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Study of two parameter formula for gamma band energies in Xe-Pt nuclei

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ABSTRACT : Two parameter formula is applied for gamma band energies of Xe-Pt, (68d"Nd"126) nuclei. This formula works better for soft as well as the deformed nuclei having fairly constant parameters $\Im_0$ and α with $R_{4/2}$. The variation of energy $E(2^+_\gamma)$ with number of valence proton and valence neutron (N_pN_n) is also discussed.

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INTRODUCTION

Our understanding of nuclear structure is framed within the context of number of idealized benchmarks. The study of gamma band energies of the A = 120-200 mass nuclei has been a subject of current interest. It include the axial rotor [1], the anhrmonic vibrator [2], γ-soft deformed nuclei [3-4] and new critical symmetries [5-8] for phase transitional regions. The several systematic studies shown that transitional nuclei exhibit the triaxial features. Zamfir and Casten [9] have discussed a number of signatures of γ-softness and γ-rigid in nuclei. The transitional region has been divided in three categories. In first the γ-soft region between the vibrator and deformed γ-soft nuclei. This region lie between SU(5) to O(6) symmetry. In second the γ-rigid region between the vibrator and axially symmetric rotor, this region lie between SU(5) to SU(3) symmetry. In third the triaxial γ-rigid region between the vibrator and rigid triaxial rotor, characterized by fixed γ values between 0° and 30°. McCutchan et al. [10] discussed the staggering in γ-band energies and the transition between different structural symmetries in nuclei. Chhail Bihari et al. [11-12] studied a new signature of the triaxial region in even nuclei and the evidence of rigid triaxiality in some even nuclei. They studied that whether the proposed even nuclei are γ-rigid at low energy and low spin by taking in mind that $\Delta E1 = E(3^+_1) - \{E2^+_1 - E2^+_2\} = 0$ not only for the γ-rigid rotor but also for the axial rotor. Both γ-axially symmetric and γ-rigid asymmetric nuclei follow the rotor formula $E \propto J(J + 1)$, and also for both $\Delta E1$ is very small as compared to the $\Delta E2$. There are several empirical formulae that describe the gamma band energies. The simplest expression for rotational spectra is

$$E = \frac{h^2}{2\Im} J(J + 1) \qquad ...(1)$$

where $\Im$ be the moment of inertia (*MI*) and J be the spin of nuclei. The Bohr-Mottelson [1] adapted the geometrical series expansion

$$E = AJ(J + 1) + BJ^2 (J + 1)^2 \qquad ...(2)$$

which is obtained as a first approximation from a classical calculation of centrifugal stretching. This equation is not enough to describe the experimental spectra with high spin values. An improved energy formula is of the form

$$E = AJ(J + 1) + BJ^2 (J + 1)^2 + CJ^3 (J + 1)^3 \qquad ...(3)$$

However, such a series tends to diverge for high spins and thus cannot be very useful. Further Brentano [13] noted that *MI* depends upon J and E,

$$\Im = \Im_0 (1 + \alpha J + \beta E) \qquad ...(4)$$

By dropping the energy-dependent term in eq. (4) and then putting it in eq. (1) he got two parameter formula

$$E = \frac{1}{\Im_0(1+\alpha J)} J(J + 1) \qquad ...(5)$$

This is also called the soft rotor formula (SRF). The purpose of the present work is to explain whether this two parameter formula is successful to explain gamma band energies of the A = 120-200 mass region. The variation of $E(2^+_\gamma)$ energy with number of valence proton and valence neutron (N_pN_n) is also discussed.

CALCULATION

The two parameter formula is

$$E = \frac{1}{\Im_0(1+\alpha J)} J(J + 1)$$

where $\Im_0$ and α be two parameters. The values of $\Im_0$ and α are calculated by using 2^+_γ and 4^+_γ energies in even sequence and 3^+_γ and 5^+_γ energies in odd sequence. The value of $\Im_0$ and α are calculated by least square fit method. The calculated gamma band energy of few even-even nuclei are presented in Table 1. The experimental data for this calculation of gamma band energies are taken www.bnl.nndc.gov [14].

