International Journal of Theoretical & Applied Sciences, **1**(1): 72-81(2009)

Dispersion of axisymmetric waves in thermomicrostretch elastic plate

Rajneesh Kumar and Geeta Partap

Department of Mathematics, Kurukshetra University, Kurukshetra, Haryana
Department of Mathematics, Dr. B. R. Ambedkar National Institute of Technology, Jalandhar, Punjab

ABSTRACT

The dispersion of axisymmetric waves in thermomicrostretch elastic plate subjected to stress free conditions is investigated in the context of Green and Lindsay (G-L) theory of thermoelasticity. Mathematical modeling of the problem of obtaining dispersion curves leads to coupled differential equations. The model has been simplified by using Helmholtz decomposition technique and the resulting equations have been solved by using variable separable method to obtain the secular equations for both symmetric and skew-symmetric wave mode propagation. These vibration modes are found to be dispersive and dissipated in character. At short wavelength limits, the secular equations reduce to Rayleigh surface wave frequency equations. The dispersion curves and attenuation coefficients are computed analytically and presented graphically.

Keywords: Microstretch, Thermoelastic, Secular equations, Phase velocity, Attenuation coefficient.

INTRODUCTION

In classical theory of elasticity, the points of the material have translational degrees of freedom and the transmission of the load across a differential element of the surface is described by a force vector only. However, in the theory of micropolar elasticity, there is an additional degree of freedom characterized by rotation of material points, and there is an additional kind of stress called couple stress. Thus, in the classical theory of elasticity, the effect of couple stress is neglected.

Eringen [1] introduced the theory of microstretch elastic solids.This theory is a generalization of the theory of micropolar elasticity [2]. The material points of microstretch solids can stretch and contract independently of their translations and rotations. Thus, in these solids, the motion is characterized by seven degrees of freedom namely three for translation, three for rotation and one for stretch.

The transmission of the load across a differential element of the surface of a microstretch elastic solid is described by a force vector, a couple stress vector and a microstretch vector. The theory of microstretch elastic solid differs from the theory of micropolar elasticity in the sense that there is an additional degree of freedom called stretch and there is an additional kind of stress called microstretch vector. The materials like porous elastic material filled with gas or inviscid fluid, asphalt, composite fibers etc. lie in the category of microstretch elastic solids.

Eringen [3] extended the theory of microstretch elastic solids to include heat conduction. In the framework of the theory of thermomicrostretch elastic solids, Eringen established a uniqueness theorem for the mixed boundary-initial value problem. The theory was illustrated with the solution of one-dimensional waves and compared with lattice dynamical results. The asymptotic behavior of solutions and an existence result were presented by Bofill and Quintanilla [4].

Singh [5] studied the reflection and refraction of plane waves at a liquid/thermomicrostretch elastic solid interface. Singh [6] studied reflection of plane waves from free surface of microstretch elastic solid. Kumar and Deswal [7] studied wave propagation through a cylindrical bore contained in a microstretch elastic medium. Kumar and Deswal [8] examined surface wave propagation through a cylindrical bore in a microstretch generalized thermoelastic medium without energy dissipation. Singh and Kumar [9] discussed the problem of reflection and refraction of plane waves at an interface between two dissimilar micropolar elastic solid half-space with stretch. Iesan and Quintanilla [10] investigated thermal stresses in microstretch elastic plates. Svanadze and Cicco [11] analyzed fundamental solution in the theory of thermomicrostretch elastic solids. Tomar and Garg [12] investigated reflection and transmission of waves from a plane interface between two microstretch solid half-spaces. Iesan and Scalia [13] discussed propagation of singular surfaces in thermo-microstretch continua with memory. Singh and Tomar [14] investigated Rayleigh-Lamb waves in a microstretch elastic plate cladded with liquid layers. Sharma, Kumar and Sharma [15] studied propagation of Rayleigh waves in micro-stretch thermoelastic continua under inviscid fluid loadings.

The present investigation is concerned to study the dispersion of axisymmetric waves in an infinite homogeneous isotropic thermomicrostretch elastic plate of finite thickness.

Basic equations

Following Eringen [16] and Green and Lindsay [17], the equations of motion and the constitutive relations in a homogeneous isotropic generalized thermomicrostretch elastic solid in the absence of body forces, body couples, stretch force and heat sources are given as

$$(\lambda+2\mu+K)\nabla(\nabla\cdot\vec{u})-(\mu+K)\nabla\times\nabla\times\vec{u}+K\nabla\times\vec{\phi}-\nu(1+\tau_1\frac{\partial}{\partial t})\nabla T+\lambda_0\nabla\phi^* = \rho\frac{\partial^2\vec{u}}{\partial t^2}, \quad ..(1)$$

$$(\alpha+\beta+\gamma)\nabla(\nabla\cdot\vec{\phi})-\gamma\,\nabla\times(\nabla\times\vec{\phi})+K\nabla\times\vec{u}-2K\vec{\phi}=\rho\, j\frac{\partial^2\vec{\phi}}{\partial t^2}, \quad ..(2)$$

$$\alpha_0\nabla^2\phi^*+\nu_1(T+\tau_1\frac{\partial T}{\partial t})-\lambda_1\phi^*-\lambda_0\nabla\cdot\vec{u}=\frac{\rho\, j_0}{2}\frac{\partial^2\phi^*}{\partial t^2}, \quad \ldots(3)$$

$$K^*\nabla^2 T=\rho\, C^*(\frac{\partial T}{\partial t}+\tau_0\frac{\partial^2 T}{\partial t^2})+\nu T_0\frac{\partial}{\partial t}(\nabla\cdot\vec{u})+\nu_1 T_0\frac{\partial}{\partial t}\phi^*, \quad \ldots(4)$$

$$t_{ij}=\lambda u_{r,r}\delta_{ij}+\mu(u_{i,j}+u_{j,i})+K(u_{j,i}-\varepsilon_{ijr}\phi_r)-\nu(T+\tau_1\frac{\partial T}{\partial t})\delta_{ij}+\lambda_0\delta_{ij}\phi^*, \quad \ldots(5)$$

