Sinh\left[2h\sqrt{\alpha_n^2+\frac{P_m}{k}}(z+h)\right]}{2hCosh\left[2h\sqrt{\alpha_n^2+\frac{P_m}{k}}\right]\langle 2hh_1+1\rangle+2h\sqrt{\alpha_n^2+\frac{P_m}{k}}.Sinh\left[2h\sqrt{\alpha_n^2+\frac{P_m}{k}}\right]}\right]\frac{e^{p_m t}-1}{(4h^2-1)\alpha_n^2+\frac{4h^2 p_m}{k}}$$

$$+\sum_{n=1}^{\infty}\alpha_n^2 H_n J_1(\alpha_n r).Cosh\left[\alpha_n(z+h)\right]$$

$$+\sum_{n=1}^{\infty}\alpha_n^2 R_n J_1(\alpha_n r)\left\{.Cosh\left[\alpha_n(z+h)\right]+\alpha_n(z+h).Sinh\left[\alpha_n\left(z+\frac{h}{2}\right)\right]\right\} \qquad (19)$$

$$U_z = \frac{2Qk}{a^2\lambda}\sum_{n=1}^{\infty}\frac{\overline{f}(\alpha_n)J_0(\alpha_n r)}{p_m J_0^2(\alpha_n a)}$$

$$\left[\frac{2kh\left(\alpha_n^2+\frac{P_m}{k}\right).Cosh\left[2h\sqrt{\alpha_n^2+\frac{P_m}{k}}(z+h)\right]}{2hCosh\left[2h\sqrt{\alpha_n^2+\frac{P_m}{k}}\right]\langle 2hh_1+1\rangle+2h\sqrt{\alpha_n^2+\frac{P_m}{k}}.Sinh\left[2h\sqrt{\alpha_n^2+\frac{P_m}{k}}\right]}\right]x\frac{e^{p_m t}-1}{(4h^2-1)\alpha_n^2+\frac{4h^2 p_m}{k}}$$

$$-\sum_{n=1}^{\infty}\alpha_n^2 H_n J_0(\alpha_n r).Sinh\left[\alpha_n(z+h)\right]$$

$$+\sum_{n=1}^{\infty}\alpha_n^2 R_n J_0(\alpha_n r)\left\{2(1-2v)Sinh\left[\alpha_n(z+h)\right]-\alpha_n(z+h).Cosh\left[\alpha_n(z+h)\right]\right\} \qquad (20)$$

Now using the equation (16) to (18) in the equation (11) to (14) one obtain the expression for stresses as,

$$\sigma_{rr} = \frac{2QGk}{a^2\lambda}\sum_{n=1}^{\infty}\frac{\overline{f}(\alpha_n)}{J_0^2(\alpha_n a)}\left(\alpha_n J_0(\alpha_n r)-\frac{J_1(\alpha_n r)}{r}\right)$$

$$\left\{\frac{k\sqrt{\alpha_n^2+\frac{P_m}{k}}.Sinh\left[2h\sqrt{\alpha_n^2+\frac{P_m}{k}}(z+h)\right]}{2h.Cosh\left[2h\sqrt{\alpha_n^2+\frac{P_m}{k}}\right]\langle 2hh_1+1\rangle+2h\sqrt{\alpha_n^2+\frac{P_m}{k}}.Sinh\left[2h\sqrt{\alpha_n^2+\frac{P_m}{k}}\right]}\right\}\frac{e^{p_m t}-1}{(4h^2-1)\alpha_n^2+\frac{4h^2 p_m}{k}}$$

$$-K\sum_{n=1}^{\infty}\frac{2\overline{f}(\alpha_n)}{P_m}\frac{J_0(\alpha_n r)}{J_0^2(\alpha_n a)}$$

$$\frac{k\sqrt{\alpha_n^2+\frac{P_m}{k}}.Sinh\left[2h\sqrt{\alpha_n^2+\frac{P_m}{k}}(z+h)\right]}{2h.Cosh\left[2h\sqrt{\alpha_n^2+\frac{P_m}{k}}\right]\langle 2hh_1+1\rangle+2h\sqrt{\alpha_n^2+\frac{P_m}{k}}.Sinh\left[2h\sqrt{\alpha_n^2+\frac{P_m}{k}}\right]}\left(e^{p_m t}-1\right)$$

$$+\sum_{n=1}^{\infty}\alpha_n^2 H_n\left(\alpha_n J_0(\alpha_n)-\frac{J_1(\alpha_n r)}{r}\right).Cosh\left[\alpha_n(z+h)\right]+\alpha_n^2 R_n$$