Table 1 : Experimental and calculated (present work) gamma band energy for few even-even nuclei in keV.

Nuclei	Energies	2^+	3^+	4^+	5^+	6^+	7^+	8^+	9^+	10^+
^{126}Ba	Exp.	873.2	1162.0	1324.7	1672.2	1658.1	2285.3	2479.0	2975.1	3177.1
	PWork	873.2	1162.0	1324.7	1672.3	1800.8	2385.1	2333.3	2964.0	2831.1
^{128}Ce	Exp.	869.4	1138.5	1312.1	1663.4	1847.1	2298.3	2466.3	3001.3	3143.6
	PWork	869.4	1138.5	1312.1	1663.4	1780.9	2193.5	2255.2	2725.2	2731.8
^{146}Ce	Exp.	1381.9	1576.7	1810.3	2139.7	2365.6	2741.1	2943.5	3353.2	3529.4
	PWork	1381.9	1576.7	1810.3	2139.7	2365.6	2741.1	2943.5	3353.2	3529.4
^{152}Sm	Exp.	1085.9	1233.8	1371.7	1559.5	1728.3	1945.8	2139.7	2375.5	
	PWork	1085.9	1233.8	1371.7	1559.5	1777.0	1948.0	2202.5	2353.4	2635.1
^{156}Gd	Exp.	1154.1	1247.9	1355.3	1506.8	1644.8	1849.6	2010.8	2249.4	2442.5
	PWork	1154.1	1247.9	1355.3	1506.8	1727.5	1854.1	2125.7	2222.4	2532.9
^{160}Dy	Exp.	998.4	1057.5	1147.7	1261.0	1392.8	1548.6	1717.0	2118.0	
	PWork	998.4	1057.5	1147.7	1261.0	1456.3	1545.6	1788.6	1849.0	2128.9
^{162}Dy	Exp.	888.1	962.9	1060.9	1182.7	1324.4	1490.3	1670.5	1878.0	2087.4
	PWork	888.1	962.9	1060.9	1182.7	1357.2	1463.4	1672.9	1758.9	1995.1
^{162}Er	Exp.	900.6	1001.9	1128.2	1286.3	1459.7	1669.2	1872.8	2133.9	2346.7
	PWork	900.7	1001.9	1128.2	1286.3	1458.7	1615.3	1806.4	1955.9	2160.1
^{164}Er	Exp.	860.3	946.3	1058.2	1197.6	1358.6	1544.9	1744.7	1977.1	2184.1
	PWork	860.3	946.4	1058.3	1197.6	1362.6	1496.5	1684.3	1807.6	2012.0
^{166}Er	Exp.	785.0	859.3	956.2	1075.2	1215.9	1376.0	1555.7	1751.4	1964.0
	PWork	785.0	859.4	956.2	1075.2	1228.5	1338.5	1517.0	1613.6	1811.2
^{170}Yb	Exp.	1145.7	1225.3	1329.3	1459.7	1601.3	1780.5	2009.3	2170.0	2412.3
	PWork	1145.7	1225.3	1329.3	1459.8	1689.9	1788.6	2077.2	2139.4	2473.5
^{180}W	Exp.	1117.2	1232.6	1360.5	1535.6	1702.9	2133.0	2274.0	2423.8	2589.1
	PWork	1117.2	1232.7	1360.5	1535.6	1747.8	1908.0	2158.3	2299.5	2576.8
^{182}W	Exp.	1221.4	1331.2	1442.8	1620.5	1762.9	1971.1	2180.5	2480.2	2770.7
	PWork	1221.4	1331.3	1442.8	1620.5	1841.2	1999.3	2267.0	2399.4	2702.0
^{186}Os	Exp.	767.4	910.4	1070.4	1275.6	1491.2	1750.8	2015.5	2317.4	2624.9
	PWork	767.5	910.5	1070.5	1275.6	1421.4	1654.0	1781.6	2036.5	2145.1