$$m_{ij}=\alpha\phi_{r,r}\delta_{ij}+\beta\phi_{i,j}+\gamma\phi_{j,i}+b_0\varepsilon_{mji}\phi^*_{,m} \quad \ldots(6)$$

$$\lambda_i^*=\alpha_0\phi^*_{,i}+b_0\varepsilon_{ijm}\phi_{j,m} \quad . \quad \ldots(7)$$

where $\lambda,\mu,\alpha,\beta,\gamma,K,\alpha_0,b_0,\lambda_0,\lambda_1$ are material constants, ρ is the density, j is the microinertia, j_0 is the microinertia of microelements, t_{ij} and m_{ij} are the components of stress and couple stress tensors respectively, $\vec{u}=(u_r,u_\theta,u_z)$ is the displacement vector, $\vec{\phi}=(\phi_r,\phi_\theta,\phi_z)$ is the microrotation vector and ϕ^* is the scalar point microstretch function, λ_i^* is the microstress tensor, T is the temperature change, T_0 is uniform temperature, $\nu=(3\lambda+2\mu+K)\alpha_{t_1}$, $\nu_1=(3\lambda+2\mu+K)\alpha_{t_2}$, α_{t_1} and α_{t_2} are the coefficients of linear thermal expansion and K^* is the coefficient of thermal conductivity, C^* is specific heat at constant strain, δ_{ij} is Kronecker delta, τ_0 and τ_1 are thermal relaxation times. The comma notation denotes spatial derivatives.

Formulation of the problem

We consider an infinite homogeneous isotropic thermally conducting microstretch elastic plate bounded by two parallel surfaces free of tractions at $z = \pm d$, z = 0 being the midplane of the plate. The circular cylindrical co-ordinates (r, θ, z) have been used to describe the response of the plate of thickness 2d as shown in Fig.1.The plate is axisymmetric with the z-axis as the axis of the symmetry.

We take r – z plane as the plane of incidence.

For two dimensional problem, we take

$\vec{u} = (u_r, 0, u_z)$ and $\vec{\phi} = (0, \phi_\theta, 0)$. …(8)

To facilitate the solution, we introduce the following dimensionless quantities

$$r' = \frac{\omega^* r}{c_1},\ z' = \frac{\omega^* z}{c_1},\ u_r' = \frac{\rho\omega^* c_1}{\nu T_0}u_r,\ u_z' = \frac{\rho\omega^* c_1}{\nu T_0}u_z,\ t' = \omega^* t,\ \phi_\theta' = \frac{\rho c_1^2}{\nu T_0}\phi_\theta,\ \phi^{*\prime} = \frac{\rho c_1^2}{\nu T_0}\phi^*,\ T' = \frac{T}{T_0},$$

$$\tau_0' = \omega^*\tau_0,\ \tau_1' = \omega^*\tau_1,\ t_{ij}' = \frac{1}{\nu T_0}t_{ij},\ m_{ij}' = \frac{\omega^* m_{ij}}{c_1 \nu T_0},\ h' = \frac{c_1 h}{\omega^*},\ p = \frac{K}{\rho c_1^2},\ p_1 = \frac{\lambda_1}{\rho c_1^2},\ p_0 = \frac{\lambda_0}{\rho c_1^2},$$

$$\delta^2 = \frac{c_2^2}{c_1^2},\ \delta_1^2 = \frac{c_3^2}{c_1^2},\ \delta_2^2 = \frac{c_4^2}{c_1^2},\ \delta^* = \frac{K c_1^2}{\gamma\omega^{*2}},\ \delta_1^* = \frac{\rho c_1^4}{\alpha_0\omega^{*2}},\ \bar{\nu} = \frac{\nu_1}{\nu},\ \lambda_i^{*\prime} = \frac{\omega^*\lambda_i^*}{c_1 \nu T_0}. \quad \text{…(9)}$$

where $c_1^2 = \frac{\lambda + 2\mu + K}{\rho}$, $c_2^2 = \frac{\mu + K}{\rho}$, $c_3^2 = \frac{\gamma}{\rho j}$, $c_4^2 = \frac{2\alpha_0}{\rho j_0}$, $\in = \frac{\nu^2 T_0}{\rho^2 C^* c_1^2}$, $\omega^* = \frac{\rho C^* c_1^2}{K^*}$, ω^* is the characteristic frequency of the medium. We introduce the potential functions ϕ and ψ through the relations

$$u_r = \frac{\partial\phi}{\partial r} + \frac{\partial\psi}{\partial z},\ u_z = \frac{\partial\phi}{\partial z} - \frac{\partial\psi}{\partial r} - \frac{\psi}{r}, \quad \text{…(10)}$$

Using equations (8) – (10) in equations (1) - (4) and after suppressing the primes for convenience, we obtain

$$(\nabla^2 - \frac{\partial^2}{\partial t^2})\phi + p_0\phi^* - T(1 + \tau_1\frac{\partial}{\partial t}) = 0, \quad \text{…(11)}$$

$$(\nabla^2\psi - \frac{\psi}{r^2}) - \frac{p\phi_\theta}{\delta^2} - \frac{1}{\delta^2}\frac{\partial^2\psi}{\partial t^2} = 0, \quad \text{…(12)}$$

$$(\nabla^2 - \frac{1}{r^2})\phi_\theta + \delta^*(\nabla^2 - \frac{1}{r^2})\psi - 2\delta^*\phi_\theta - \frac{1}{\delta_1^2}\frac{\partial^2\phi_\theta}{\partial t^2} = 0, \quad \text{…(13)}$$

$$\nabla^2\phi^* - p_1\delta_1^*\phi^* - p_0\delta_1^*\nabla^2\phi + \bar{\nu}\delta_1^* T(1 + \tau_1\frac{\partial}{\partial t}) - \frac{1}{\delta_2^2}\frac{\partial^2\phi^*}{\partial t^2} = 0, \quad \text{…(14)}$$

$$\nabla^2 T - (\dot{T} + \tau_0\ddot{T}) = \in\nabla^2\dot{\phi} + \bar{\nu}\in\frac{\partial\phi^*}{\partial t}. \quad \text{…(15)}$$

where $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$ is the Laplacian operator.