$$\left\langle 2v\alpha_n J_0(\alpha_n r).Cosh\left[\alpha_n(z+h)\right]+\left(\alpha_n J_0(\alpha_n r)-\frac{J_1(\alpha_n r)}{r}\right)\right.$$
$$\left.\left(Cosh\left[\alpha_n(z+h)+\right]\alpha_n(z+h).Sinh\left[\alpha_n(z+h)\right]\right)\right\rangle \tag{21}$$

$$\sigma_{\theta\theta}=\frac{2QGk}{a^2\lambda}\left\{\sum_{n=1}^{\infty}\frac{\overline{f}(\alpha_n)J_1(\alpha_n r)}{p_m.J_0^2(\alpha_n a).r}\right.$$
$$\left[\frac{(-\alpha_n)k\sqrt{\alpha_n^2+\frac{P_m}{k}}.Sinh\left[2h\sqrt{\alpha_n^2+\frac{P_m}{k}}(z+h)\right]}{2h.Cosh\left[2h\sqrt{\alpha_n^2+\frac{P_m}{k}}\right]\langle 2hh_1+1\rangle+2h\sqrt{\alpha_n^2+\frac{P_m}{k}}.Sinh\left[2h\sqrt{\alpha_n^2+\frac{P_m}{k}}\right]}\right]\frac{e^{p_m t}-1}{\left(4h^2-1\right)\alpha_n^2+\frac{4h^2p_m}{k}}$$
$$-K\sum_{n=1}^{\infty}\frac{2\overline{f}(\alpha_n)}{P_m}\frac{J_0(\alpha_n r)}{J_0^2(\alpha_n a)}$$
$$\frac{k\sqrt{\alpha_n^2+\frac{P_m}{k}}.Sinh\left[2h\sqrt{\alpha_n^2+\frac{P_m}{k}}(z+h)\right]}{2h.Cosh\left[2h\sqrt{\alpha_n^2+\frac{P_m}{k}}\right]\langle 2hh_1+1\rangle+2h\sqrt{\alpha_n^2+\frac{P_m}{k}}.Sinh\left[2h\sqrt{\alpha_n^2+\frac{P_m}{k}}\right]}\left(e^{p_m t}-1\right)$$
$$+\alpha_n^2H_n\left(\frac{J_1(\alpha_n r)}{r}\right)Cosh\left[\alpha_n(z+h)\right]+$$
$$\alpha_n^2R_n\left\langle 2v\alpha_n J_0(\alpha_n r).Cosh\left[\alpha_n(z+h)\right]+\left(\frac{J_1(\alpha_n r)}{r}\right)\left[Cosh\left[\alpha_n(z+h)\right]+\alpha_n(z+h)Sinh\left[\alpha_n(z+h)\right]\right]\right\rangle \tag{22}$$

$$\sigma_{zz}=\left(\frac{2QGk}{a^2\lambda}\right)\left\{\sum_{n=1}^{\infty}\overline{f}(\alpha_n)\frac{J_0(\alpha_n r)}{J_0^2(\alpha_n a)}\right.$$
$$\left[\frac{4h^2k\left(\alpha_n^2+\frac{P_m}{k}\right)^{3/2}.Sinh\left[2h\sqrt{\alpha_n^2+\frac{P_m}{k}}(z+h)\right]}{2hCosh\left[2h\sqrt{\alpha_n^2+\frac{P_m}{k}}\right]\langle 2hh_1+1\rangle+2h\sqrt{\alpha_n^2+\frac{P_m}{k}}.Sinh\left[2h\sqrt{\alpha_n^2+\frac{P_m}{k}}\right]}\right]\frac{\left(e^{p_m t}-1\right)}{\left[\left(4h^2-1\right)\alpha_n^2+\frac{4h^2p_m}{k}\right]}$$
$$-\sum_{n=1}^{\infty}\frac{\overline{f}(\alpha_n)J_0(\alpha_n r)}{p_m J_0^2(\alpha_n a)}$$
$$\left\{\frac{k\sqrt{\alpha_n^2+\frac{P_m}{k}}.Sinh\left[2h\sqrt{\alpha_n^2+\frac{P_m}{k}}(z+h)\right]}{2h.Cosh\left[2h\sqrt{\alpha_n^2+\frac{P_m}{k}}\right]\langle 2hh_1+1\rangle+2h\sqrt{\alpha_n^2+\frac{P_m}{k}}.Sinh\left[2h\sqrt{\alpha_n^2+\frac{P_m}{k}}\right]}\right\}\left(e^{p_m t}-1\right)$$
$$-\sum_{n=1}^{\infty}\alpha_n^3H_nJ_0(\alpha_n r).Cosh\left[\alpha_n(z+h)\right]+$$
$$\left.\sum_{n=1}^{\infty}\alpha_n^3R_nJ_0(\alpha_n r)\left\langle(1-2v).Cosh\left[\alpha_n(z+h)\right]-\alpha_n(z+h).Sinh\left[\alpha_n(z+h)\right]\right\rangle\right\} \tag{23}$$