RESULTS AND DISCUSSION

First we check the constancy of $\Im_0$ and α parameters. In Fig.1,2. we see the variation of $\Im_0$ and α parameters with energy ratio $R_{4/2}(= E(4_g^+)/E(2_g^+))$ for even sequence of energy. The large value of $\Im_0$ shows the small energy difference between the calculated and experimental energy levels. The α parameter shows the constant behavior with $R_{4/2}$. Similar variation is shown for odd sequence of energy. We summarize the results in Fig.3 in terms of differences between the experimental and calculated gamma band energies

$$\frac{dE(J)}{E(J)} = [E(J)_{th} - E(J)_{exp}]/E(J)_{exp} \quad ...(6)$$

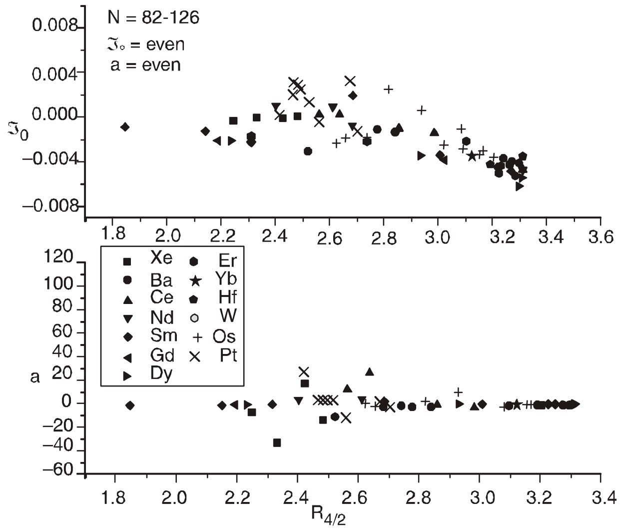

Fig.1. The value of fitted even parameters $\Im_0$ and α as a function of $R_{4/2}$ from the least square fit method.

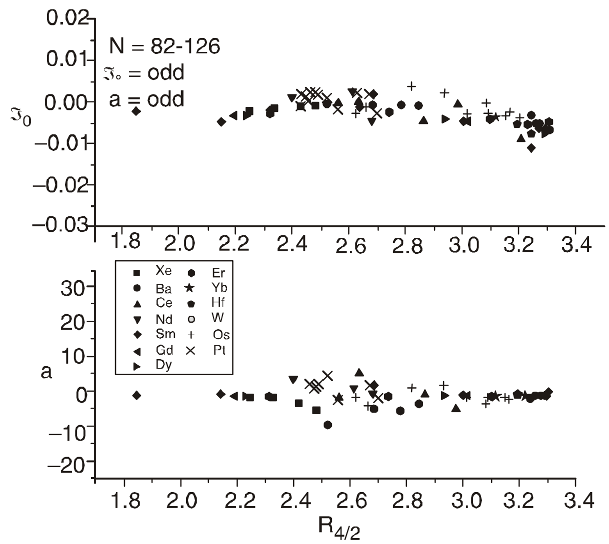

Fig.2. The value of fitted odd parameters $\Im_0$ and α as a function of $R_{4/2}$ from the least square fit method.

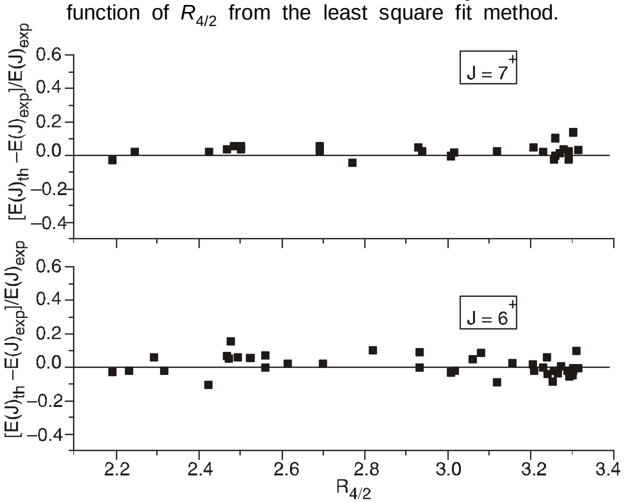

Fig.3. Relative difference between experimental and calculated gamma energies as a function of $R_{4/2}$ for $J=6^+$, 7^+.