Boundary Conditions

We consider the following mechanical and thermal boundary conditions at surfaces $z = \pm d$

3.1.1 Mechanical Conditions

The non-dimensional mechanical boundary conditions at $z = \pm d$ are given as follows

$$t_{zz} = 0,\ t_{zr} = 0,\ m_{z\theta} = 0,\ \lambda_{,z}^* = 0. \quad \text{…(16)}$$

3.1.2 Thermal Conditions

The thermal boundary conditions at $z = \pm d$ are given by

$$T_{,z} + hT = 0. \quad \text{…(17)}$$

where h is the surface heat transfer coefficient . Here $h \to 0$ corresponds to thermally insulated boundaries and $h \to \infty$ refers to isothermal one.

Formal solution of the problem

We assume the solutions of equations (11) – (15) of the form

$$(\phi,\psi,\phi_\theta,T,\phi^*) = \left[f(z)J_0(\xi r), g(z)J_1(\xi r), w(z)J_1(\xi r), h(z)J_0(\xi r), \eta(z)J_0(\xi r)\right]e^{-i\omega t}, \qquad \ldots(18)$$

where ω is the circular frequency, ξ is the wave number and $J_0(\xi r)$ and $J_1(\xi r)$ are the Bessel functions of order zero and one respectively.

Using equation (18) in equations (11) - (15) and solving the resulting differential equations, the expressions for ϕ, ψ, ϕ_θ, T and ϕ^* are obtained as

$$\phi = (A_1 Cosm_1 z + A_2 Cosm_2 z + A_3 Cosm_3 z + B_1 Sinm_1 z + B_2 Sinm_2 z + B_3 Sinm_3 z) J_0(\xi r) e^{-i\omega t}, \qquad \ldots(19)$$

$$\psi = (A_4 Cosm_4 z + B_4 Sinm_4 z + A_5 Cosm_5 z + B_5 Sinm_5 z) J_1(\xi r) e^{-i\omega t}, \qquad \ldots(20, 21)$$

$$T = [S_1(A_1 Cosm_1 z + B_1 Sinm_1 z) + S_2(A_2 Cosm_2 z + B_2 Sinm_2 z) \qquad + S_3(A_3 Cosm_3 z + B_3 Sinm_3 z)] J_0(\xi r) e^{-i\omega t}, \ldots(22)$$

$$\phi^* = [V_1(A_1 Cosm_1 z + B_1 Sinm_1 z) + V_2(A_2 Cosm_2 z + B_2 Sinm_2 z) \qquad + V_3(A_3 Cosm_3 z + B_3 Sinm_3 z)] J_0(\xi r) e^{-i\omega t} \ \ldots(23)$$

where $m_i^2 = \xi^2 (c^2 a_i^2 - 1)$, $i = 1,2,3,4,5$; $b^2 = \xi^2 (\frac{c^2}{\delta^2} - 1)$, $k_0 = \tau_0 + i\omega^{-1}$, $k_1 = i\omega^{-1}$,

$$a_4^2 + a_5^2 = \frac{1}{\delta^2} + \frac{1}{\delta_1^2} + \frac{\delta^*(p - 2\delta^2)}{\omega^2 \delta^2} \ , \ a_4^2 a_5^2 = \frac{1}{\delta^2}(\frac{1}{\delta_1^2} - \frac{2\delta^*}{\omega^2}),$$

$$\sum a_i^2 = 1 + k_0 + \frac{1}{\delta_2^2} - \frac{p_1 \delta_1^*}{\omega^2} + \frac{p_0^2 \delta_1^*}{\omega^2} - i\omega \in k_1 k_0 \ ,$$

$$\sum a_i^2 a_j^2 = k_0 + \frac{1 + k_0}{\delta_2^2} - \frac{\delta_1^*}{\omega^2}\left[(1 + k_0) p_1 - p_0^2 k_0 + i\omega \in k_0 k_1 (\bar{\nu} p_0 - p_1) - i\omega \in k_0 k_1 \bar{\nu}(\bar{\nu} - p_0)\right] - \frac{i\omega \in k_0 k_1}{\delta_2^2}$$

$$a_1^2 a_2^2 a_3^2 = k_0 (\frac{1}{\delta_2^2} - \frac{p_1 \delta_1^*}{\omega^2}) - \frac{i k_0 k_1 \in \delta_1^* \bar{\nu}^2}{\omega}, \qquad V_i = \frac{-\delta_1^*\left[(\bar{\nu} - p_0) m_i^2 - \bar{\nu}\xi^2 (c^2 + \frac{p_0}{\bar{\nu}} - 1)\right]}{m_i^2 - \xi^2\left[c^2(\frac{1}{\delta_2^2} - \frac{\delta_1^*}{\omega^2}(\bar{\nu} p_0 - p_1)) - 1\right]}, \qquad i, j = 1,2,3$$

$$S_i = \frac{\left[m_i^2 - \xi^2 (c^2 - 1)\right]\left[m_i^2 - \xi^2 (c^2 (\frac{1}{\delta_2^2} - \frac{\delta_1^*}{\omega^2}(\bar{\nu} p_0 - p_1)) - 1\right] + \delta_1^* p_0 \left[(\bar{\nu} - p_0) m_i^2 - \bar{\nu}\xi^2 (c^2 + \frac{p_0}{\bar{\nu}} - 1)\right]}{i\omega k_1 [m_i^2 - \xi^2 (c^2 (\frac{1}{\delta_2^2} - \frac{\delta_1^*}{\omega^2}(\bar{\nu} p_0 - p_1)) - 1)]},$$