$$\sigma_{rz} = \left(\frac{2QGk}{a^2\lambda}\right)\sum_{n=1}^{\infty}\frac{\overline{f}(\alpha_n)\,J_1(\alpha_n r)}{J_0^2(\alpha_n a)}$$

$$\left[\frac{(-\alpha_n)2kh\left[\left(\alpha_n^2+\frac{P_m}{k}\right)\right].Cosh\left[2h\sqrt{\alpha_n^2+\frac{P_m}{k}}(z+h)\right]}{2hCosh.\left[2h\sqrt{\alpha_n^2+\frac{P_m}{k}}\right]\langle 2hh_1+1\rangle+2h\sqrt{\alpha_n^2+\frac{P_m}{k}}.Sinh\left[2h\sqrt{\alpha_n^2+\frac{P_m}{k}}\right]}\right]\left[\frac{\left(e^{p_m t}-1\right)}{\left[\left(4h^2-1\right)\alpha_n^2+\frac{4h^2 p_m}{k}\right]}\right]$$

$$+\sum_{n=1}^{\infty}\alpha_n^3 H_n J_1(\alpha_n r).Cosh\left[\alpha_n(z+h)\right]-$$

$$\sum_{n=1}^{\infty}\alpha_n^3 R_n J_1(\alpha_n r)\left\langle 2v.Sinh\left[\alpha_n(z+h)\right]+\alpha_n(z+h).Cosh\left[\alpha_n(z+h)\right]\right\rangle\Big\}$$

(24)

where

$$H_n = \frac{1}{J_0(\alpha_n r)\alpha_n^3}\left[\frac{k\sqrt{\alpha_n^2+\frac{P_m}{k}}.Sinh\left[2h\sqrt{\alpha_n^2+\frac{P_m}{k}}\right]\left(e^{p_m t}-1\right)}{2hCosh.\left[2h\sqrt{\alpha_n^2+\frac{P_m}{k}}\right]\langle 2hh_1+1\rangle+2h\sqrt{\alpha_n^2+\frac{P_m}{k}}.Sinh\left[2h\sqrt{\alpha_n^2+\frac{P_m}{k}}\right]}\right] \quad (25)$$

$$\left[\frac{1}{\left[\left(4h^2-1\right)\alpha_n^2+\frac{4h^2 p_m}{k}\right]}-1\right]$$

$$R_n = 0 \quad (26)$$

V. SPECIAL CASE

Setting $f(r) = (r^2 - a^2)^2$ (27)

Applying finite Hankel transform to the equation (27) one obtain

$$\overline{f}(\alpha_n) = \int_0^a r\left(r^2-a^2\right)J_0(\alpha_n r)\,dr,$$

$$\overline{f}(\alpha_n) = \frac{8a\left\{\left(8-a^2\alpha_n^2\right)J_1(\alpha_n a)-4a\,\alpha_n J_0(\alpha_n a)\right\}}{\alpha_n^5} \quad (28)$$

VI. CONCLUSION

In this paper, we have discussed completely the transient thermal stresses in a thick circular plate subjected to asymmetric temperature field on the Axial direction of the cylindrical co-ordinate system. The Laplace and Hankel transforms technique are used to obtain the numerical results. The temperature, displacement and thermal stresses are obtained may be applied to the design of useful structures or machines in engineering applications.

REFERENCES

[1]. Grysa, K. and Kozlowski, Z. (1982). One dimensional problems of temperature and heat Flux determination at the surfaces of a thermoelastic slab, part i: the analytical solutions, *Nucl. Engg. Des.,* Vol. **74**, pp.1-14.
[2]. Noda, N. (1983). Transient Thermal Stress Problem in a Transversely Isotropic Finite Circular Cylinder Under Three-Dimensional Temperature Field, *Journal of Thermal Stresses.* Vol. **6**, Issue 1, pp. 57-71.

[3]. Deshmukh, K.C. (1996). A Study of some aspect of inverse Thermoelastic problem. Ph.D. Thesis.
[4]. Khobragade, N. W. and Kumar, Navneet (2011). Analysis of Coupled Thermal Stresses in a Axisymmetric Hollow Cylinder, *International journal of Latest trend in mathematics.* Vol. **1**, issue 2, pp. 29-32.
[5]. Kumar, Navneet and N.W. Khobragade, (2012). "Integral transform methods for inverse problem of heat conduction with known boundary of a thin rectangular object and its stresses, *Journal of thermal sciences.* Vol. **21**, Issue 5, pp 459-465.
[6]. Yoshihiro Ochiai, (1996). "Axisymmetric thermal stress analysis in the steady state with heat generation in the region by boundary element method, *Journal of thermal stresses.* Vol. **19**, Issue, 3 pp 273-286.