Each point in the Fig.3 is plotted according to its energy ratio ($R_{4/2}$) for $J = 6^+$ and 7^+ spin. The quality of fit of nuclei on structure can be seen at a glance.

Gupta *et. al.* [15] grouped the A = 120-200 nuclei into four quadrants. First quadrant (Q-I) of $82 \leq N \leq 104$ is of Z = 50-82 shell space, with particle like proton-bosons and hole like neutron-bosons forming the p-p subspace. Second quadrant (Q-II) is of $82 \leq N \leq 104$ of Z = 50-82 shell space, with hole like proton-bosons and particle like neutron-bosons forming the *h-p* subspace. The third quadrant (Q-III) of $104 \leq N < 126$ of Z = 50-82 shell space, with hole like proton-bosons and hole like neutron-bosons forming the *h-h* subspace. The fourth hole like neutron-bosons forming the *p-h* subspace. In brief, quadrant I and quadrant (Q-IV) of $N < 82$ of Z = 50-82, N = 50-82 shell space with particle like proton-bosons and III for *p-p* and *h-h* bosons, and II, IV for *p-h* and *h-p* bosons respectively. We divided this region into four quadrants to see the behavior of various nuclei in different region. In $N < 82$ we take ^{128}Ba nuclei having $R_{4/2}$=2.6 means nuclei is γ-soft in nature. This $N < 82$ region is neutron deficient and lying away from the β-stability line. Therefore most of nuclei in this region are of γ-soft in nature. The calculated ground band energy shows good agreement with experimental values upto $J = 10^+$ but after these spin values calculated energy decreases (Fig.4).

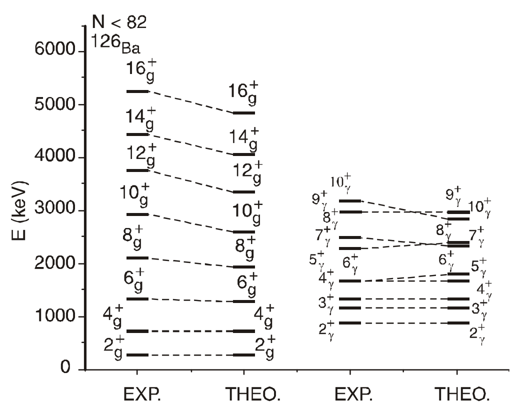

Fig.4. Comparison of experimental energies with fits of SRF formula for ^{128}Ba nuclei.

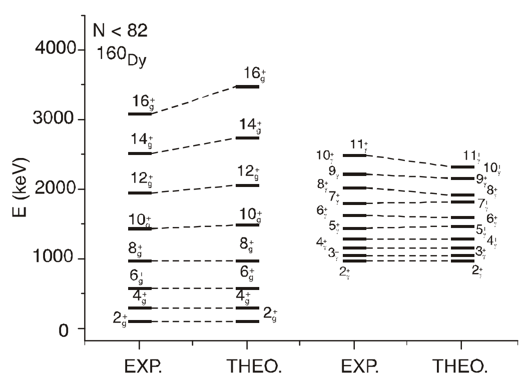

Fig.5. Comparison of experimental energies with fits of SRF formula for ^{180}Dy nuclei.

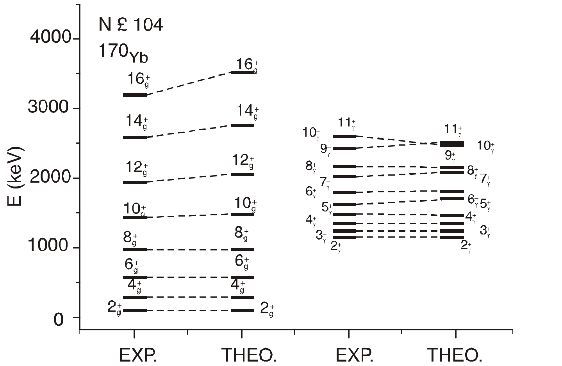

Fig.6. Comparison of experimental energies with fits of SRF formula for ^{170}Yb nuclei.