Derivation of the secular equations

Using the boundary conditions (16) and (17) on the surfaces $z = \pm d$ of the plate and with the help of equations (19) – (23), we obtain a system of ten simultaneous equations. This system of ten simultaneous equations has a non-trivial solution if the determinant of the coefficients of amplitudes $[A_1, A_2, A_3, A_4, A_5, B_1, B_2, B_3, B_4, B_5]^T$ vanishes. We obtain the following secular equations after applying lengthy algebraic reductions and manipulations.

$$
\begin{aligned}
&\left[\frac{T_1}{T_4}\right]^{\pm 1} - \frac{m_1(V_1S_3 - V_3S_1)}{m_2(V_2S_3 - V_3S_2)}\left[\frac{T_2}{T_4}\right]^{\pm 1} + \frac{m_1(V_1S_2 - V_2S_1)}{m_3(V_2S_3 - V_3S_2)}\left[\frac{T_3}{T_4}\right]^{\pm 1} + \frac{RV}{SU}\frac{(f_5 - f_4)}{(m_5 f_5T_4 - m_4 f_4T_5)} \\
&\frac{m_1S_1(V_3 - V_2)}{m_2m_3(V_2S_3 - V_3S_2)}\left\{\left[\frac{T_2T_3}{T_4^2T_5}\right]^{\pm 1} - \frac{m_2S_2(V_3 - V_1)}{m_1S_1(V_3 - V_2)}\left[\frac{T_1T_3}{T_4^2T_5}\right]^{\pm 1} + \frac{m_3S_3(V_2 - V_1)}{m_1S_1(V_3 - V_2)}\left[\frac{T_1T_2}{T_4^2T_5}\right]^{\pm 1}\right\} \\
&+ \frac{Q}{P}\frac{(m_4 - m_5(\frac{T_5}{T_4})^{\pm 1})}{[m_5 f_5 - m_4 f_4(\frac{T_5}{T_4})^{\pm 1}]}\left\{\begin{array}{l}\frac{V}{U}\frac{f_5 f_4(S_3 - S_2)}{(V_2S_3 - V_3S_2)}\left[\left[\frac{T_1}{T_4}\right]^{\pm 1} - \frac{m_1(S_3 - S_1)}{m_2(S_3 - S_2)}\left[\frac{T_2}{T_4}\right]^{\pm 1} + \frac{m_1(S_2 - S_1)}{m_3(S_3 - S_2)}\left[\frac{T_3}{T_4}\right]^{\pm 1}\right] \\ + \frac{R}{S}\left[\left[\frac{T_1}{T_4}\right]^{\pm 1} - \frac{m_1V_2(V_1S_3 - V_3S_1)}{m_2V_1(V_2S_3 - V_3S_2)}\left[\frac{T_2}{T_4}\right]^{\pm 1} + \frac{m_1V_3(V_1S_2 - V_2S_1)}{m_3V_1(V_2S_3 - V_3S_2)}\left[\frac{T_3}{T_4}\right]^{\pm 1}\right]\end{array}\right\} \\
&= \frac{-Q^2m_1m_4m_5[(V_1 - V_2)(S_2 - S_3) - (V_2 - V_3)(S_1 - S_2)](f_5 - f_4)}{P^2(V_2S_3 - V_3S_2)[m_5 f_5 - m_4 f_4(\frac{T_5}{T_4})^{\pm 1}]} \\
&- \frac{Q^2}{P^2}\frac{RV}{SU}\frac{(m_5 f_4T_4 - m_4 f_5T_5)}{(m_5 f_5T_4 - m_4 f_4T_5)}\frac{V_1(S_3 - S_2)}{(V_2S_3 - V_3S_2)}\left\{\begin{array}{l}\left[\frac{T_1}{T_4}\right]^{\pm 1} - \frac{m_1V_2(S_3 - S_1)}{m_2V_1(S_3 - S_2)}\left[\frac{T_2}{T_4}\right]^{\pm 1} \\ + \frac{m_1V_3(S_2 - S_1)}{m_3V_1(S_3 - S_2)}\left[\frac{T_3}{T_4}\right]^{\pm 1}\end{array}\right\} \qquad (24)
\end{aligned}
$$

For stress free thermally insulated boundaries $(h \to 0)$ of the plate.

$$
\begin{aligned}
&\left[\frac{T_1}{T_4}\right]^{\pm 1} - \frac{m_1S_2(V_3 - V_1)}{m_2S_1(V_3 - V_2)}\left[\frac{T_2}{T_4}\right]^{\pm 1} + \frac{m_1S_3(V_2 - V_1)}{m_3S_1(V_3 - V_2)}\left[\frac{T_3}{T_4}\right]^{\pm 1} + \frac{RV}{SU}\frac{(m_5 f_4T_4 - m_4 f_5T_5)}{m_4m_5(f_5 - f_4)}\frac{m_1(V_2S_3 - V_3S_2)}{m_2m_3S_1(V_3 - V_2)} \\
&\times\left\{\left[\frac{T_2T_3}{T_4}\right]^{\pm 1} - \frac{m_2(V_1S_3 - V_3S_1)}{m_1(V_2S_3 - V_3S_2)}\left[\frac{T_1T_3}{T_4}\right]^{\pm 1} + \frac{m_3(V_1S_2 - V_2S_1)}{m_1(V_2S_3 - V_3S_2)}\left[\frac{T_1T_2}{T_4}\right]^{\pm 1}\right\} + \frac{P}{Q}\frac{(m_4T_5 - m_5T_4)}{m_4m_5(f_5 - f_4)}\times \\
&\left\{\begin{array}{l}\frac{U}{V}\frac{m_1 f_5 f_4(S_3 - S_2)}{m_2m_3S_1(V_3 - V_2)}\left[\left[\frac{T_2T_3}{T_4}\right]^{\pm 1} - \frac{m_2(S_3 - S_1)}{m_1(S_3 - S_2)}\left[\frac{T_1T_3}{T_4}\right]^{\pm 1} + \frac{m_3(S_2 - S_1)}{m_1(S_3 - S_2)}\left[\frac{T_1T_2}{T_4}\right]^{\pm 1}\right] + \\ \frac{R}{S}\frac{m_1V_1(V_2S_3 - V_3S_2)}{m_2m_3S_1(V_3 - V_2)}\left[\left[\frac{T_2T_3}{T_4}\right]^{\pm 1} - \frac{m_2V_2(V_1S_3 - V_3S_1)}{m_1V_1(V_2S_3 - V_3S_2)}\left[\frac{T_1T_3}{T_4}\right]^{\pm 1} + \frac{m_3V_3(V_1S_2 - V_2S_1)}{m_1V_1(V_2S_3 - V_3S_2)}\left[\frac{T_1T_2}{T_4}\right]^{\pm 1}\right]\end{array}\right\} \\
&= \frac{V_2(S_3 - S_1)[1 - \frac{m_4 f_4}{m_5 f_5}(\frac{T_5}{T_4})^{\pm 1}] f_5P^2}{Q^2m_3m_4S_1(V_3 - V_2)(f_5 - f_4)}\left\{\left[\frac{T_1T_3}{T_4^2}\right]^{\pm 1} - \frac{m_1V_1(S_3 - S_2)}{m_2V_2(S_3 - S_1)}\left[\frac{T_2T_3}{T_4^2}\right]^{\pm 1} - \frac{m_3V_3(S_2 - S_1)}{m_2V_2(S_3 - S_1)}\left[\frac{T_1T_2}{T_4^2}\right]^{\pm 1}\right\} \\
&+ \frac{P^2}{Q^2}\frac{RV}{SU}\frac{S_2(V_3 - V_1)}{S_1(V_3 - V_2)m_2m_3m_4m_5}\left[\frac{T_1T_2T_3}{T_4^2T_5}\right]^{\pm 1}\left\{1 - \frac{S_1(V_3 - V_2)}{S_2(V_3 - V_1)} - \frac{S_3(V_2 - V_1)}{S_2(V_3 - V_1)}\right\} \qquad (25)
\end{aligned}
$$

For stress free isothermal boundaries $(h \to \infty)$ of the plate.

where

$P = b^2 - \xi^2 + \dfrac{p\xi^2}{\delta^2}$, $Q = -2\xi(1 - \dfrac{p}{2\delta^2})$, $f_i = b^2 - m_i^2$, $i = 4,5$; $R = i\xi\, b_0$, $S = \dfrac{\gamma\delta^2}{p}$,

$U = \alpha_0$, $V = \dfrac{b_0 i\xi\,\delta^2}{p}$, $T_i = \tan m_i d$, i = 1,2,3,4,5 .

Here the superscript +1 refers to skew-symmetric and -1 refers to symmetric modes. Equations (24) and (25) are the most general dispersion relations involving wave number and phase velocity of various modes of propagation in a microstretch generalized thermoelastic plates under the considered situations.

Particular cases

5.1.1 Micropolar elastic plate

In the absence of stretch effect, we have R = V = U = $V_1 = V_3 = 0$, $V_2 = 1$ and $S_2 = 0$, $S_i = i\omega^{-1}k_1^{-1}(m_i^2 - a^2), i = 1,3$; where $a^2 = \xi^2(c^2 - 1)$ and the secular equations (24) and (25) can be reduced accordingly.

Thermoelastic plate

In the absence of micropolarity effect (K = p = 0), we have $m_4^2 = b^2$, $m_5^2 = \xi^2(\frac{c^2}{\delta_1^2} - 1)$, and the secular equations (24) and (25) for insulated and isothermal boundaries can be reduced accordingly.

5.1.3 Elastic plate

In the absence of thermal effect ($K^* = C^* = \nu = 0$), the secular equations (24) and (25) can be reduced accordingly.

Regions of the secular equation

In order to explore various regions of the secular equations, here we consider the equation (24) as an example for the purpose of discussion. Depending upon whether $m_1, m_2, m_3, m_4, m_5, b$ being real, purely imaginary or complex, the frequency equations (24) and (25)are correspondingly altered as follows:

Region I

When the characteristic roots are of the type, $b^2 = -b'^2$, $m_k^2 = -\alpha_k^2$, k = 1,2,3,4,5 so that $b = ib'$, $m_k = i\alpha_k$, k = 1, 2, 3,4,5 are purely imaginary or complex numbers. This ensures that the superposition of partial waves has the property of exponential decay. In this case, the secular equations are written from equations (24) and (25) by replacing circular tangent functions of m_k, k = 1, 2, 3,4,5 with hyperbolic tangent functions of α_k, k = 1, 2, 3,4,5.

Region II

This region is characterized by δ < c < 1. In this case, we have $m_k^2 = -\alpha_k^2$, k = 1,2,3 ; $m_k^2 = m_k^2$ for k = 4,5 and the secular equations can be obtained from equations (24) and (25) by replacing circular tangent functions of m_k, k = 1, 2, 3 with hyperbolic tangent functions of α_k, k = 1, 2, 3.

Region III

In this case, the characteristic roots are given by m_k^2, k = 1, 2,3,4,5 and the secular equations are given by equations (24) and (25).

Waves of short wavelength

Some information on the asymptotic behavior is obtainable when the transverse wavelength with respect to the thickness of the plate is quite small, so that $\xi d >> 1$. Then the characteristic roots b, m_k, k = 1,2,3,4,5 lie in Region I and secular equations (24) and (25) can be reduced accordingly.

Numerical results and discussion

With the view of illustrating theoretical results obtained in the preceding sections and comparing these in the context of G-L theory of thermoelasticity, we now present some numerical results. The material chosen for this purpose is Magnesium crystal (microstretch thermoelastic solid), the physical data for which is given below

Micropolar parameters are

$\rho = 1.74 \times 10^3 Kg/m^3$,

$\lambda = 9.4 \times 10^{10} N/m^2$,

$\mu = 4.0 \times 10^{10} N/m^2$,

$K = 1.0 \times 10^{10} N/m^2$,

$\gamma = 0.779 \times 10^{-9} N$,

$j = 0.2 \times 10^{-19} m^2$,

$j_0 = 0.185 \times 10^{-19} m^2$.