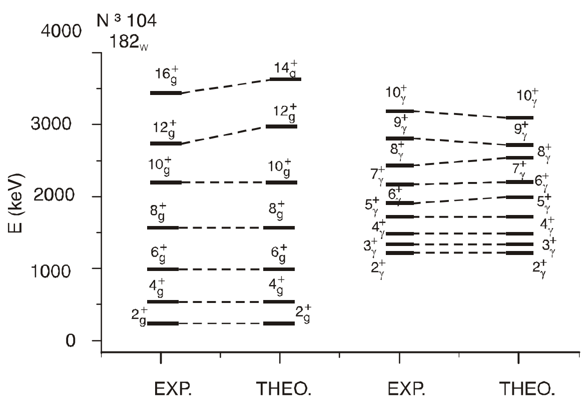

Fig.7. Comparison of experimental energies with fits of SRF formula for ^{182}W.

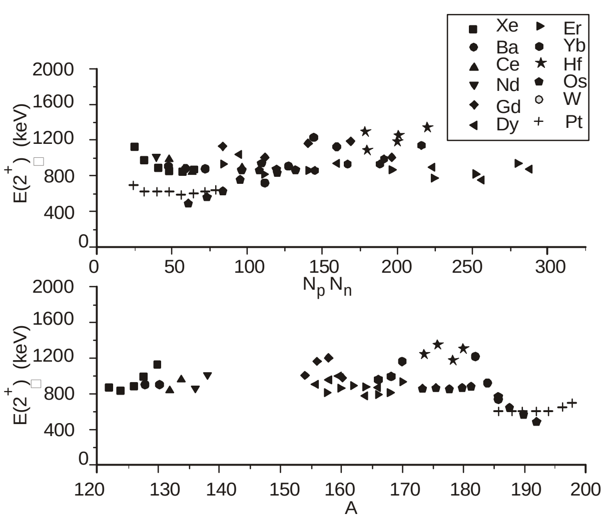

Fig.8. The variation of $E(2^+_\gamma)$ energy with N_pN_n and mass number A.

The calculated gamma band energy also shows good agreement at low spin, but at high spin the energy band crosses to each other. In N > 82 for ^{180}Dy the $R_{4/2}$=3.2, this nuclei lies in deformed region (Fig.5). The calculated ground band energy show good agreement with experimental values upto J = 12^+ but after this the values show small rise. The gamma band energy of ^{180}Dy nuclei also show the good agreement with the experimental values. Next in N $\leq$ 104 for ^{170}Yb nuclei and N>104 for ^{182}W nuclei show good agreement for both ground band energy and gamma band energy values at low and high spin values (Fig.6,7). Therefore with increasing neutron number the deformation increases. Hence the agreement between the calculated and experimental energy values is more accurate. Next in Fig. 8 we study the behavior of $E(2^+_\gamma)$ energy with N_pN_n and mass number A. At low value of N_pN_n the $E(2^+_\gamma)$ energy decreases with increasing the N_pN_n and there after gamma band energy shows constant behavior. The Pt nuclei lie below the trend because these nuclei are not deformed at low value of N_pN_n.

CONCLUSION

To summarize, we try to show soft rotor formula is successful to explain the gamma band energy and also helpful to find the new energy levels of *Xe-Pt* nuclei. We observe that two parameter formula is more relevant for those nuclei having $R_{4/2} \geq 3.1$. For light nuclei it gives good result below the back bending. For deformed nuclei it gives good agreement for ground band energy upto J = 14^+ and for gamma band energy $J = 10^+$. The gamma band energy show constant behavior with N_pN_n and mass number A. The even and odd $\Im_0$ and α parameters also shows constant trend with energy ratios. It means energy calculated by SRF formula shows close agreement with the experimental energy values.

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