Thermal parameters are

$\tau_0 = 6.131 \times 10^{-13}$ sec, $\tau_1 = 8.765 \times 10^{-13}$ sec, $\in = 0.028$,

$T_0 = 298^0 K$,

$C^* = 1.04 \times 10^3 J/Kg \deg$,

$K^* = 1.7 \times 10^6 J/m \sec \deg$, $\nu = 2.68 \times 10^6 N/m^2 \deg$,

$\nu_1 = 2.0 \times 10^6 N/m^2 \deg$

and stretch parameters are

$\lambda_0 = 0.5 \times 10^{10} N/m^2$,

$\lambda_1 = 0.5 \times 10^{10} N/m^2$, $\alpha_0 = 0.779 \times 10^{-9} N$,

$b_0 = 0.5 \times 10^{-9} N$, $d = 0.01 m$.

A FORTRAN program has been developed for the solution of equation (24) to compute phase velocity c for different values of n by using the relations $\tan\theta = \tan(n\pi + \theta)$ and $m_i^2 = \xi^2(c^2 a_i^2 - 1)$

The phase velocity and attenuation coefficient of symmetric and skew-symmetric modes of wave propagation in the context of G-L theory of thermoelasticity have been computed for various values of wave number from dispersion equation (24) for stress free thermally insulated boundaries and have been represented graphically for different modes (n = 0 to n = 1) in Figs. 2 - 5. The solid curves correspond to thermomicrostretch elastic plate (MSTE), dotted curves refer to microstretch elastic plate (MSE) and broken-line curves correspond to micropolar thermoelastic plate (MTE).

Phase velocity

The phase velocities of higher modes of propagation, symmetric and skew-symmetric attain quite large values at vanishing wave number, which sharply slashes down to become steady and asymptotic to the reduced Rayleigh wave velocity with increasing wave number.

For symmetric modes of wave propagation, we observe from Fig.2 that (a) The phase velocities of lowest mode of propagation become dispersionless i.e. remain constant with variation in wave number in case of micropolar thermoelastic plate (MTE) whereas the phase velocities of lowest mode of propagation become dispersionless i.e. remain constant with variation in wave number in case of micropolar thermoelastic plate (MTE) and microstretch elastic plate (MSE) for wave number $\xi \geq 4.2$ and $\xi \leq 9.2$; phase velocities are lowest for MSE and highest for MTE for wave number $\xi \leq 1.2$, phase velocities are lowest and highest for MSTE and MTE respectively for wave number lying between 1.2 and 2.2, phase velocities are highest for MSE and lowest for MSTE for wave number $\xi \geq 2.2$ and $\xi \leq 3.2$, and phase velocities are highest for MSTE and lowest for MSE for wave number $\xi \geq 3.2$ and $\xi \leq 4.2$ as evident from Fig.2 (b) for n = 1, the phase velocity profiles in respect of MSTE,MSE and MTE coincide.

From Fig. 4, It is observed for skew-symmetric modes of wave propagation that (*i*) for lowest mode n = 0, phase velocities are highest for MTE and lowest for MSTE For wave number $\xi \leq 0.5$, phase velocities are lowest and highest for MSE and MTE respectively for wave number lying between 0.5 and 0.8, phase velocities are highest for MSTE and lowest for MSE for wave number $\xi \geq 0.8$ and $\xi \leq 1.2$,phase velocities are lowest and highest for MSE and MSTE respectively for wave number lying between 1.2 and 2.2, phase velocities are highest for MTE and lowest for MSE for wave number $\xi \geq 2.2$ and $\xi \leq 3.2$, phase velocity profiles for MSTE and MTE coincide and phase velocity are lowest in respect of MSE for wave number $\xi \geq 3.2$. (*ii*) for n = 1, the phase velocity profiles in respect of MSTE and MTE coincide, phase velocity in MSE is more than MSTE and MTE for wave number $\xi \leq 5.2$, phase velocity for MSE is less than in case of MSTE and MTE for wave number $\xi \geq 5.2$.

Attenuation coefficients

In general, wave number and phase velocity of the waves are complex quantities, therefore, the waves are attenuated in space. If we write

$$c^{-1} = s^{-1} + i\omega^{-1}q \qquad \ldots(26$$

then $\xi = K_1 + iq$, where $K_1 = \omega / s$ and q are real numbers. This shows that s is the propagation speed and q is attenuation coefficient of waves. Upon using (26) in the FORTRAN program developed for the solution of equation (24) to compute phase velocity c, attenuation coefficient q for different modes of wave propagation can be obtained.

The attenuation coefficients of symmetric and skew-symmetric modes have been plotted in the context of thermomicrostretch elastic plate (MSTE), microstretch elastic plate (MSE) and micropolar thermoelastic plate (MTE) in Fig. 3 and Fig. 5 respectively.

(a) For lowest symmetric mode (n = 0) in respect of MSTE, the magnitude of attenuation coefficient has negligible variation with wave number in regions $0.2 \leq \xi \leq 2.2$ and $4.2 \leq \xi \leq 9.2$, the magnitude of attenuation coefficient have maxima upto 12.04 in region $0.2 \le \xi \le 4.2$ at $\xi = 3.2$, (b) For first symmetric mode (n = 1) in respect of MSTE, the attenuation coefficient has negligible variation with wave number in region $0.2 \le \xi \le 6.2$, the attenuation coefficient increases from .41 to 51.31 as wave number increases from 6.2 to 7.2, the attenuation coefficient decreases from 51.31 to 20.06 as wave number increases from 7.2 to 8.2, the attenuation coefficient increases from 20.06 to 119.2 as wave number increases from 8.2 to 9.2.© For lowest symmetric mode (n = 0) in respect of MSE, the magnitude of attenuation coefficient has negligible variation with wave number in regions $0.2 \le \xi \le 5.2$ and $7.2 \le \xi \le 9.2$, the magnitude of attenuation coefficient have maxima upto 59.50 in region $5.2 \le \xi \le 7.2$ at $\xi = 6.2$ (d) For first symmetric mode (n = 1) in respect of MSE, the magnitude of attenuation coefficient is observed to be quite high for wave number $\xi = 0.2$ in region $0.2 \le \xi \le 1.2$,the attenuation coefficient has negligible variation with wave number in region $1.2 \le \xi \le 4.2$, the magnitude of attenuation coefficient have maxima upto 64.46 in region $4.2 \le \xi \le 6.2$ at $\xi = 5.2$,the attenuation coefficient increases from .2411 to 55.71 as wave number increases from 6.2 to 7.2, the attenuation coefficient decreases from 55.71 to 30.91 as wave number increases from 7.2 to 8.2, the attenuation coefficient increases from 30.91 to 94.73 as wave number increases from 8.2 to 9.2. (e) For lowest symmetric mode (n = 0) in respect of MTE, the magnitude of attenuation coefficient has negligible variation with wave number in regions $0.2 \le \xi \le 1.2$ and $4.2 \le \xi \le 9.2$, the magnitude of attenuation coefficient attains values 7.613 and 9.525 at $\xi = 2.2$ and at $\xi = 3.2$ respectively in region $1.2 \le \xi \le 4.2$ (f) For first symmetric mode (n = 1) in respect of MTE, the attenuation coefficient has negligible variation with wave number in regions $0.2 \le \xi \le 4.2$ and $6.2 \le \xi \le 7.2$ the magnitude of attenuation coefficient shoots upto 165.4 at $\xi = 5.2$ in region $4.2 \le \xi \le 6.2$ and attains high values 114.1 and 138.4 at $\xi = 8.2$ and at $\xi = 9.2$ respectively in region $7.2 \le \xi \le 9.2$.

For skew-symmetric modes of wave propagation in case of MSTE, we observe that (i) for lowest mode (n = 0) the attenuation coefficient possesses nearly constant value in the region $0.2 \le \xi \le 2.2$, the magnitude of attenuation coefficient decreases to 12.92 at $\xi = 3.2$ and increases to 71.74 at $\xi = 4.2$ and further decreases to 16.69 at $\xi = 5.2$ in the region $2.2 \le \xi \le 6.2$, the attenuation coefficient has negligible variation with wave number in region $6.2 \le \xi \le 9.2$ (ii) for first mode (n = 1) the attenuation coefficient has negligible variation with wave number in regions $0.2 \le \xi \le 1.2$ and $6.2 \le \xi \le 9.2$, the attenuation coefficient attains values 13.66 and 8.533 at $\xi = 2.2$ and $\xi = 3.2$ respectively in region $1.2 \le \xi \le 3.2$, the magnitude of attenuation coefficient shoots upto 320.4 at $\xi = 4.2$ and decreases to 20.15 in region $3.2 \le \xi \le 6.2$.(iii) For lowest skew-symmetric mode (n = 0) in respect of MSE, the attenuation coefficient has negligible variation with wave number in region $0.2 \le \xi \le 9.2$ (iv) For first skew-symmetric mode (n = 1) in respect of MSE, the attenuation coefficient does not vary with wave number (v) For lowest skew-symmetric mode (n = 0) in respect of MTE, the magnitude of attenuation coefficient shoots from 89.86 at $\xi = 0.2$ to 343.6 at $\xi = 1.2$ and slashes to 15.36 at $\xi = 2.2$ in the region $0.2 \le \xi \le 2.2$, the magnitude of attenuation coefficient attains values 10.79, 50.85 and 26.15 at $\xi = 3.2$, $\xi = 4.2$ and $\xi = 5.2$ respectively in region $2.2 \le \xi \le 6.2$, the attenuation coefficient has negligible variation with wave number in regions $6.2 \le \xi \le 9.2$ (vi) For first skew-symmetric mode (n = 1) in respect of MTE, the attenuation coefficient has negligible variation with wave number in regions $0.2 \le \xi \le 1.2$ and $6.2 \le \xi \le 9.2$, the magnitude of attenuation coefficient have maximum value upto 53.28 in region $1.2 \le \xi \le 6.2$ at $\xi = 5.2$.

CONCLUSIONS

(i)The dispersion of axisymmetric waves in thermomicrostretch elastic plate subjected to stress free thermally insulated and isothermal boundary is studied in the context of Green and Lindsay (G-L) theory of thermoelasticity (ii) The secular equations for both symmetric and skew-symmetric vibration modes have been obtained (iii) At short wavelength limits, the secular equations for symmetric and skew-symmetric waves in stress free thermally insulated and isothermal thermomicrostretch elastic plate reduce to Rayleigh surface wave frequency equations (iv) The phase velocities of higher modes of propagation, symmetric and skew-symmetric attain quite large values at vanishing wave number, which sharply slashes down to become steady and asymptotic to the reduced Rayleigh wave velocity with increasing wave number

REFERENCES

[1] A.C. Eringen, Micropolar elastic solids with stretch, Ari Kitabevi Matbassi, Istanbul,**24**: 1 – 18(1971).

[2] A.C. Eringen, *J. Math. Mech.*, **15**, 909 (1966).

[3] A.C.Eringen, *Int. J. Engng. Sci.*, **28**: 1291(1990).

[4] F. Bofill and R. Quintanilla, *International Journal of Engineering Science*, **33**: 2115(1995).

[5] B. Singh, *Int. J. Engng. Sci.*, **39**: 583(2001).

[6] B. Singh, *Proc. Indian Acad. Sci. (Earth Planet Sci.)*, **111**: 29(2002).

[7] R. Kumar and S. Deswal, *Journal of Sound and Vibration*, **250**: 711(2002) .

[8] R. Kumar and S. Deswal, *Proc. Nat. Acad. Sci.*, **73**(A): 315(2003) .

[9] B. Singh and R. Kumar, *Proc. Nat. Acad. Sci. India*, **74(**A): 123(2004) .

[10] D. Iesan and R. Quintanilla, *Int. J. Engng. Sci.*, **43**: 885(2005) .

[11] M. Svanadze and S. De Cicco, *Int. J. Engng. Sci.*, **43**: 417(2005).

[12] S.K Tomar and M. Garg, *Int. J. Engng. Sci.*, **43**: 139(2005) .

[13] D. Iesan and A. Scalia, *Int. J. Engg. Sci.*, **44**: 845(2006).

[14] D. Singh and S.K. Tomar, *Journal of Sound and Vibration*, **302**: 313(2007)

[15] J.N. Sharma, S. Kumar and Y.D. Sharma, *J. Thermal Stresses*, **31**: 18(2008).

[16] A. C. Eringen, *Microcontinuum field theories I: Foundations and Solids* (Springer – Verlag, New York, 1999).

[17] 17. A.E.Green and K.A.Lindsay, *J. Elasticity*, **2**: 1(1972).

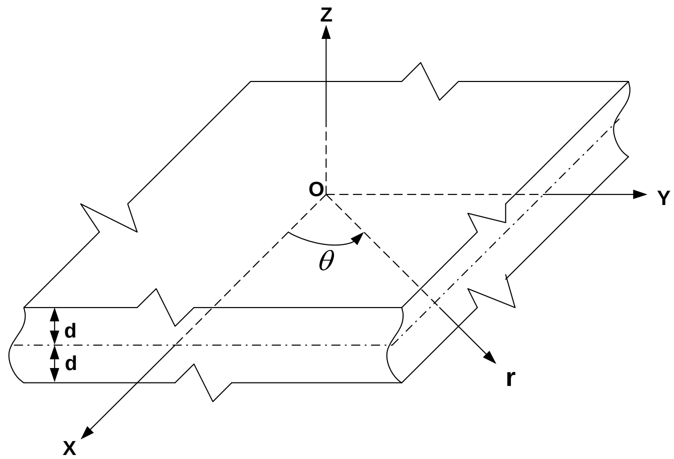

Fig.1 Geometry of the problem

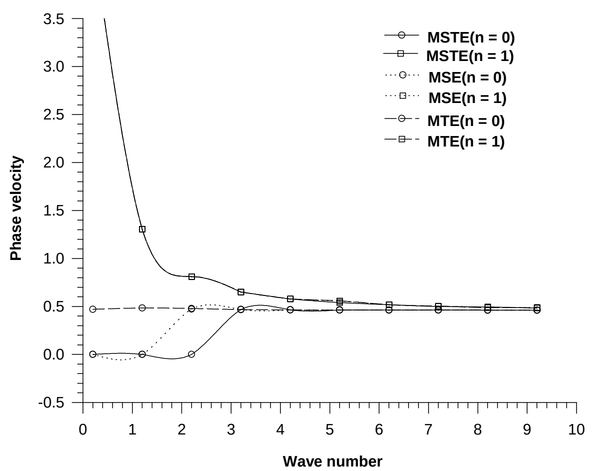

Fig.2 Variation of phase velocity of symmetric modes of wave propagation

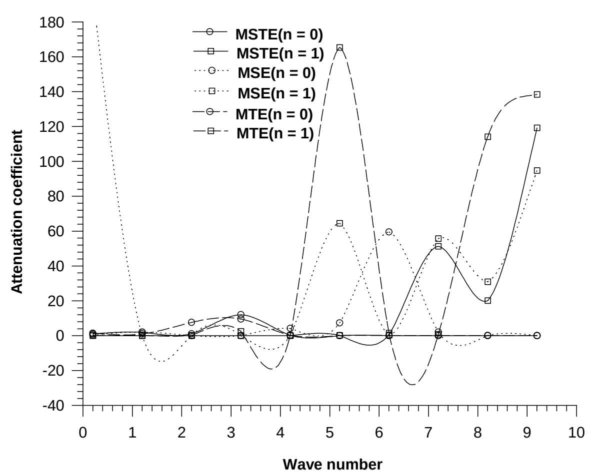

Fig.3 Variation of attenuation coefficient of symmetric modes of wave propagat

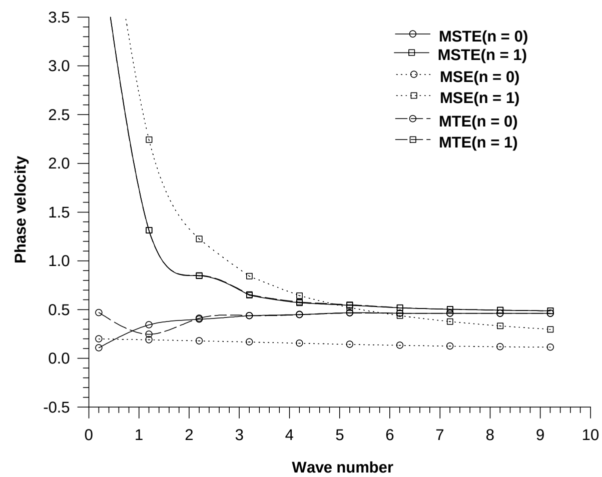

Fig.4 Variation of phase velocity of skew-symmetric modes of wave propagation

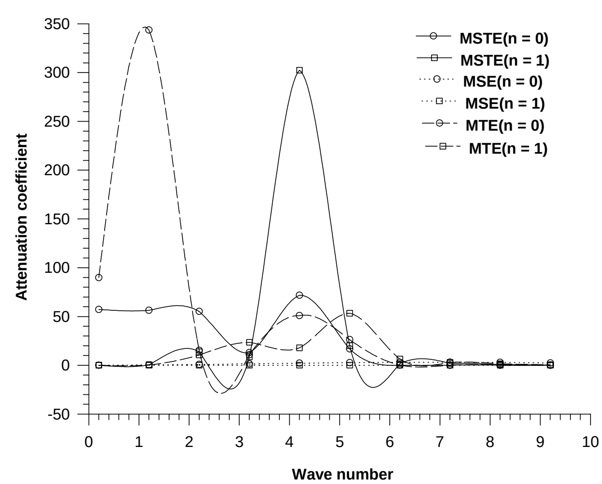

Fig.5 Variation of attenuation coefficient of skew-symmetric modes of wave propagati